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SURVEYING MANUAL

A MANUAL OF FIELD AND OFFICE METHODS FOR THE USE OF STUDENTS IN SURVEYING

BY

WILLIAM D. PENCE, C.E.
M. Am. Soc. C. E.
SOMETIME PROFESSOR OF RAILWAY ENGINEERING, UNIVERSITY OF WISCONSIN

AND

MILO S. KETCHUM, C.E.
M. Am. Soc. C. E.
PROFESSOR-IN-CHARGE OF CIVIL ENGINEERING, UNIVERSITY OF PENNSYLVANIA

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1915

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PREFACE TO FIRST EDITION.

In preparing this manual the following points have been kept especially in view: (1) To provide a simple and comprehensive text designed to anticipate and supplement, rather than replace, the usual elaborate treatise. (2) To bring the student into immediate familiarity with approved surveying methods. (3) To cultivate the student's skill in the rare arts of keeping good field notes and making reliable calculations.

It is believed that the discussions of the different instruments, their use and theory, at the beginning of the several chapters is unusually simple, especially in the relations of the elementary lines.

The several series of practice problems at the conclusion of the respective chapters are arranged so as to give the student familiarity with the use of the instrument before taking up its theory and adjustments, this order being more effective than the reverse. The interest of the student may be stimulated and his gain in skill promoted by giving him practice with level and transit very early in the course, after which the scope of the work may be much more flexible both for student and instructor.

Since the list of problems is more extended than can be covered in the time usually available for surveying field practice, some range is permitted in the choice of work from year to year and under varying local conditions. By using some discrimination in selecting the more important problems for actual field work, the others may be covered sufficiently by class room discussions.

The consistent treatment of errors of surveying receives attention throughout the book. The methods of work both in the field and office are designed to reveal and, as far as possible, to eliminate blunders and errors, and the tests of precision are borrowed from the most rational current practice. The distribution of residual errors falling within the permissible limits likewise receives due consideration.

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PREFACE.

An important innovation in this manual is the liberal use of field note and other forms executed according to the standard required of the student in like work. The high value of such samples in developing the student's skill in this important detail of field work has been well established. It will be seen that the forms are prescribed in liberal numbers in the earlier stages of the work while the student is engaged in fixing a standard of quality, but that farther on he is required more and more to devise his own forms. A valuable feature of this system is the liberal amount of practice obtained in freehand lettering which has marked effect on the drafting and other work.

It is suggested that the student should be trained to be self-reliant by requiring him to verify his own results before submitting them for criticism. Likewise he should be encouraged to be genuine by placing him on his honor.

This somewhat informal guide to field and office methods is issued primarily for the use of the authors' classes, but it is hoped that others as well may find it of value in presenting principles to the beginner, and in cultivating his spirit and manual skill.

December, 1900.

W. D. P.
M. S. K.

PREFACE TO THIRD EDITION.

This third edition is issued to meet the call for the manual at various technical institutions and from practicing engineers. With a view to increase the value of the book both for teaching and reference purposes, various revisions and additions have been made. Among these are the full scale sample of field notes on the inset sheet at the back of the book.

The authors desire to acknowledge the valuable suggestions and criticisms received from various sources.

August, 1902.

W. D. P.
M. S. K.

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PREFACE TO FOURTH EDITION.

In this enlarged edition the book has been revised, nearly all the cuts have been redrawn, natural, logarithmic and trigonometric tables have been added, and the entire book has been reset and recast. The book has been critically revised and many changes and revisions have been made to increase its usefulness. The extensive use of the book has been due to the following important features, which are retained in the present edition.

1. Clear, definite and concise descriptions of surveying instruments and surveying methods.
2. Clear, definite and concise instructions for surveying field practice.
3. Carefully arranged and well executed forms of field notes.
4. The field practice unit has been taken as the squad; the idea being to give definite individual training.
5. By giving allowable limits of precision the critical instinct of the student is developed and he is taught to use consistent accuracy.
6. The main idea has been to train and develop engineering students by a study of surveying, rather than to teach the trade of surveying.

The thanks of the authors are due the McGraw-Hill Book Company for the use of the tables giving squares, cubes, square roots, cube roots and properties of circles, taken from Harger and Bonney's "Highway Engineers' Handbook"; and to the J. B. Lippincott Company for the use of the tables of five-place logarithms of numbers and of logarithmic functions of angles taken from Suplee's "Five Place Logarithms," and the tables of five place natural functions of angles taken from Suplee's "Mechanical Engineers' Reference Book."

The thanks of the authors are also due the Publication Committee of the American Railway Engineering Association for permission to print an abstract of "Conventional Signs and Symbols" adopted by the Association.

The authors desire to express their appreciation for the many suggestions received.

April, 1915.

W. D. P.
M. S. K.

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FIELD AND OFFICE TABLES.

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SPECIFICATIONS FOR A GOOD ENGINEER.

"A good engineer must be of inflexible integrity, sober, truthful, accurate, resolute, discreet, of cool and sound judgment, must have command of his temper, must have courage to resist and repel attempts at intimidation, a firmness that is proof against solicitation, flattery or improper bias of any kind, must take an interest in his work, must be energetic, quick to decide, prompt to act, must be fair and impartial as a judge on the bench, must have experience in his work and in dealing with men, which implies some maturity of years, must have business habits and knowledge of accounts. Men who combine these qualities are not to be picked up every day. Still they can be found. But they are greatly in demand, and when found, they are worth their price; rather they are beyond price, and their value can not be estimated by dollars."—*Chief Engineer Starling's Report to the Mississippi Levee Commissioners.*

"Be sure you are right, and then go ahead."—*D. Crockett.*

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CHAPTER I.

GENERAL INSTRUCTIONS.

FIELD WORK.

Habitual Correctness.—Habitual correctness is a duty. Error should be looked upon as *probable*, and every precaution taken to verify data and results. Unchecked work may always be regarded as doubtful. A discrepancy which is found by the maker in time to be corrected by him before any damage is done is not necessarily discreditable, provided the error is not repeated. However, *habitual error* is not only discreditable but dishonorable as well, and nothing except intentional dishonesty injures the reputation of the engineer more quickly or permanently.

Consistent Accuracy.—The degree of precision sought in the field measurements should be governed strictly by the dictates of common sense and experience. Due consideration of the purposes of the survey and of the time available will enable one to avoid extreme precision when ordinary care would suffice, or crudeness when exactness is required, or inconsistency between the degrees of precision observed in the several parts of the survey. It is a very common practice of beginners, and of many experienced engineers as well, to carry calculated results far beyond the consistent exactness.

Speed.—Cultivate the habit of doing the field work quickly as well as accurately. True skill involves both quantity and quality of results. However, while the habit of rapid work can and should be acquired, the speed attempted in any given problem should never be such as to cast doubt upon the results. Slowness due to laziness is intolerable.

Familiarity with Instructions.—The instructions for the day's work should be read over carefully, and preliminary steps, such as the preparation of field note forms, should be taken so as to save time and make the work in

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the field as effective as possible. The ability and also the desire to understand and obey instructions are as essential as the skill to execute them.

Inferior Instruments.—Should a poor instrument or other equipment be assigned, a special effort should be made to secure excellent results. In actual practice, beginners often have to work with defective instruments, but they should never seek, nor are they permitted, to justify poor results by the character of the field equipment. The student should therefore welcome an occasional opportunity to secure practice with poor instruments.

Alternation of Duties.—The members of each party should alternate in discharging the several kinds of service involved in the field problems, unless otherwise instructed. Training in the subordinate positions is essential whether the beginner is to occupy them in actual practice or not, for intelligent direction of work demands thorough knowledge of all its details.

Field Practice Decorum.—The decorum of surveying field practice should conform reasonably to that observed in other laboratory work.

THE CARE OF FIELD EQUIPMENT.

RESPONSIBILITY.—The student is responsible for the proper use and safe return of all equipment. All cases of breakage, damage, loss or misplacement must be reported promptly. The equipment should be examined when assigned and a report made at once of any injury or deficiency found, so that responsibility may be properly fixed.

PRECAUTIONS.—Careful attention to the following practical suggestions will save needless wear to the equipment and reduce the danger of accidents to a minimum, besides adding to the quality and speed of the work.

Tripod.—Inspect the tripod legs and shoes. The leg is of the proper tightness if, when lifted to an elevated position, it sinks gradually of its own weight. The tripod shoes should be tight and have reasonably sharp points.

Setting Up Indoors.—In setting up the instrument indoors press the tripod shoes firmly into the floor, preferably with each point in a crack. Avoid disturbing other instruments in the room.

Instrument Case.—Handle the instrument gently in removing it from and returning it to the case. It is always

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best to place the hands beneath the leveling base in handling the detached instrument. Considerable patience is sometimes required to close the lid after returning the instrument; if properly placed the lid closes freely.

Mounting the Instrument.—See that the instrument is securely attached to the tripod before shouldering it. Undue haste in this particular sometimes results in costly accidents. When screwing the instrument on the tripod head, it should be turned in a reverse direction until a slight jar is felt, indicating that the threads are properly engaged.

Sunshade.—Always attach the sunshade regardless of the kind of weather. The sunshade is a part of the telescope tube and the adjustment of a delicate instrument may sometimes be affected by its absence. In attaching or removing the sunshade or object glass cap, always hold the telescope tube firmly with one hand and with the other twist the shade or cap *to the right* to avoid unscrewing the object glass cell.

Carrying the Instrument.—Do not carry the instrument on the shoulder in passing through doors or in climbing fences. Before shouldering the instrument, the principal motions should be slightly clamped; with the transit, clamp the telescope on the line of centers; and with the level, when the telescope is hanging down. In passing through timber with low branches, give special attention to the instrument. Before climbing a fence, set the instrument on the opposite side with tripod legs well spread.

Setting Up in the Field.—When setting up in the field, bring the tripod legs to a firm bearing with the plates approximately level. Give the tripod legs additional spread in windy weather or in places where the instrument may be subject to vibration or other disturbance. On side-hill work place one leg up hill. With the level, place two tripod shoes in the general direction of the line of levels.

Exposure of Instrument.—Do not expose the instrument to rain or dampness. In threatening weather the water proof bag should be taken to the field. Should the instrument get wet, wipe it thoroughly dry before returning it to the case. Protect the instrument from dust and dirt, and avoid undue exposure to the burning action of the sun. Avoid subjecting it to sudden changes of temperature. In cold weather when bringing an instrument indoors cover the instrument with the bag or return it to the case immediately to protect the lenses and graduations from condensed moisture.

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Guarding the Instrument.—Never leave an instrument unguarded in exposed situations such as in pastures, near driveways, or where blasting is in progress. Never leave an instrument standing on its tripod over night in a room.

Manipulation of Instrument.—Cultivate from the very beginning the habit of delicate manipulation of the instrument. Many parts, when once impaired, can never be restored to their original condition. Rough and careless treatment of field instruments is characteristic of the unskilled observer. Should any screw or other part of the instrument work harshly, call immediate attention to it so that repairs may be made. Delay in such matters is very destructive to the instrument.

Foot Screws.—In leveling the instrument, the foot screws should be brought just to a snug bearing. If the screws are too loose, the instrument rocks, and accurate work can not be done; if too tight, the instrument is damaged, and the delicacy and accuracy of the observations are reduced. Much needless wear of the foot screws may be avoided if the plates are brought about level when the instrument is set up. With the level, a pair of foot screws should be shifted to the general direction of the back or fore sight before leveling up.

Eyepiece.—Before beginning the observations, focus the eyepiece perfectly on the cross-hairs. This is best done by holding the note book page, handkerchief, or other white object a foot or so in front of the object glass so as to illuminate the hairs; and then, by means of the eyepiece slide, focus the microscope on a speck of dust on the cross-hairs near the middle of the field. To have the focusing true for natural vision, the eye should be momentarily closed several times between observations in order to allow the lenses of the eye to assume their normal condition. The omission of this precaution strains the eye and is quite certain to cause parallax. After the eyepiece is focused on the cross-hairs, test for parallax by sighting at a well defined object and observing whether the cross-hairs seem to move as the eye is shifted slightly.

Clamps.—Do not overstrain the clamps. In a well designed instrument the ears of the clamp screw are purposely made small to prevent such abuse. Find by experiment just how tight to clamp the instrument in order to prevent slipping, and then clamp accordingly.

Tangent Screws.—Use the tangent screws for slight motions only. To secure even wear the screws should

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be used equally in all parts of their length. The use of the wrong tangent movement is a fruitful source of error with beginners.

Adjusting Screws.—Unless the instrument is assigned expressly for adjustment, do not disturb the adjusting screws.

Magnetic Needle.—Always lift the needle before shouldering the instrument. Do not permit tampering with the needle. If possible, avoid subjecting the needle to magnetic influence, such as may exist on a trolley car. Should the needle become reversed in its polarity or require remagnetization, it may be removed from the instrument and brought into the magnetic field of a dynamo or electric motor for several minutes, the needle being jarred slightly during the exposure; or a good horseshoe magnet may be used for the same purpose. The wire coil counterbalance on the needle will usually require shifting after the foregoing process.

Lenses.—Do not remove or rub the lenses of the telescope. Should it be *absolutely necessary* to clean a lens, use a very soft rag with caution to avoid scratching or marring the polished surface. Protect the lenses from flying sand and dust, which in time seriously affect the definition of the telescope.

Plumb Bob.—Do not abuse the point of the plumb bob and avoid needless knots in the plumb bob string.

Cleaning Tripod Shoes.—Remove the surplus soil from the tripod shoes before bringing the instrument indoors.

Leveling Rods.—Leveling rods and stadia boards should not be leaned against trees or placed where they may fall. Avoid injury to the clamps, target and graduations Do not mark the graduations with pencil or otherwise. Avoid needless exposure of the rod to moisture or to the sun.

Flag Poles.—Flag poles should not be unduly strained and their points should be properly protected.

Chains and Tapes.—Chains should not be jerked. Avoid kinks in steel tapes, especially during cool weather. When near driveways, in crowded streets, etc., use special care to protect the tape. Band tapes will be done up in 5-foot loops, figure 8 form, unless reels are provided. Etched tapes should be wiped clean and dry at the end of the day's work.

Axes and Hatchets.—Axes and hatchets will be employed for their legitimate purposes only. Their wanton use in clearing survey lines is forbidden, and their use at all,

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for such purpose, on private premises must be governed *strictly* by the rights of the owner.

Stakes.—The consumption of stakes should be controlled by reasonable economy, and surplus stakes returned to the general store. For the protection of mowing machines in meadows, etc., hub stakes should be driven flush with the surface of the ground, and other stakes should be left high enough to be visible. Whenever practicable, stakes which may endanger machines should be removed after serving the purpose for which they were set.

FIELD NOTES.

Scope of Field Notes.—The notes should be a complete record of each day's work in the field. In addition to the title of the problem and the record of the data observed, the field notes should include the date, weather, organization of party, equipment used, time devoted to the problem, and any other information which is at all likely to be of service in connection with the problem. No item properly belonging to the notes should be trusted to memory. Should the question arise as to the desirability of any item, it is always safe to include it. The habit of rigid self criticism of the field notes should be cultivated.

Character of Notes.—The field notes should have character and force. As a rule, the general character of the student's work can be judged with considerable certainty by the appearance of his field notes. A first-class page of field notes always commands respect, and tends to establish and stimulate confidence in the recorder. The notes should be arranged systematically.

Interpretation of Notes.—The field notes should have one and only one reasonable interpretation, and that the correct one. They should be perfectly legible and easily understood by anyone at all familiar with such matters.

Original Notes.—Each student must keep complete notes of each problem. Field notes must not be taken on loose slips or sheets of paper or in other note books, but the *original record* must be put in the prescribed field note book *during the progress of the field work.*

Field Note Book.—The field record must be kept in the prescribed field note book. For ease of identification the name of the owner will be printed in bold letters at the top of the front cover of the field note book.

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Pencil.—To insure permanency all notes will be kept with a hard pencil, preferably a 4H. The pencil should be kept well sharpened and used with sufficient pressure to indent the surface of the paper somewhat.

Title Page.—An appropriate title page will be printed on the first page of the field note book.

Indexing and Cross Referencing.—A systematic index of the field notes will be kept on the four pages following the title page. Related notes on different pages will be liberally and plainly cross referenced. The pages of the note book will be numbered to facilitate indexing.

Methods of Recording Field Notes.—There are three general methods of recording field notes, namely: (1) by sketch, (2) by description or narration, and (3) by tabulation. It is not uncommon to combine two or perhaps all three of these methods in the same problem or survey.

Form of Notes.—All field notes must be recorded in a field note book ruled as shown below, except where circumstances require modification. If no form is given, the student will devise one suited to the particular problem.

Lettering.—Field notes will be printed habitually in the "Engineering News" style of freehand lettering, as treated in Reinhardt's "Freehand Lettering." The body of the field notes will be recorded in the slanting letter and the headings will be made in the upright letter. The former slants to the right 1 : 2.5 and the so-called upright letter is made to slant to the left slightly, say 1 : 25. Lower case letters will be used in general, capitals being employed for initials and important words, as required. In the standard field note alphabet the height of lower case letters a, c, e, i, m, n, etc., is $\frac{3}{40}$ inch, and the height of lower case b, d, f, g, h, etc., and of all capital letters and all numerals is $\frac{5}{40}$ ($\frac{1}{8}$) inch; lower case t is made four units ($\frac{4}{40}$) inch high. This standard accords with best current practice and is based upon correct economic principles. Sample pages of field notes with letters and figures drawn full size are given on page 9. The student is expected to make the most of this opportunity to secure a liberal amount of practice in freehand lettering.

Field Note Sketches.—Sketches will be used liberally in the notes and will be made *in the field*. If desired, a ruler may be used in drawing straight lines, but the student is urged to acquire skill at once in making good plain freehand sketches. The field sketches should be bold and clear, in fair proportion, and of liberal size so as to avoid con-

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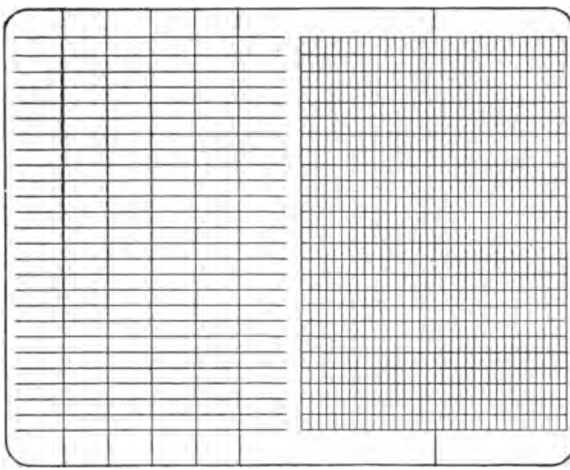
fusion of detail. The exaggeration of certain details in a separate sketch sometimes adds greatly to the clearness of the notes. The sketches should be supplemented by descriptive statements when helpful, and important points of the sketch should be lettered for reference. The precise scaling of sketches in the field note book, while sometimes necessary is usually undesirable owing to the time consumed. It is also found that undue attention to the drafting of the sketch is very apt to occupy the mind and cause



omissions of important numerical data. Since recorded figures and not the size of the field sketch itself must usually be employed in the subsequent use of the notes, it is important to review the record *before leaving the field* to detect omissions or inconsistencies. Making sketches on loose sheets or in other books and subsequently copying them into the regular field book is very objectionable practice and will not be permitted in the class work. Copies of field notes or sketches are never as trustworthy as the original record made *during the progress* of the field work. In very rapid surveys where legibility of the original record must perhaps suffer somewhat, it is excellent practice to transcribe the notes at once to a neighboring page, thus preserving the original rough notes for future reference. The original has more weight as evidence, but the neat copy

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ANGLES OF TRIANGLE 5-6-7

Station	Value of Angle				
	1st Meas.	2nd Meas.	Mean		
5	88°50′	88°51′	88°50′30″		
6	47°47′	47°47′	47°47′00″		
7	43°23′	43°23′	43°23′00″		
			180°00′30″		

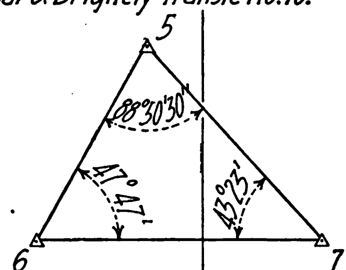
(Difference in measurements not to exceed 1′)
(Error not to exceed 1′)

Left Hand Page.

Observers, J. Doe & R. Roe.

WITH ENGINEERS' TRANSIT.

Nov. 15, 1914, (2 hours). Warm and quiet
Used Hellar & Brightly Transit No. 10.



Right Hand Page.

made before the notes are "cold" is of great help in interpreting them.

Numerical Data.—The record of numerical data should be consistent with the precision of the survey. In observations of the same class a uniform number of decimal places should be recorded. When the fraction in a result is exactly one-half the smallest unit or decimal place to be observed, record the even unit. Careful attention should be given to the *legibility of numerals*. This is a matter in which the beginner is often very weak. This defect can be corrected best by giving studious attention and practice to both the form and vertical alinement of tabulated numerals.

Erasures.—Erasures in the field notes should be avoided. In case a figure is incorrectly recorded, it should be crossed out and the correct entry made near by. The neat cancellation of an item in the notes inspires confidence, but evidence of an erasure or alteration casts doubt upon their genuineness. When a set of notes becomes so confused that erasure seems desirable, it should be transcribed, usually on another page. Rejection of a page of notes should be indicated by a neat cross mark, and cross reference should be made between the two places.

Office Copies.—Office copies of field notes will be submitted promptly, as required. These copies must be actual transcripts from the original record contained in the field note book of the individual submitting the copy. When office copies are made, a memorandum of the fact should be entered on the page of the field note book. When so specified, the office copies will be executed in india ink.

Criticism of Field Notes.—The field notes must be kept in shape for inspection at any time, and be submitted on call. All calculations and reductions must be kept up to date. The points to which chief attention should be directed in the criticism of the field notes are indicated in the following schedule. The student is expected to criticise his own notes and submit them in as perfect condition as possible. For simplicity the criticisms will be indicated by stamping on the note book page the reference letters and numbers shown in the schedule.

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SCHEDULE OF POINTS FOR THE CRITICISM OF FIELD NOTE BOOKS.

A. SUBJECT MATTER.

(1) **General:**

(a) Descriptive title of problem.
(b) Date.
(c) Weather.
(d) Organization of party.
(e) Equipment used.
(f) Time devoted to the problem.
(g) Indexing and cross referencing.
(h) Page numbering.
(i) Title page.
(j) Identification of field note book.

(2) **Record of Data:**

(a) Accuracy.
(b) Completeness.
(c) Consistency.
(d) Arrangement.
(e) Originality.

B. EXECUTION.

(1) **Lettering:**

(a) Style. ("Engineering News")

(b) Size. (a, c, e, i, etc., 3/40 inch high; b, d, f, g, etc., A, B, C, etc., and 1, 2, 3, etc., 5/40 (1/8) inch high; t, 4/40 inch.)

(c) Slant. (In body of notes, "slanting," 1:2.5 right; in headings, "upright," about 1:25 to left.)

(d) Form. (See Reinhardt's "Freehand Lettering.")

(e) Spacing. (Of letters in words; of numerals; of words; balancing in column or across page.)

(f) Alinement. (Horizontal; vertical.)

(g) Permanency. (Use sharp hard pencil with pressure.)

(2) **Sketches.**

(a) To be bold, clear and neat.
(b) To be ample in amount.
(c) To be of liberal size.
(d) To be in fair proportion.
(e) To be made freehand.
(f) To be made in the field.

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OFFICE WORK.

Importance of Office Work.—Capable office men are comparatively rare. Skill in drafting and computing is within the reach of most men who will devote proper time and effort to the work. Men who are skillful in both field and office work have the largest opportunity for advancement.

Calculations.—All calculations and reductions of a permanent character must be shown in the field note book in the specified form. Cross references between field data and calculations should be shown. Consistency between the precision of computed results and that of the observed data should be maintained. Computed results should be verified habitually, and the verified results indicated by a check mark. Since most computers are prone to repeat the same error, it is desirable in checking calculations to employ independent methods and to follow a different order. A fruitful source of trouble is in the transcript of data, and this should be checked first when reviewing doubtful calculations. Skilled computers give much attention to methodical arrangement, and to contracted methods of computing and verifying results. Familiarity with the slide rule and other labor saving devices is important. (See Chapter X, Methods of Computing.)

Drafting Room Equipment.—The student is responsible for the proper use and care of drafting room furniture and equipment provided for his use.

Drafting.—The standard of drafting is that indicated in Reinhardt's "Technic of Mechanical Drafting."

Drafting Room Decorum.—The decorum of the student in the drafting room will conform to that observed in first-class city drafting offices.

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CHAPTER II.

THE CHAIN AND TAPE.

METHODS OF FIELD WORK.

Units of Measure.—In the United States the foot is used by civil engineers in field measurements. Fractions of a foot are expressed decimally, the nearest 0.1 being taken in ordinary surveys, and the nearest 0.01 foot (say ⅛ inch) in more refined work.

In railroad and similar "line" surveys by which a station stake is set every 100 feet, the unit of measure is really 100 feet instead of the foot. The term "station" was originally applied only to the actual point indicated by the numbered stake, but it is now universal practice in this country to use the word station in referring to either the point or the 100-foot unit distance. A fractional station is called a "plus" for the reason that a plus sign is used to mark the decimal point for the 100-foot unit, the common decimal point being reserved for fractions of a foot. The initial or starting stake of such a survey is numbered 0.

The 100-foot chain is commonly called the "engineers' chain" to distinguish it from the 66-foot or 100-link chain which is termed the "surveyors' chain" because of its special value in land surveys involving acreage. The latter is also called the Gunter chain after its inventor, and is otherwise known as the four-rod or four-pole chain. British engineers use the Gunter chain for both line and land surveys. The "surveyors'" or Gunter chain, while no longer used in actual surveying, is described in this book for the reason that the United States rectangular surveys were made throughout with the 66-foot chain.

In the Spanish-American countries the vara is generally used in land surveys. The Castilian vara is 32.8748 inches long, but the state of California has adopted 32.372 inches, and Texas 33⅓ inches, as the legal length of the vara.

While the metric system is used exclusively, or in part, in each of the several United States government surveys, except those for public lands, little or no progress has been made towards its introduction in other than government surveys.

Linear Measuring Instruments.—Two general types of linear measuring devices are used by surveyors, viz., the common chain and the tape. There are several kinds of each, according to the length, material, and method of graduation.

Fig. 1.

The common chain is made up of a series of links of wire having loops at the ends and connected by rings so as to afford flexibility. The engineers' chain is shown in (a), Fig. 1, the illustration being that of a 50-foot chain, or one-

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half the length generally used. The surveyors' or Gunter chain is shown in (b), Fig. 1. In the common chain the end graduation is the center of the cross bar of the handle, and every tenth foot or link is marked by a notched brass tag. In the 100-foot or 100-link chain the number of points on the tag indicates the multiple of ten units from the nearer end, and a circular tag marks the middle of the chain. The chain is done up hour-glass shape, as shown in the cut.

Chaining pins made of steel wire are used in marking the end of the chain or tape in the usual process of linear measurement. A set of pins usually numbers eleven, as indicated at (c), Fig. 1. The pins are carried on a ring made of spring steel wire.

The flat steel band, shown in (d) and (e), Fig. 1, is the best form of measuring device for most kinds of work. The band tape is usually 100 feet long. The end graduations of the band tape are usually indicated by brass shoulders, which should point in the same direction, as shown in (f), Fig. 1. The 100-foot band tape is commonly graduated every foot of its length, and the end foot to every 0.1 foot, every fifth foot being numbered on a brass sleeve. Brass rivets are most commonly used in graduating this tape. The band tape may be rolled up on a special reel, as indicated in (d) and (e), although some engineers dispense with the reel and do up the tape in the form of the figure 8 in loops of five feet or so.

The steel tapes shown in (g) and (h) have etched graduations. This style of tape is commonly graduated to 0.01 foot or ⅛ inch. It is more fragile than the band tape and is commonly used on more refined work. The form of the case shown in (h) has the advantage of allowing the tape to dry if wound up while damp.

The "metallic" tape (i), Fig. 1, is a woven linen line having fine brass wire in the warp.

The steel tape is superior to the common chain chiefly because of the permanency of its length. The smoothness and lightness of the steel tape are often important advantages, although the latter feature may be a serious drawback at times. The tape is both easier to break and more difficult to mend than the common chain.

Tapes for measuring base lines with great precision have recently been made of Invar steel. Invar steel has a very small coefficient of expansion. Invar steel tapes are very expensive.

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Chaining.—In general, the horizontal distance is chained. Two persons, called head and rear chainmen, are required. The usual process is as follows:

The line to be chained is first marked with range poles. The head chainman casts the chain out to the rear, and after setting one marking pin at the starting point and checking up the remaining ten pins on his ring, steps briskly to the front. The rear chainman allows the chain to pass through his hands to detect kinks and bent links. Just before the full length is drawn out, the rear chainman calls "halt," at which the head chainman turns, shakes out the chain and straightens it on the true line under the direction of the rear chainman. In order to allow a clear sight ahead, the front chainman should hold the chain handle with a pin in his right hand well away from his body, supporting the right elbow on the right knee, if desired. The rear chainman holds the handle in his left hand approximately at the starting point and motions with his right to the head chainman, his signals being distinct both as to direction and amount. Finally, when the straight and taut chain has been brought practically into the true line, the rear chainman, slipping the handle behind the pin at the starting point with his left hand, and steadying the top of the pin with his right, calls out "stick." The head chainman at this instant sets his pin in front of the chain handle and responds "stuck," at which signal *and not before* the rear chainman pulls the pin.

Both now proceed, the rear chainman giving the preliminary "halt" signal as he approaches the pin just set by the head chainman. The chain is lined up, stretched, the front pin set, and the rear pin pulled on signal, as described for the first chain length. This process is repeated until the head chainman has set his tenth pin, when he calls "out" or "tally," at which the rear chainman walks ahead, counting his pins as he goes and, if there are ten, transfers them to the head chainman who also checks them up and replaces them on his ring. A similar check in the pins may be made at any time by remembering that the sum, omitting the one in the ground, should be ten. This safeguard should be taken often to detect loss of pins. The count of tallies should be carefully kept.

When the end of the line is reached, the rear chainman steps ahead, and reads the fraction at the pin, noting the units with respect to the brass tags on the chain. The number of pins in the hand of the rear chainman indicates

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the number of applications of the chain since the starting or last tally point. A like method is used in case intermediate points are to be noted along the line.

On sloping ground the horizontal distance may be obtained either by leveling the chain and plumbing down from the elevated end, or by measuring on the slope and correcting for the inclination. In ordinary work the former is preferred, owing to its simplicity. In "breaking chain" up or down a steep slope, the head chainman first carries the full chain ahead and places it carefully on the true line. A plumb bob, range pole or loaded chaining pin should be used in plumbing the points up or down. The segments of the chain should be in multiples of ten units, as a rule, and the breaking points should be "thumbed" by both chainmen to avoid blunders. Likewise, special caution is required to avoid confusion in the count of pins during this process.

The general method of measuring with the band tape is much the same as with the common chain. The chief difference is due to the fact that the handle of the tape extends beyond the end graduation, so that it is more convenient for the head chainman to hold the handle in his left hand and rest his left elbow on his left knee, setting the pin with his right hand. Another difference is in the method of reading fractions. It is best to read the fraction *first by estimation*, as with the chain, making sure of the *feet;* then shifting the tape along one foot, getting an exact decimal record of the fraction by means of the end foot graduated to tenths; the nearest 0.01 foot is estimated, or in especially refined work, read by scale.

In railroad and similar line surveys, chaining pins are usually dispensed with and the ends of the chain are indicated by numbered stakes. The stake marked 0 corresponds to the pin at the starting point, and the station stakes are marked thence according to the number of 100-foot units laid off.

Perpendiculars.—Perpendiculars may be erected and let fall with the chain or tape by the following methods:

(a) By the 3 : 4 : 5 method, shown in (a), Fig. 2, in which a triangle having sides in the ratio stated, is constructed.

(b) By the chord bisection method, shown in (b), Fig. 2, in which a line is passed from the bisecting point of the chord to the center of the circle, or vice versa.

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(c) By the semicircle method, shown in (c), Fig. 2, in which a semicircle is made to contain the required perpendicular.

The first method corresponds to the use of the triangle in drafting. Good intersections are essential in the second and third methods. Results may be verified either by using another process, or by repeating the same method with the measurements or position reversed, as indicated in (d), Fig. 2.

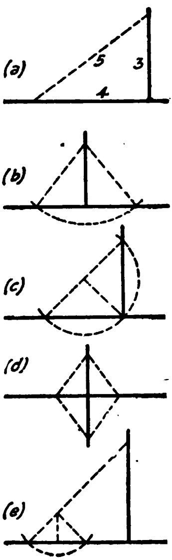

Fig. 2

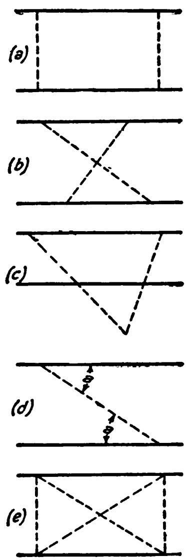

Fig. 3.

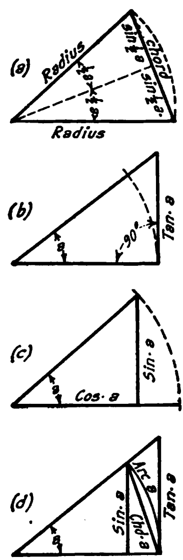

Fig. 4.

In locating a perpendicular from a remote point, the ratio method shown in (e), Fig. 2, may be used; or a careful trial perpendicular may be erected at a point estimated by placing the heels squarely on line and swinging the arms to the front, then proving by precise method.

Parallels.—Parallels may be laid off with the chain in various ways, a few of the simpler of which are:

(a) By equal distances, as in (a), Fig. 3, in which two equal distances are laid off, usually at right angles to the given line.

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(b) By similar triangles, as in (b) and (c), Fig. 3. The ratio may, of course, have any value.

(c) By alternate angles, as in (d), Fig. 3, in which two equal angles are laid off in alternation.

The first method is adapted to laying off a rectangle, as in staking out a building, in which case a good check is found in the equality of the diagonals. Precision of alinement is important, especially where a line is prolonged.

Angles.—Angles may be determined by linear measurements in the following ways:

(a) By the chord method, shown in (a), Fig. 4, in which the radius is laid off on the two lines forming the angle, and the chord measured.

(b) The tangent method, shown in (b), Fig. 4, in which a perpendicular is erected at one end of the radius, and the length of the perpendicular intercepted by the two lines measured.

(c) The sine-cosine method, (c), Fig. 4, which is better suited to constructing than to measuring angles.

The chord method is usually the most satisfactory. The tangent method may be applied to the bisected angle when its value approaches a right angle. Measurement of the supplementary angle affords an excellent check. A 100-foot radius is commonly used, although good results may be had with the 50-foot tape. Careful alinement is of the first importance in angular measurements.

It is sometimes necessary to determine angles, at least approximately, when no tables are at hand. Fair results may be had on smooth ground by measuring the actual arc struck off to a radius of 57.3 feet.

For very small angles, the sine, chord, arc and tangent, (d), Fig. 4, are practically equal. Thus, sin 1° is .017452 and tan 1°, .017455, or either (say) .01745, or 1¾ per cent. Also, arc 1′ is .000291, or (say) .0003 (three zeros three); and, arc 1″ is .00000485, (say) .000005 (five zeros five).

Location of Points.—Points are located in surveying field practice in the following seven ways.

(a) By rectangular coordinates, that is, by measuring the perpendicular distance from the required point to a given line, and the distance thence along the line to a given point, as in (a), Fig. 5.

(b) By focal coordinates or tie lines, that is, by measuring the distances from the required point to two given points, as in (b), Fig. 5.

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(c) By polar coordinates, that is, by measuring the angle between a given line and a line drawn from any given point of it to the required point; and also the length of this latter line, as in (c), Fig. 5.

(d) By modified polar coordinates, that is, by a distance from one known point and a direction from another, as in (d), Fig. 5.

(e) By angular intersection, that is, by measuring the angles made with a given line by two other lines starting from given points upon it, and passing through the required point, as in (e), Fig. 5.

(f) By resection, that is, by measuring the angles made with each other by three lines of sight passing from the required point to three points, whose positions are known, as in (f), Fig. 5.

(g) By diagonal intersection, that is, by two lines joining two pairs of points so as to intersect in the required point, as in (g), Fig. 5.

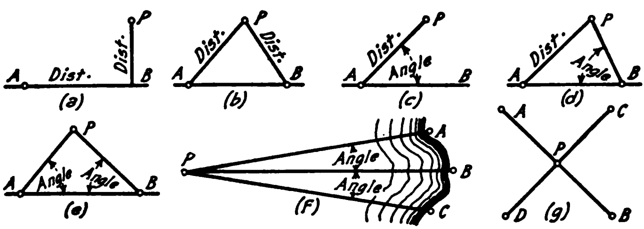

Fig. 5.

In each of these methods, except (f), the point is determined by the intersection of either two right lines, or two circles, or a right line and a circle.

Methods (a) and (b) are best suited to chain surveys; (c) and (d) are used most in the location of railroad curves; (e) and (f) are employed chiefly in river and marine surveys for the location of soundings, the latter being commonly known as the "three-point problem"; the last method, (g), is much used for "referencing out" transit points in railroad and similar construction surveys.

Location of Objects.—The location of buildings and topographic objects usually involves one or more of the foregoing methods of locating a point.

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In Fig. 6, (a), (b), (c), and (d) suggest methods of locating a simple form, and (e) and (f) illustrate more complex cases.

Tie Line Surveys.—For many purposes tie line surveys, made with the chain or tape alone, are very satisfactory. The skeleton of such surveys is usually the triangle, the detail being filled in by the methods just outlined. Much time may be saved by carefully planning the survey. A few typical applications of the tie line method are shown in Fig. 7.

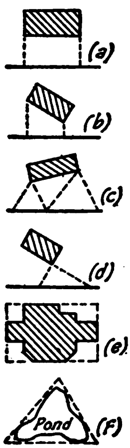

Fig. 6.

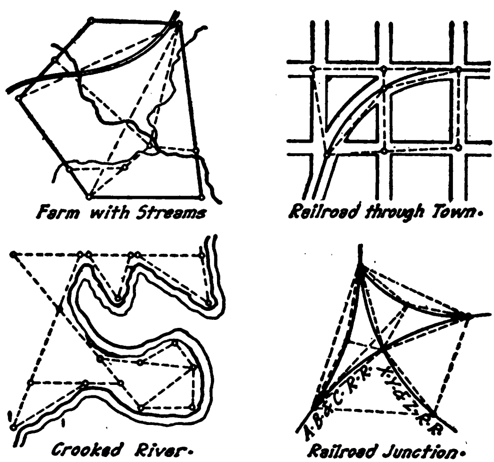

Fig. 7.

Ranging in Lines.—The range or flag pole is usually painted with alternate feet red and white, and the lower end is shod or spiked. A temporary form of range pole, called a picket, is sometimes cut from a straight sapling.

In flagging a point, the spike of the pole is placed on the tack and the pole plumbed by holding it symmetrically between the tips of the fingers of the two hands, the flagman being squarely behind the pole.

In hilly or timbered country the two land corners or other points between which it is desired to range in a line, are often invisible one from the other. In many cases two intermediate points C′ and D′, (a), Fig. 8, may be found, from

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which the end points B and A, respectively, are visible; so that after a few successive linings in, each by the other, the true points, C and D, are found.

Otherwise, as shown at (b), Fig. 8, a random line may be run from A towards B. The trial line is chained and marked, the perpendicular from B located, and points interpolated on the true line.

If the desired line is occupied by a hedge or other obstruction, an auxiliary parallel line may be established in the adjacent road or field, after one or two trials, as in (c), Fig. 8.

A line may be prolonged past an obstacle by rectangular offsets or by equilateral triangles.

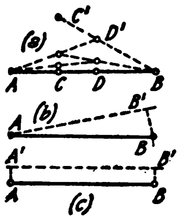

Fig. 8.

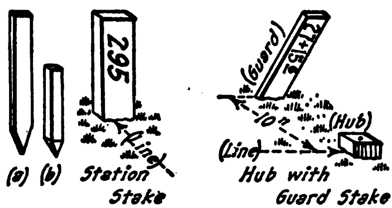

Fig. 9.

Signals.—There is little occasion for shouting in surveying field work if a proper system of sight signals is used. Each signal should have but one meaning and that a perfectly distinct one. Signals indicating motion should at once show clearly both the direction and amount of motion desired. Some of the signals in common use are as follows:

(a) "Right" or "left,"—the arm is extended distinctly in the desired direction and the motion of the forearm and hand is graduated to suit the lateral motion required.

(b) "Up" or "down,"—the arm is extended laterally and raised or lowered distinctly with motions to suit the magnitude of the movement desired. Some levelers use the left arm for the "up" signal and the right for "down."

(c) "Plumb the pole (or rod),"—If to the right, that arm is held vertically with hand extended and the entire body, arm included, is swung distinctly to the right, or vice versa.

(d) "All right,"—both arms are extended full length horizontally and waved vertically.

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(e) "Turning point" or "transit point,"—the arm is swung slowly about the head.

(f) "Give line,"—the flagman extends both arms upward, holding the flag pole horizontally, ending with the pole in its vertical position. If a precise or tack point is meant, the signal is made quicker and sharper.

(g) Numerals are usually made by counted vertical swings with the arm extended laterally. A station number is given with the right hand and the plus, if any, with the left; or a rod reading in like manner. The successive counts are separated by a momentary pause, emphasized, if desired, by a slight swing with both hands.

Stakes and Stake Driving.—A flat stake is used to mark the stations in a line survey, and a square stake or hub to mark transit stations, (a) and (b), Fig. 9. The station stake is numbered on the rear face, and the hub is witnessed by a flat guard stake driven slanting 10 inches or so to the left, Fig. 9. The numerals should be bold and distinct, and made with keel or waterproof crayon, pressed into the surface of the wood.

Having located a point approximately with the flag pole, the stake should be driven truly plumb in order that the final point may fall near the center of its top. In driving a stake, the axeman should watch for signals. It is better to draw the stake by a slanting blow than to hammer the stake over after it is driven. Good stake drivers are scarce.

PROBLEMS WITH THE CHAIN AND TAPE.

General Statement.—Each problem is stated under the following heads:

(a) *Equipment.*—In which are specified the articles and instruments assigned or required for the proper performance of the problem. A copy of this manual and of the regulation field note book, with a hard pencil to keep the record, form part of the equipment for every problem assigned.

(b) *Problem.*—In which the problem is stated in general terms. The special assignments will be made by program.

(c) *Methods.*—In which the methods to be used in the assigned work are described more or less in detail. In some problems alternative methods are suggested, and in others the student is left to devise his own.

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PROBLEM A1. LENGTH OF PACE.

(a) *Equipment.*—(No instrumental equipment required.)

(b) *Problem.*—Investigate the length of pace as follows: (1) the natural pace; (2) an assumed pace of 3 feet; and (3) the effect of speed on the length of the pace.

(c) *Methods.*—(1) On an assigned course of known length count the paces while walking at the natural rate. Observe the nearest 0.1 pace in the fraction at the end of the course. Secure ten consecutive results, with no rejections, varying not more than 2 per cent. (2) Repeat (1) for an assumed 3-foot pace. (3) Observe (in duplicate) time and paces for four or five rates from very slow to very fast, with paces to nearest 0.1 and time to nearest second. Record data and make reductions as in the form.

PROBLEM A2. DISTANCES BY PACING.

(a) *Equipment.*—(No instrumental equipment required.)

(b) *Problem.*—Pace the assigned distances.

(c) *Methods.*—(1) Standardize the pace in duplicate on measured base. (2) Pace each line in duplicate, results differing not more than 2 per cent. Record and reduce as in form.

PROBLEM A3. AXEMAN AND FLAGMAN PRACTICE.

(a) *Equipment.*—Flag pole, axe, 4 flat stakes, 1 hub, tacks.

(b) *Problem.*—Practice the correct routine duties of axeman and flagman.

(c) *Methods.*—(1) Number three station stakes to indicate representative cases and drive them properly. (2) Drive a hub flush with ground and tack it; number a witness stake and drive it properly. (3) Arrange program of signals with partner, separate 1,000 feet or so and practice same. (4) Signal say five station numbers to each other and afterwards compare notes. Make concise record of the foregoing steps.

PROBLEM A4. RANGE POLE PRACTICE.

(a) *Equipment.*—4 flag poles.

(b) *Problem.*—Given two hubs approximately 1,000 feet apart, interpolate a flag pole say 100 feet from one hub,

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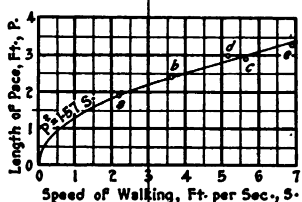
INVESTIGATION OF LENGTH OF PACE.

Kind of Pace	No.	Paces per 400 Ft. Paces	Mean Paces	Length of Pace Ft.	Remarks	
					Sept. 13, Smooth ground.	Clear With the Wind.
Natural	1	138.0				Against "
	2	137.4			"	"
	3	139.0				" "
	4	137.6			"	"
	5	138.0				" "
	6	139.0			"	"
	7	137.3				" "
	8	139.0			"	"
	9	138.0				" "
	10	139.3	138.26	2.89		" "
		(12.6)				
3-Foot	1	~~131.0~~			With the Wind	
	2	132.6			Against "	
	3	133.0			"	"
	4	133.4				" "
	5	134.0			"	"
	6	133.3				" "
	7	132.0			"	"
	8	133.0				" "
	9	133.3			"	"
	10	132.6				" "
	11	133.0	133.02	3.01	"	"
		(10.2)				

Sept. 13, '14. (3 Hrs.) Clear and Cool.
LENGTH OF PACE. J. Doe, Surveyor.
EFFECT OF SPEED ON LENGTH OF PACE.

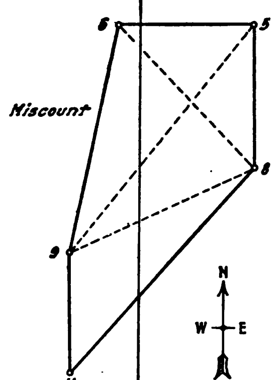
Kind of Pace	Paces in 400 Ft. Observ'd Paces	Mean Paces	Mean Pace, P Ft.	Time 400 Ft. Sec.	Speed of Pacing, S Ft. per Sec.
Very slow	214.6		(a)		(a)
" "	217.8	216.20	1.85	182	2.20
Slow	168.0		(b)		(b)
"	167.5	167.75	2.38	111	3.60
Natural	139.4		(c)		(c)
"	137.5	138.45	2.89	71	5.63
3-Foot	133.3		(d)		(d)
"	133.6	133.45	3.00	77	5.20
Fast	124.7		(e)		(e)
"	125.3	125.00	3.20	58	6.90



DISTANCES BY PACING

Line	No.	Length of Pace Per 400 Ft.	Ft.	Length of Line Observed Paces	Mean Paces	Length Ft.
	1	144.0				
	2	142.0				
		143.0	2.80			
5-6	1			134.0		
"	2			134.0	134.0	375
6-9	1			217.0		
"	2			218.5	217.8	610
9-11	1			~~126.5~~		
"	2			118.0		
"	3			118.5	118.2	331
8-11	1			279.0		
"	2			280.5	279.8	783
8-9	1			209.0		
"	2			210.0	209.5	587
5-9	1			286.0		
"	2			288.5	287.2	804
5-8	1			140.0		
"	2			141.0	140.5	393
6-8	1			188.0		
"	2			187.6	187.8	526

J. Doe, Surveyor.
Sept. 14, '14 (3 Hours) Clear & Cool.



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remove the distant pole, prolong the line by successive 100-foot sights and note the error at distant hub. Repeat process for 200-foot and 300-foot sights.

(c) *Methods.*—(1) Set distant flag pole precisely behind hub and hold spike of pole on tack of near hub; lying on ground back of near hub, line in pole 100 feet (paced) distant; remove pole from distant hub, and prolong by 100-foot sights up to distant hub, noting error to nearest 0.01 foot. (2) Repeat in reverse direction, using 200-foot sights. (3) Repeat with 300-foot sights. Avoid all bias. Record data in suitable form, describing steps concisely.

PROBLEM A5. STANDARDIZING CHAIN OR TAPE.

(a) *Equipment.*—Chain or tape assigned in any problem where standard length of chain may be of value.

(b) *Problem.*—Determine the length of the assigned chain or tape by comparison with the official standard under the conditions of actual use.

(c) *Methods.*—In standardizing tape, reproduce the conditions of actual use as regards tension, support, etc., bring one end graduation of chain or tape to coincide with one standard mark, and observe fraction at the other end with a scale. As a general rule, observe one more decimal place than is taken in the actual chaining.

PROBLEM A6. DISTANCES WITH SURVEYORS' CHAIN.

(a) *Equipment.*—Surveyors' chain, set of chaining pins, 2 plumb bobs, 2 flag poles (unless instructed otherwise).

(b) *Problem.*—On an assigned chaining course about one mile long measure distances with the surveyors' chain to the nearest 0.1 link, and repeat the measurements in the opposite direction.

(c) *Methods.*—(1) Standardize the chain before and after, as prescribed in A5. (2) Chain along the assigned course, noting the distances from the starting point to the several intermediate points and to the end station. Observe fractions to the nearest 0.1 link by estimation. (3) Repeat the chaining in the opposite direction, noting the distances from the end point, as before. The difference between the totals in the two directions should not exceed 1 : 3,000. Retain the same party organization throughout the problem. Record the data as in the prescribed form.

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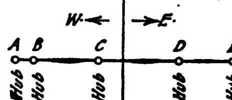
Line	Direction Chained	DISTANCES Observed Length Ch.	Diff. of Total Ch.	WITH Ratio 1:d	Coef. C Lk.
Chain	Before	1.0020			
"	After	1.0022			
A-B	E.	7.327			
A-C	"	30.306			
A-D	"	60.357			
A-E	"	79.838			
E-D	W.	19.479			
E-C	"	49.531			
E-B	"	72.506			
E-A	"	79.833	0.005	1:15970	0.06
		*L	*E=	CYL or	$C=\frac{E}{YL}$
				(See Diagram)	

Note:- The above data will be used in a subsequent problem in discussing the precision of chaining.

SURVEYOR'S CHAIN.

Head Chainman:- J. Doe; Rear Chainman, R. Roe.
Sept. 18, '14. (2 Hours) Clear and Cool.
Used Gunter Chain No. 210, Locker No. 35
Compared chain with official standard both before and after day's chaining.
Chained along Chaining Course "A", beginning at hub with tack, marked A on guard stake, located at S. edge of W. brick walk on Green St.; at E. curb line of Mathews Ave., Urbana, Ill.; thence E'ly along said S. line of N. brick side walk, observing distances to nearest 0.1 lk. to tacked hubs marked B, C, D and E, the total distance from starting point A being noted.
Chained same course in the reverse direction carrying total distances from Hub E.
Fractions of a link were estimated. Pocket rule was used in standardizing chain.



PROBLEM A7. DISTANCES WITH THE ENGINEERS' CHAIN.

(a) *Equipment.*—Engineers' chain, set of chaining pins, 2 plumb bobs, 2 flag poles (unless instructed otherwise).

(b) *Problem.*—On an assigned chaining course about one mile long measure distances with the engineers' chain to the nearest 0.1 foot, and repeat the measurements in the opposite direction.

(c) *Methods.*—(1) Standardize the chain before and after, as prescribed in A5. (2) Chain along the assigned course, noting the distances from the starting point to the several intermediate points and to the end station. Observe fractions to the nearest 0.1 foot by estimation. (3) Repeat the chaining in the opposite direction, noting the distances from the end point, as before. The difference between the totals in the two directions should not exceed 1 : 3,000. Retain the same party organization throughout the problem. Record the data as in the form.

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PROBLEM A8. DISTANCE WITH 100-FOOT STEEL TAPE.

(a) *Equipment.*—100-foot steel band tape with end foot graduated to tenths, set of chaining pins, 2 plumb bobs, 2 flag poles (unless instructed otherwise).

(b) *Problem.*—On an assigned chaining course about one mile long measure distances with the 100-foot steel band tape to the nearest 0.01 foot, and repeat the measurements in the opposite direction.

(c) *Methods.*—(1) Standardize before and after, as prescribed in A5. (2) Chain along the assigned course, noting the distances from the starting point to the several intermediate points and to the end station. In observing the fractions, first determine the foot units, then estimate the nearest 0.1 foot, then shift the tape along one foot and read the exact fraction on the end of the tape, estimating the nearest 0.01 foot. (3) Repeat the measurement in the opposite direction, noting the distances from the end point, as before. The difference between the totals in the two directions should not exceed 1 : 5,000. Retain the same party organization. Record data as in the form.

PROBLEM A9. HORIZONTAL DISTANCE ON SLOPE WITH STEEL TAPE.

(a) *Equipment.*—100-foot steel tape with etched graduations to 0.01 foot, set of chaining pins, 2 plumb bobs, 3 flag poles, axe, supply of pegs, engineers' level and rod (unless otherwise instructed).

(b) *Problem.*—Determine the horizontal distance between two assigned points on a steep slope, (1) by direct horizontal measurement, and (2) by measurement on the slope and reduction to the horizontal.

(c) *Methods.*—(1) Standardize the tape for each method, as prescribed in A5, both before and after the day's chaining. (2) In chaining down hill, *rear chainman* lines in flag pole in hand of head chainman, then holds tape end to tack on hub; *flagman* stands 50 feet or more from line opposite middle of tape and directs head chainman in leveling front end, then supports middle point of tape under direction of head chainman; *head chainman*, with spring balance attached to tape and using pole as help to steady pull, brings tension to 12 pounds; *recorder* plumbs down front end, and sets pin slanting sidewise. After checking the pin, proceed

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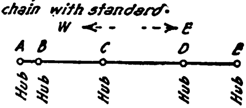
Line	Direction Chained	DISTANCES Observed Length Ft.	Diff. of Total Ft.	WITH Ratio 1:d	Coef. C Ft.
Chain	Before	100.10			
"	After	100.12			
A-B	E.	484.0			
A-C	"	2002.2			
A-D	"	3987.5			
A-E	"	5274.6			
E-D	W.	1286.9			
E-C	"	3272.4			
E-B	"	4790.2			
E-A	"	5274.3	0.3	1:17580	0.04
		$L = 52.743$	" $E =$	$C\sqrt{L}$ or	" $C = \frac{E}{\sqrt{L}}$
			(See Diagram)		

Note: The above data will be used in a subsequent problem in discussing the precision of chaining.

ENGINEER'S CHAIN.

Head Chainman, R. Roe. Rear Chainman, J. Doe.
Sept. 19, '14. (2 Hours) Cloudy & Cool.
Used 100 Ft. Chain No. 63, Locker No. 35.
Compared chain with official standard both before and after day's chaining.
Chained along chaining course "A", beginning at hub with tack, marked A on guard stake, located at S. edge of N. brick walk on Green St. at E. curb line of Mathews Ave., Urbana, Ill.; thence E'ly along said S. line of N. brick walk, observing distances to nearest 0.1 Ft. to tacked hubs B, C, D and E, the total distances from starting point A being noted.
Chained same course in the reverse direction, carrying total distances from Hub E.
Fractions of a Foot were estimated. A pocket rule was used to compare chain with standard.

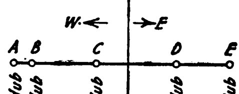


Line	Direction Chained	DISTANCES Observed Length Ft.	Diff. of Total Ft.	WITH Ratio 1:d	Coef. C. Ft.
Tape	Before	100.01			
"	After	100.008			
A-B	E.	484.58			
A-C	"	2003.79			
A-D	"	3991.69			
A-E	"	5279.48			
E-D	W.	1287.83			
E-C	"	3275.72			
E-B	"	4794.96			
E-A	"	5279.57	0.09	1:58660	0.012
		$L = 52.7957$	" $E =$	$C\sqrt{L}$ or	" $C = \frac{E}{\sqrt{L}}$
			(See Diagram)		

Note:— The above data will be used in a subsequent problem in discussing the precision of chaining

100 FT STEEL TAPE *Head Chain — R. Roe, Rear Chain — J. Doe*

Sept. 20, '14. (2 hours) Clear, moderate.
100 Ft. Roe Steel Tape No. 312, Locker 35
Compared tape with official standard both before and after day's chaining measuring fraction with engineer's scale
Chained along Chaining Course "A", previously chained with Gunter and 100 Ft. Chains, described on pp. of Field note book, observing continuous distances to hubs B, C, D and E to nearest 0.01 Ft
Chained course in reverse direction.
Fractions of a Foot were estimated to nearest 0.01 Ft. on the end Foot of the tape which was graduated to tenths of a Foot.



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with the next 100 feet. In chaining *up hill*, follow same general method, using plumb bob at rear end. In leveling the tape the tendency will be to get the down hill end too low. Chain the line in duplicate, retaining the same organization. (3) Chain the line again in duplicate, tape lying on the ground, pull 12 pounds, pins set plumb, fraction direct to nearest 0.01 foot. Set temporary pegs flush with ground every 100 feet and also at intermediate sudden changes of slope, for levels. Determine differences of elevation between successive pegs, unless the leveling data are supplied to the party. Record data and make reductions and comparisons as in the form.

PROBLEM A10. ANGLES OF A TRIANGLE WITH TAPE.

(a) *Equipment.*—100-foot steel tape, 50-foot metallic tape, set of chaining pins, 2 plumb bobs, 2 flag poles, five-place tables of trigonometric functions (each member of party to have tables).

(b) *Problem.*—Measure the angles of an assigned triangle with the steel tape and also with the metallic tape, the error of closure not to exceed 3 minutes.

(c) *Methods.*—(1) Measure each angle with the steel tape by both the chord and tangent methods, 100-foot radius, the difference in the two results not to exceed 2 minutes. If the angle is near 90°, the tangent method may be applied to the bisected angle. (2) After securing satisfactory check on an angle with the steel tape, make a rapid but careful measurement with the metallic tape, radius 50 feet. The results may be taken to the nearest half minute. (3) Measure at least one angle, preferably on smooth ground, by laying out an arc with radius of 57.3 feet, setting pins every few feet, and measuring the actual arc. Give close attention to alinement throughout. Record data and make reductions as in the form.

PROBLEM A11. SURVEY OF FIELD WITH STEEL TAPE.

(a) *Equipment.*—100-foot steel tape, set of chaining pins, 2 plumb bobs, 4 flag poles, five-place table of functions.

(b) *Problem.*—Make survey of an assigned field with tape, collecting all data required for plotting the field and calculating its area by the "perpendicular," "three-side," and "angle" methods.

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HORIZONTAL DISTANCE Δ14 – Δ17 (STEEP SLOPE) WITH STEEL TAPE.

H'd Chain, J. Doe. Rr. Chain, R. Roe.
Recorder, B. F. Keen. Flagman, G. W. Sure.
Sept. 21, '14. (3 Hours) Cloudy; moderate.
Used 100-Ft. K.&E. etched tape, No. 416, Locker 26, with spring balance.

A. By Direct Horizontal Measurement.

No.	Observed Length Ft.	Mean Length Ft.	Cor. for Standard Ft.	Reduced Length Ft.	Diff. (E) Coef. (C) Ratio (1:d)
Tape 1	99.995				
" 2	99.997	99.996			E = 0.04
1	761.45				c = 0.015
2	761.49	761.47	−0.03	761.44	1:25380
		410 00.0			(See Diagram)
		−.030			

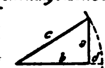
1st. Method: Standardized tape (before and after), supported at ends and middle, plumbing ends down, pull 12 pounds. Chained line in duplicate, leveling tape by estimation, pull 12 lbs., plumbing down high end, marking points by chaining pins leaning sidewise.

B. By Measurement on the Slope and Reduction to the Horizontal

No.	Observed Length Ft.	Mean Length Ft.	Cor. for Standard Ft.	Reduced Length Ft.	Diff. (E) Coef. (C) Ratio (1:d)
Tape 1	100.007				
2	100.008	100.008			E = 0.02
1	761.81				c = 0.007
2	761.79	761.80	+0.06	761.86	1:38590
Correction for Inclination				−0.47	(See Diagram)
Reduced Horizontal Measurement				761.39	
Result by First Method				761.44	
Difference between Results				0.05	= E
Mean of Two Results				761.42	c = 0.018
					1:15230

2nd Method: Compared tape with standard (before and after), supported full length on ground, pull 12 lbs. Chained line in duplicate, tape supported on ground, pull 12 lbs., ends marked by chaining pins. Drove temporary pegs every 100 Ft. for levels. Ran levels over line with following results:

c	a	a²	d
Ft.	Ft.		Ft.
100	+3.5	12.25	0.06
100	+3.7	13.69	0.07
100	+4.1	16.81	0.08
100	+2.8	7.84	0.04
100	+3.3	10.89	0.05
100	+4.5	20.25	0.10
100	+3.1	9.61	0.05
62	+1.6	2.56	0.02
762			0.47 = Σd



With a small, $d = \frac{a^2}{2c}$ (Nearly)

ANGLES OF TRIANGLE 5-6-8 WITH TAPE.

Surveyors, J. Doe and R. Roe.
Sept. 22, '14. (2 Hours) Clear and warm.
Used Roe 100-Ft. Steel Tape, No. 362. and Lufkin 50-Ft. Metallic Tape, No. 411, Lkr. 35.

Station	Trigo. Functions Name	Value	Computed Angle Half	Whole	Mean.
6	Sin. ½ (6)	0.4050	23°53'.5	47°47'	
"	Tan. (6)	1.1023		47°47'	47°47'
8	Sin. ½ (8)	0.3696	21°41'.5	43°23'	
"	Tan. (8)	0.9450		43°23'	43°23'
5	Sin. ½ (5)	0.6995	~~44°23'~~	~~88°46'~~	
"	Tan. ½ (5)	0.9796	~~44°24'.5~~	~~88°49'~~	
"	Sin. ½ (5)	0.6997	44°24'	88°48'	
	Tan. ½ (5)	0.9799	44°25'	88°50'	88°49'
					179°59'
		Error of Closure =			01'
		Permissable Error =			03'

Though not needed in problem, noted the lengths of tapes by standard, 100.01 and 50.01 Ft., respectively.
Measured each angle by chord method and checked by tangent method, using radius of 100 Ft. with steel tape. In measuring ∠5, (nearly 90°) the tangent method was applied to the bisected angle. Each angle was verified before proceeding to next, a difference of 2' being allowed in each.

Check with 50 Ft. Metallic Tape

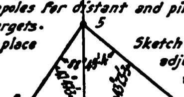
Angle	Sin. ½ A.	½ A.	A.
6	0.4051	23°54'	47°48'
8	0.3696	21°41'.5	43°23'
5	0.6999	44°25'	88°50'
			180°01'

After an angle was thus verified, a rapid but careful measurement was made with metallic tape, by chord method only, using 50-Ft. radius.

Assigning equal weight to the three results by steel tape, the most probable values are:

Angle 6	47°47'.3
" 8	43°23'.3
" 5	88°49'.4
	180°00'

Used Flagpoles for distant and pins for close targets.
Used 5-place table.



Sketch shows adjusted values.

Rough test of ∠6, radius = 57.3 Ft. Arc = 47.80 Ft. ∠6 = 47°48'.

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(c) *Methods.*—(1) Standardize the tape once. (2) Examine the field carefully and plan the survey. (3) Measure the required angles with tape. (4) Locate the perpendiculars. (5) Chain all necessary lines, and also take distances to feet of perpendiculars. Follow the form.

PROBLEM A12. AREA OF FIELD BY PERPENDICULAR METHOD.

(a) *Equipment.*—Five-place logarithms.

(b) *Problem.*—Calculate the area of the assigned field by the perpendicular method, using the data collected in Problem A11.

(c) *Methods.*—(1) Prepare form for calculations; transcribe data, and carefully verify transcript. (2) Calculate double areas of the several triangles by contracted multiplication, perpendicular method, preserving a consistent degree of precision. (3) Make the same calculations with logarithms, as a check. (4) Combine the verified results, as shown in the form.

PROBLEM A13. AREA OF FIELD BY THREE-SIDE METHOD.

(a) *Equipment.*—Five-place logarithms.

(b) *Problem.*—Calculate the area of the assigned field by the three-side method, using data collected in A11.

(c) *Methods.*—(1) Prepare form for calculation; transcribe data, and carefully verify transcript. (2) Calculate the areas of the several triangles by logarithms, three-side method, preserving proper units in the results. (3) Carefully review the calculations, and combine the verified results, as in the form.

PROBLEM A14. AREA OF FIELD BY ANGLE METHOD.

(a) *Equipment.*—Five-place logarithms.

(b) *Problem.*—Calculate the area of the assigned field by the "two sides and included angle" method, using the data collected in A11.

(c) *Methods.*—(1) Prepare form, transcribe data, and verify copy. (2) Calculate the double areas of the several triangles by contracted multiplication, angle method, preserving consistent accuracy in results. (3) Make same cal-

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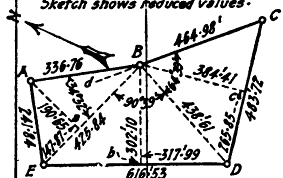
SURVEY OF FIELD A-B-C-D-E WITH TAPE. (DATA FOR AREA AND PLAT.)

Angle	Sin ½A	½A	A	Proof
ABE	·2968	17°16′	34°32′	
EBD	·7131	45°29′·5	90°59′	190°10′
DBC	·5347	32°19′·5	64°39′	180°00′
ABd	·0888	5°06′	10°12′	10°10′

Line	Observ'd Length Ft.	Cor. for Standard Ft.	Reduced Length Ft.
Sept. 25 Tape	99·992		
Sept. 26 Tape	99·980		
AB	336·83	−0·07	336·76
BC	465·07	−0·09	464·98
CD	483·82	−0·10	483·72
DE	616·65	−0·12	616·53
EA	241·89	−0·05	241·84
BE	425·93	−0·09	425·84
BD	438·70	−0·09	438·61
Ae	190·69	−0·04	190·65
Ee	147·90	−0·03	147·87
Bb	302·16	−0·06	302·10
Db	318·05	−0·06	317·99
Bc	381·49	−0·08	381·41
Dc	265·90	−0·05	265·85

Head Chainman, R. Roe.
Rear Chainman, J. Doe.
Sept. 25, '14. (3 Hours) Cloudy & Cool.
Used Roe 100 Ft. Steel Tape No 361, Locker #35
Standardized tape before only.
Let fall perpendiculars Ae, Bb and Bc by first estimating positions of e, b and c, then erecting precise perpendiculars and shifting as required. Set pegs at points e, b and c.
Measured angles ABE, EBD and DBC with tape by chord method, 100 Ft. radius, and checked by measuring angle between AB and Bd (line CB prolonged)
Sept. 26, (2 Hours) Drizzling & Cold. Chained each line carefully once.
Sketch shows reduced values.

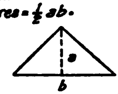


Sept. 27, '14. Computer, J. Doe.

COMPUTATION OF AREA OF FIELD A-B-C-D-E, PERPENDICULAR METHOD.

Triangle	Line	Base, b Ft.	Altitude, a Ft.	Multiplication	Logarithms	Double Areas Sq. Ft.	
ABE	BE Ae	425·84	190·65	425·84 56·091 42584 38326 256 21 81187	2·62925 2·28024 4·90949 (81190)	Data From p.p. Transcript checked. 81 190	(Result to nearest 10 Sq. Ft.)
BDE	De Bb	616·53	302·10	616·53 01·203 184959 1233 62 186254	2·78995 2·48015 5·27010 (186 250)	186 250	" " " " " "
BCD	CD Bc	483·72	381·41	483·72 14·183 145116 38698 484 193 5 184496	2·68459 2·58139 5·26598 (184 500)	184 500 2)451 940 225 970 =	" " " " " " (Result to nearest 0·001 Ac.) 5·188 Ac.

Area = ½ ab.



Note. To reduce sq. ft. to acres, divide by 43.560. Special methods are given below

1 Acre = 43.560 square Feet = 10 square chains.

$$1\ \text{Sq. Ft.} = \frac{1}{6 \times 6 \times 11 \times 11 \times 10}\ \text{Ac.} = \frac{1}{43560}\ \text{Ac.}$$

= 0·0000229596 Ac.

Use long division or one of the methods shown in the application opposite.

Short Division	Contracted Div'n		Contracted Mult'n
6)225970 6)37661·667 11)6276·944 11)570·631 10)51·876 5·1876	225970 217800 8170 4356 3814 3485 329 305 24 26	43560 5·1876	225970 0759220000·0 45194 4519 2034 113 16 5·1876

culations by logarithms, as a check. (4) Combine the checked results. Follow the form.

PROBLEM A15. AREA OF FIELD FROM PLAT.

(a) *Equipment.*—Drafting instruments, paper, etc., planimeter (as assigned).

(b) *Problem.*—Determine the area of the assigned field directly from the plat.

(c) *Methods.*—(1) Make an accurate plat of the field from the notes secured in A11, using a prescribed scale. (2) Determine the area of the field by resolving the polygon into an equivalent triangle. (3) Determine the area from the plat by the polar planimeter and by one of the following "home-made" planimeters: "bird shot" planimeter, "jack knife" planimeter, cross-section paper, parallel strip, weighing, etc. (4) Prepare on the plat a tabulated comparison of results secured by the several methods. (5) Finish the plat, as required.

PROBLEM A16. SURVEY OF FIELD WITH CURVED BOUNDARY.

(a) *Equipment.*—100-foot tape, 50-foot metallic tape, set of chaining pins, 2 plumb bobs, 4 flag poles.

(b) *Problem.*—Make survey with tape of an assigned tract having a curved boundary, collecting all data required for plotting the field and calculating its area.

(c) *Methods.*—(1) Standardize the tape once to nearest 0.01 foot. (2) Examine the tract carefully and plan the survey so as to secure a simple layout of base lines designed to give short offsets to the curved boundaries. (3) Locate the perpendiculars, if any, and chain all lines; on the curved sides, take offsets so as to secure a definite location, and as a rule take equal intervals on the same line. Follow the form.

PROBLEM A17. AREA OF FIELD WITH CURVED BOUNDARY.

(a) *Equipment.*—(No instrumental equipment required).

(b) *Problem.*—Calculate the area of the assigned field with curved boundary by "Simpson's one-third rule," using the data collected in Problem A16.

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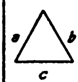
Sept. 28, '14. Computer, J. Doe.

COMPUTATION OF AREA OF FIELD A-B-C-D-E, 3 SIDE METHOD.

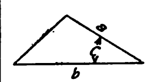
Triangle	Sides Line	Sides Length Ft.	s = ½(a+b+c) Ft.	(s−a) Ft.	(s−b) Ft.	(s−c) Ft	Area of Triangle √s(s−a)(s−b)(s−c) Logarithms	Areas Sq. Ft.	
ABE	AB=a	336.76	502.22				2.70089		
	BE=b	425.84		165.46			2.21869		
	EA=c	241.84			76.38		1.88298		
	2)	1004.44				260.38	2.41561		
	s =	502.22					2) 9.21817		
							4.60908	40 650	(To nearest 10 sq. ft.)
BDE	BD=a	438.61	740.49				2.86952		
	DE=b	616.53		301.88			2.47983		
	EB=c	425.84			123.96		2.09328		
	2)	1480.98				314.65	2.49783		
		740.49					2) 9.94046		
							4.97023	93 380	
BCD	BC=a	464.98	693.66				2.84115		
	CD=b	483.72		228.68			2.35923		
	DB=c	438.61			209.94		2.32210		
	2)	1387.31				255.05	2.40662		
		693.66					2) 9.92910		
							4.96455	92 160	
Data	from pp.		Transcript	checked.			5.35447	226 190	= 5.193 Ac.
							−4.63909 ←	÷ 43 560	
							0.71538		



Sept. 22, '14. Computer, J. Doe.

COMPUTATION OF AREA OF FIELD A-B-C-D-E, ANGLE METHOD.

Triangle	Part	Value Ft. or °	Multiplication a Sin. C	Multiplication abSin. C	Logarithms	Double Areas Sq. Ft.		Area = ½ab sin. C.
ABE	AB=a	336.76	336.76	190.91	2.52732			
	BE=b	425.84	986650	48.524	2.62925	Data from	pp.	
	ABE=C	34°32'	16838	76364	9.75350	Transcript	checked.	
			2021	3818	4.91007			
			202	955				
			27	153				
			3	8				
			19091	81298	(81300)	81 300	(Result to	nearest 10 Sq. Ft.)
BDE	BE=a	425.84	425.84	425.78	2.62925			
	BD=b	438.61	58999.0	16.834	2.64208			
	EBD=C	90°59'	38326	170312	9.99994			
			3833	12773	5.27127			
			83	3406				
			34	255				
			2	4				
			425.78	186 750	(186 750)	186 750	" "	" " " "
BCD	BD=a	438.61	438.61	396.38	2.64208			
	BC=b	464.98	17309.0	89.464	2.66743			
	DBC=C	64°39'	39475	158552	9.95603			
			132	23783	5.26554			
			31	1586				
			396.38	357				
				31				
				184309	(184 310)	184 310	" "	" " " "
						2) 452 360		
						226 180 =	5.192 Ac.	
					5.35446		(Result to	nearest 0.001 Ac.)
					−4.63909 ←	−(÷ 43560)		
					0.71537	(5.192 Ac.)		



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(c) *Methods.*—(1) Prepare form for calculation; transcribe data in convenient form for calculation, and carefully check copy. (2) Calculate the area of the polygon formed by the base lines, preferably by the perpendicular method. (3) Calculate the areas of the curved figures by "Simpson's one-third rule," which is as follows: "Divide the base line into an *even number of equal parts* and erect ordinates at the points of division; then add together the first and last ordinates, twice the sum of all the other odd ordinates, and four times the sum of all the even ordinates; multiply the sum by one-third of the common distance between ordinates." (The field notes might have been taken with special reference to the rule, but it is better to take from the notes the largest *even number* of equal segments, assuming the remaining portion to be trapezoid or triangle.) (4) Give signs to the several results by reference to the field sketch, and combine them algebraically to get the net area, as shown in the accompanying form.

PROBLEM A18. AREA OF FIELD WITH CURVED BOUNDARY FROM PLAT.

(a) *Equipment.*—Drafting instruments, paper, etc., planimeter (as assigned).

(b) *Problems.*—Determine the area of the field with curved boundary directly from the plat.

(c) *Methods.*—(1) Make an accurate plat of the field from the notes obtained in A16, using a prescribed scale. (2) Determine its area directly from plat by two methods mentioned in (3) of A15, other than those used in that problem. (3) Prepare on the plat a tabulated comparison of the results by the several methods. (4) Finish the plat, as required.

PROBLEM A19. PASSING AN OBSTACLE WITH TAPE.

(a) *Equipment.*—100-foot steel tape, set of chaining pins, plumb bobs, 4 flag poles.

(b) *Problem.*—Prolong an assigned line through an assumed obstacle by one method and prove by another, finally checking on a precise point previously established.

(c) *Methods.*—Given two hubs, *A* and *B*, 200 feet apart prolong line and establish *C* 200 feet from *B*: (1) by constructing a 200-foot square in one direction; and (2) by lay-

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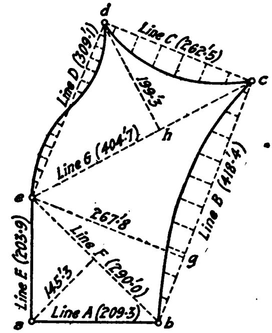
Survey of Field with Curved Boundary Line.

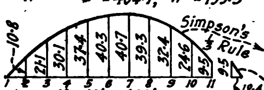
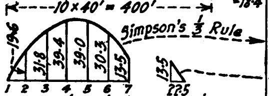
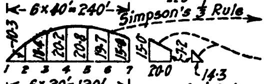
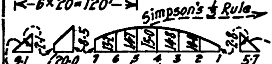
Head Chainman, R. Roe. Rear Chainman, J. Doe.
Oct. 2, '14. (3 Hours) Clear and warm.
Tape No. 361, Locker No. 35 = 100'.01
Sketch shows observed lengths. Final area result corrected for standard.

Offset L. Ft.	Dist. Ft.	Offset R. Ft.	Offset L. Ft.	Dist. Ft.	Offset R. Ft.
0	262.5	= d			
13.5	240				
30.3	200		e =	309.1	0
39.0	160			300	2.1
39.4	120			280	8.5
31.8	80			260	13.2
19.6	40			240	14.7
0	0			220	15.0
	Line C	(c to d)		200	14.8
				180	10.0
0	418.4	= c		160	2.8
9.5	400		0	154.3	0
24.6	360		7.2	140	
32.4	320		15.0	120	
39.3	280		19.7	100	
40.7	240		20.8	80	
40.3	200		20.2	60	
37.4	160		18.4	40	
30.1	120		10.3	20	
21.2	80		0	0	= d
10.8	40			Line D	(d to e)
0	0	= b	Tape =	100'.01	
	Line B (Read Up)	(b to c)	Oct. 3, '14	, Clear (Read Up)	& warm



Computation of Area of Field with Curved Boundary.

Oct. 3, '14. Computer, J. Doe.

Part	Data for Calculation of Areas	Indicated Calculations	+Areas	−Areas
		Data From pp. Transcript Checked.		
abe	Triangle, Base = 290'.0, Alt. = 145'.3	$\frac{1}{2}(290.0\times145.3)$	21068	
bce	" " = 418'.4, " = 267'.8	$\frac{1}{2}(418.4\times267.8)$	56024	
cde	" " = 404'.7, " = 199'.3	$\frac{1}{2}(404.7\times199.3)$	40328	
Line B		$\frac{40}{3}[(0+9.5)+2(21.1+37.4+40.7+32.4)+4(10.8+30.1+40.3+39.3+24.6)]$		11375
		$\frac{1}{2}(9.5\times18.4)$		87
Line C		$\frac{40}{3}[(0+13.5)+2(31.8+39.0)+4(19.6+39.4+30.3)]$		6831
		$\frac{1}{2}(13.5\times22.5)$		152
Line D		$\frac{20}{3}[(0+15.0)+2(18.4+20.8)+4(10.3+20.2+19.7)]$		1961
		$\frac{1}{2}[20(15.0+7.2)+(7.2\times14.3)]$		273
" "		$\frac{20}{3}[(2.8+8.5)+2(14.8+14.7)+4(10.0+15.0+13.2)]$	1487	
		$\frac{1}{2}(2.8\times5.7)$	8	
		$\frac{1}{2}[(2.1\times9.1)+20(2.1+8.5)]$	116	
			119031	20679
Chain = 100'.01		Chain Cor. {98352, 2000.0, +20}	20679	
True Area = Computed Area × $(1+0.0001)^2$	$(1+0.0001)^2=(1+0.0002)$ (nearly)		98352	
			+20	
			98372	= 2.258 Ac.

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ing off a 200-foot equilateral triangle on the opposite side using pins to mark points thus established. (3) Prolong the line by each method to the hub *D*, 200 feet from *C*, and record discrepancies in line. (4) Interpolate a point at *C* on true line between *B* and *D*, and note errors of prolongation at *C*. Record as in the form.

PROBLEM A20. OBSTRUCTED DISTANCE WITH TAPE.

(a) *Equipment.*—100-foot steel tape, set of chaining pins, 2 plumb bobs, 4 flag poles.

(b) *Problem.*—Determine the distance between two assigned points through an assumed obstruction to both vision and measurement, using two independent methods, and finally chain the actual distance.

(c) *Methods.*—(1) Standardize the tape. (2) Determine the distance between the assigned points by constructing a line parallel to the given line, and equal or bearing a known relation to it. (3) Secure a second result by running a random line from one hub past the other so that a perpendicular less than 100 feet long may be let fall, measuring the two sides and calculating the hypothenuse. (4) After securing two results differing by not more than 1: 1,000, chain the actual distance. Follow the form.

PROBLEM A21. RUNNING IN CURVE WITH TAPE.

(a) *Equipment.*—100-foot steel tape, 50-foot metallic tape, set of chaining pins, 2 plumb bobs, 3 hubs, 6 flat stakes, marking crayon, tacks, five-place table of functions.

(b) *Problem.*—Lay out two lines making an assigned angle with each other, and connect them with a prescribed curve by the "chord offset" method.

(c) *Methods.*—(1) Calculate the *radius*, *R*, for the given *degree of curve*, *D*. (2) Calculate the *tangent distance*, *T*, for the given *radius*, *R*, and *angle of intersection*, *I*. (3) Calculate the *chord offset*, *d*, and *tangent offset*, *t*, for the known *radius*, *R*, *chord*, *c* and *degree*, *D*. (4) At the given point intersection (*P. I.*), *A*, lay off the given angle, *I*, by the chord method. (5) From the *P. I.* lay off *T* along the two tangent lines and locate point tangent (*P. T.*) and point curve (*P. C.*), setting hubs at *P. C.* and *P. T.*, with guard stake at each hub. (6) Run in the curve, by chord offsets, beginning at *P. C.* and checking at *P. T.* Calling *P.*

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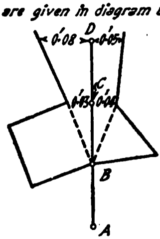
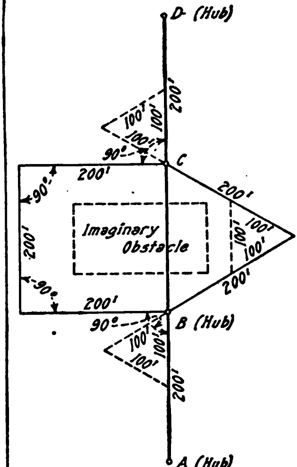
PASSING AN OBSTACLE

Oct. 4, '14, (2 Hours) Clear and warm.
Tape No. 361, Locker No. 35, Length = 100.01.
Given three hubs, (set on true line by transit), B 200 Ft. from A, and D 400 Ft. beyond B, all on smooth ground.
Assumed obstacle as shown in sketch, and then (ignoring D) passed obstacle by 200 Ft. equilateral triangle to right and by 200 Ft. square to left. Resumed line by each method and prolonged to point D. Used pins marked by slips of paper to indicate points.
Also interpolated C on BD carefully by eye.
Results are given in diagram below.



WITH STEEL TAPE.

Chainmen, J. Doe and R. Roe.



OBSTRUCTED DISTANCE

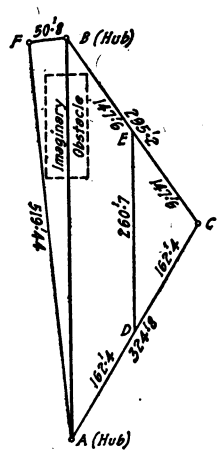
Oct. 5, '14 (2 Hours) Cloudy and cool.
Tape No. 361, Locker No. 35, Length 99.99
Given two hubs A and B an unknown distance apart, on smooth ground.
Assumed an obstruction to vision and measurement, as shown in sketch.
Selected point C visible from A and B, chained CA and CB, observing nearest 0.1 Ft., and bisected CA at D and CB at E. Chained DE. Then calculated AB by doubling ED. 260.7 × 2 = 521.4
Ran random line from A as close as practicable to obstruction so as to reduce BF to a minimum. Let fall perpendicular BF from B on random line. Measured BF and FA to nearest 0.1 Ft. Calculated hypothenuse AB.
$AB = \sqrt{50.8^2 + 519.4^2} = 521.0$
Finally, after securing the above results, chained the actual distance AB. The three results are sumarized below.

Method.	Obs. Dist.	Std. Cor.	Red. Dist.
By similar triangles	521.4	−0.1	521.3
By right triangle	521.0	−0.1	520.9
By actual measurement	521.5	−0.1	521.4

Total range = 1:1040.

WITH STEEL TAPE.

Chainmen, J. Doe and R. Roe.



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C. Station 0, establish Station 1 by laying off tangent offset, *t*, and chord, *c* Having one station on the curve, the next is located by prolonging the chord and forming an isosceles triangle having the chord offset as a base. Check on the *P. T.*, noting the discrepancy of distance and line. Also establish the tangent again by tangent offset and observe the error of line. Follow the form.

PROBLEM A22. DISCUSSION OF ERRORS OF CHAINING

(a) *Equipment.*—(No instrumental equipment, unless further data are desired, in which case Problems A6, A7 and A8 may be assigned again).

(b) *Problem.*—Investigate the errors of linear measurement with the several kinds of chains and tape, with the view to determine practical working tests or coefficients of precision for actual use.

(c) *Methods.*—Assume that the conditions in Problems A6, A7 and A8 are practically constant in the same problem, and that the actual differences between observed lengths of the several segments when chained in opposite directions, represent the normal errors with the particular chain and chainmen; then tabulate: (1) the measured lengths of all possible segments of the chaining course, either from direct observation or by subtraction; (2) the actual errors or differences between the two results, giving signs; (3) the chaining ratios, $l: d$, and the decimal expressions of the same to six places; (4) the "coefficients of precision" for each case, calculated by formula, or more quickly, taken from the diagram in the chapter on errors of surveying; (5) the mean decimal chaining ratio and its equivalent; and (6) the mean coefficient of precision. Follow the form.

PROBLEM A23. TESTING (OR ESTABLISHING) AN OFFICIAL STANDARD OF LENGTH.

(a) *Equipment.*—Standard tape (with certified length given), turnbuckle adjustments with bolts, spring balance, standard steel rule graduated to 0.01 inch, 2 thermometers, 2 microscopes, strips of wood, a watch.

(b) *Problem.*—Make a series of ten observations with a standardized steel tape for the purpose of testing (or establishing) an official standard of length, observing the nearest 0.0001 foot. (The Bureau of Standards, Washington, D. C., will standardize a tape for a small fee.)

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LOCATION OF CURVE WITH STEEL TAPE.

H'd Chain, J. Doe.
R'r Chain, R. Roe.
Axeman, B. F. Keen.
Flagman, G. W. Sure.

Oct. 6, '14. (3 hours) Clear and cool.
100 Ft. Steel Tape No. 361, Locker No. 35 = 100.00

Given hub at A and a distant hub B, to lay off a line A-C making an angle I of 80° with BA prolonged, and connect the two lines with a 20° curve, that is, a curve having a central angle of 20° subtended by a 100 ft. chord, c.

The radius was calculated thus: Since the chord of an arc is twice the sine of half the arc, chord = 2 rad. × sin. D

$$\text{rad.} = \frac{\frac{1}{2}\text{chord}}{\sin \frac{1}{2}D} = \frac{50}{0.17365} = 287.9'$$

Calculated tangent distance thus: In right triangle (O–P.C.–P.I.)

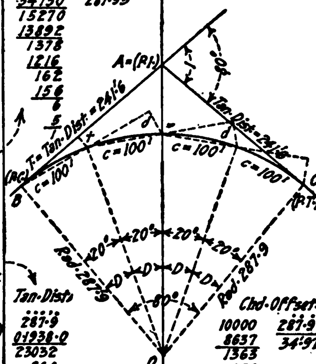
$$\text{Tan. Dist.} = \text{Rad.} \times \tan \tfrac{1}{2} I$$
$$= 287.9 \times 0.83910 = 241.6'$$

Calculated chord offset d, and tangent offset t, thus: By similar triangles R : c = c : d, $d = \frac{c^2}{R} = \frac{100^2}{287.9} = 34.97'$, $t = \frac{1}{2}d = 17.48'$
(An approx. formula is $d = 1\frac{3}{4}D = 35'$; $t = \frac{7}{8}D = 17.5'$*)*

From A (Point Intersection) laid off Tan. Dist. (T), locating Point Curve (P.C.) and Point Tangent (P.T.). Began at P.C. and ran in curve, as shown in sketch. Error of Closure at P.T. was 0.2' in line and 0.1' in distance.

Rad.

50.000	0.17365
34730	287.93
15270	
13892	
1378	
1216	
162	
156	
6	
5	
1	



Tan. Dist.

287.9
0.8391 0
23032
864
259
3
241.58

Chd. Offset.

10000	287.9
8637	34.97
1363	
1152	
211	
202	
9	

DISCUSSION OF ERRORS WITH STEEL TAPE.

Oct. 9, '14. Computer, J. Doe.

Line	Direction Chained	Observed Length Ft.	Difference, E Ft.	Chaining Ratio 1 : d	Coef. of Precision (c), Ft.
A-B	E.	484.58	E+ W−	0.000062	
B-A	W.	484.61	−0.03	1:16150	0.014
A-C	E.	2003.79		0.000029	
C-A	W.	2003.85	−0.06	1:33400	0.013
A-D	E.	3991.69		0.000012	
D-A	W.	3991.74	−0.05	1:79830	0.008
A-E	E.	5279.48		0.000017	
E-A	W.	5279.57	−0.09	1:58660	0.012
B-C	E.	1519.21		0.000019	
C-B	W.	1519.24	−0.03	1:50640	0.008
B-D	E.	3507.11		0.000006	
D-B	W.	3507.13	−0.02	1:175350	0.003
B-E	E.	4794.90		0.000012	
E-B	W.	4794.96	−0.06	1:78250	0.009
C-D	E.	1987.90		0.000005	
D-C	W.	1987.89	+0.01	1:198790	0.002
C-E	E.	3275.69		0.000009	
E-C	W.	3275.72	−0.03	1:109190	0.005
D-E	E.	1287.79		0.000030	
E-D	W.	1287.73	−0.04	1:32920	0.011
		(L. in	Σ =	0.000202	0.085
		100-ft.	Mean =	0.0000202 1:49500	0.008
		units)	$E = c\sqrt{L}$ (See Diagram)	or $c =$	$\frac{E}{\sqrt{L}}$

Data from pp. Transcript O.K.

A B C D E

Distances by Subtraction.

B-A 5279.57		E-A 5279.57		B-A 5279.57	
E-B 4794.96		E-C 3275.72		E-D 1287.83	
B-A 484.61		C-A 2003.85		D-A 3991.74	
A-C 2003.79		E-B 4794.96		A-D 3991.69	
A-B 484.58		E-C 3275.72		A-B 484.58	
B-C 1519.21		C-B 1519.24		B-D 3507.11	
E-B 4794.96		A-E 5279.48		A-D 3991.69	
E-D 1287.83		A-B 484.58		A-C 2003.79	
D-B 3507.13		B-E 4794.90		C-D 1987.90	
B-C 3275.72		A-E 5278.48		A-E 5279.48	
E-D 1287.83		A-C 2003.79		A-D 3991.69	
D-C 1987.89		C-E 3279.69		D-E 1287.79	

Designating E+ and W− (4th Column) it is seen that the returning results (except C-D) are greater. This is explained by standard tape lengths, viz., before = 100.011, after = 100.008, i.e. the tape gradually decreased in length, causing greater observed lengths.

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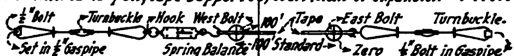
(c) *Methods.*—(If a *new* official standard is being established, one standard mark may be made permanent, and the precise distance taken to an approximate temporary point on the other bolt, the exact correction being applied after a sufficient number of results have been obtained. If the sun is shining, the tape should be protected by a wooden box or other covering throughout its length. Cloudy days or night time give best results. The observations should be made briskly so as to have slight range of temperature.

Oct. 10, '14. Cloudy and Cool. *Party: J. Doe, R. Roe, B. F. Keen, G. W. Sure.*

TEST OF 100-FT. STANDARD ———— UNIVERSITY.

Selected cloudy day with slight range of temperature during period of observations. Used Standard Tape No. 417, marked "U. S. W. & M. 215," certified length = 99.9967 ft., at 62° F. with 12-lb. pull, tape supported, coefficient of expansion = 0.0000061.



Program. *Arranged "bucksaw" adjustments, etc., as shown in sketch, tape supported on strip of wood. (a) Doe set zero of tape at east standard mark with reading glass. (b) Roe set balance at 12 lbs. (c) Keen observed fraction at west standard mark, using Starrett steel scale graduated to 0.01 in., estimating to nearest 0.001 in. with reading glass. (d) Sure recorded all data and observed time and temperature, two thermometers placed one each at 33' and 67'. Released pull between observations.*

No.	Time	Temperature				Temp. Cor.	Tape	West Fraction		Standard	Prob.	Error
	P. M.	At 33'	At 67'	Mean	62°-Mh	Ft.	Ft.	In. +	Ft. +	Ft.	d(.0001)	d^2
1	2:23	52.0°	53.0°	52.5°	9.5°	0.0058	99.9909	0.116	0.0097	100.0006	1	1
2	:28	52.0	53.0	52.5	9.5	0.0058	99.9909	.118	.0098	100.0007	0	0
3	:32	52.0	53.0	52.5	9.5	0.0058	99.9909	.116	.0097	100.0006	1	1
4	:35	52.0	53.0	52.5	9.5	0.0058	99.9909	.118	.0098	100.0007	0	0
5	:39	52.0	52.5	52.2	9.8	0.0060	99.9907	.121	.0101	100.0008	1	1
6	:46	52.0	52.5	52.2	9.8	0.0060	99.9907	.120	.0100	100.0007	0	0
7	:53	52.5	52.5	52.5	9.5	0.0058	99.9909	.119	.0099	100.0008	1	1
8	:58	52.5	52.0	52.2	9.8	0.0060	99.9907	.122	.0102	100.0009	2	4
9	3:04	52.0	52.0	52.0	10.0	0.0061	99.9906	.121	.0101	100.0007	0	0
10	:08	52.0	52.0	52.0	10.0	0.0061	99.9906	.122	.0102	100.0008	1	1

n $E_1 = 0.67\sqrt{\frac{\Sigma d^2}{n-1}} = 0.000067$ $E_m = \frac{E_1}{\sqrt{n}} = 0.000021$

Mean = 100.0007 $\Sigma d^2 = 9$

Length of Standard = 100.0007 ± 0.00002 Ft.

If isolated standard monuments are used, their foundation should go below frost line, and the monuments should be located so as to suffer as little as possible from heaving. If the standard marks are indoors, the conditions are less difficult to control.)

(1) Arrange "bucksaw" or turnbuckle adjustments, each held firmly by a bolt dropped into a piece of gaspipe driven flush with surface of ground, with spring balance and tape lined up, as shown in sketch in accompanying form; place the two thermometers at the one-third points as nearly as possible under the actual conditions of the tape. (2) With

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four men in party, No. 1 sets end graduation precisely at one standard mark by means of screw adjustments and microscope; No. 2 sets balance at 12 pounds; No. 3 observes fraction at other standard mark by means of steel scale graduated to 0.01 inch, estimating to nearest 0.001 inch (say 0.0001 foot) by microscope; and No. 4 records all data, observes time to nearest minute, and temperature to nearest 0.1 degree. Nos. 1, 2 and 3 should lie flat. Release the tension between observations. Record and reduce as in the form.

PROBLEM A24. DETERMINATION OF CONSTANTS OF A STEEL TAPE.

(a) *Equipment.*—Steel tape and other articles named in preceding problem.

(b) *Problem.*—Determine coefficients of expansion and stretch of the assigned tape.

(c) *Methods.*—(See Problem E9.)

PROBLEM A25. MAKING A STANDARD WIRE TAPE.

(a) *Equipment* —Spring balance, thermometer, etc., as in A23, and a piece of piano or other suitable steel wire.

(b) *Problem.*—Make a 100-foot or other standard tape by graduating the wire with reference to the official standard.

(c) *Methods.*—(To be devised by the student.)

PROBLEM A26. COMPARISON OF DIFFERENT MAKES AND TYPES OF CHAINS AND TAPES.

(a) *Equipment.*—Department equipment and collection of catalogs of representative instrument makers.

(b) *Problem.*—Make a critical comparison of the several types of chains and tapes made by different makers.

(c) *Methods.*—Study the different catalogs and prepare a systematic and concise report.

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CHAPTER III.

THE COMPASS.

Description.—The magnetic compass consists of a line of sight attached to a graduated circular box, at the center of which is a magnetic needle supported on a steel pivot. The compass box is attached to a tripod or jacob staff by a ball and socket joint, and is leveled by means of the plate levels. The needle should be strongly magnetized and have an agate cap to receive the point of the hardened steel pivot. The dip of the needle is counter-balanced by a small coil of wire, which can be shifted as desired. The *E* and *W* points are reversed.

In Fig. 10 are shown the usual types of magnetic compasses: (a) the vernier compass; (b) the plain compass;

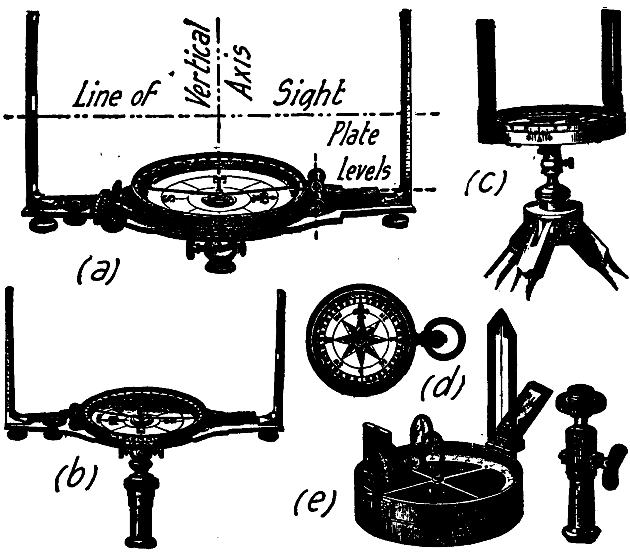

Fig. 10.—Types of Magnetic Compasses.

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(c) the vernier pocket compass with folding sights; (d) the ordinary pocket compass; (e) the prismatic compass.

Declination of the Needle.—If the needle is allowed to swing freely, its magnetic axis will come to rest in the magnetic meridian. The horizontal angle between the magnetic meridian and the true meridian at any point is called the *magnetic declination* for that point. Imaginary lines joining points on the earth's surface having the same declination are called *isogonic lines.* The isogonic line joining the points of zero declination is called the *agonic line.* Fig. 12 is an isogonic chart of the United States. Of the three agonic lines on the earth's surface, one passes through Michigan, Ohio, etc.

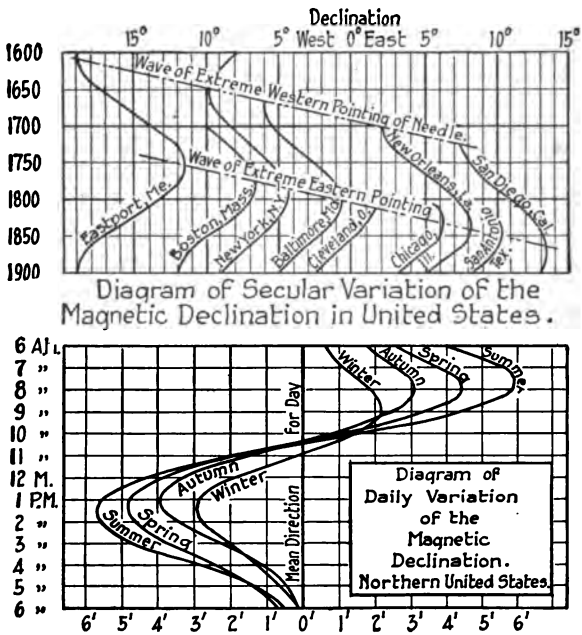

Fig. 11.

(For additional data see bulletin of Department of Commerce, U. S. Coast and Geodetic Survey, entitled "Principal Facts of the Earth's Magnetism.")

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Variation of the Declination.—The declination of the needle is not a constant at any place. The change or fluctuation is called the *variation* of the declination. The variations of the magnetic needle are of several kinds:

Fig. 12. Isogonic Chart of the United States, 1910.

secular, daily, annual, lunar, and irregular variations due to magnetic storms. The most important of these is the *secular* variation which is illustrated in the upper diagram of Fig. 11 for a series of representative points in the United States. This diagram shows that the extreme range or swing of the needle is roughly 6° or 7°, and that the period of time between extreme positions is about a century and a half. Also that the wave of magnetic influence progresses across the continent alike in successive cycles. In 1900 the needle was at its extreme western position at Eastport, Me., and at its extreme eastern position at San Diego, Cal. The 3° East isogonic line passed through western Indiana, and was moving westward at the rate of about 4′ per year. This rate of change was general throughout the central part of the United States, and is represented by the straight sections of the curve in the upper diagram of Fig. 11.

The *daily* variation of the magnetic declination is shown graphically in the lower part of Fig. 11, the scale being greatly magnified laterally. It is seen that the needle undergoes each day a vibration similar in a general way to the grand swing of three centuries or so shown in the upper diagram. The magnitude of the daily movement in northern United States ranges from 5′ in winter to nearly 12′ in summer time. The needle is in its mean daily position between 10 and 11 a. m. for all seasons. The diagram represents the normal magnetic day, of which there are perhaps five or six per month.

Local Attraction.—The pointing of the needle is affected by the close proximity of magnetic substances, such

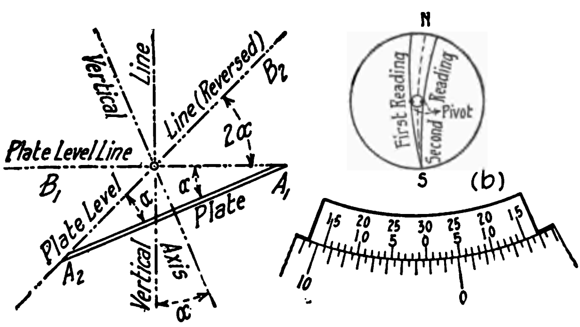

Fig. 13.

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as iron ore, wire fences, railroad rails, etc. However, local attraction does not prevent correct work, provided back and fore sights are taken without change of magnetic conditions. It is therefore especially important to avoid disturbances of the needle by the chain, axe, passing vehicles, electric wires, etc., or by articles on the person of the observer, such as keys, knife, spectacle frame, wire in the hat rim, reading glass case, etc. Also the glass cover may become electrified by friction and attract the needle, in which case it may be discharged with the moistened finger, or by breathing on it.

The Vernier.—The vernier is an auxiliary scale used to read fractional parts of the divisions of the main scale or limb. Verniers are retrograde or direct, according as the divisions on the vernier are larger or smaller than those on the limb. The vernier used on compasses for the setting off of the declination is direct, and is usually of the type shown in (c) of Fig. 13. In reading a vernier of any kind, blunders may be avoided by first estimating the fraction by eye before noting the matched lines on the two scales.

USE OF THE COMPASS.

Use.—The compass is used: (1) to determine the bearings of lines; (2) to measure the angle formed by two lines; (3) to retrace old lines. The bearing of a line is the horizontal angle between the line and a meridian through one end of it. Bearings are measured from the north or south point 90° each way. The angle between two lines is the difference in their directions as indicated by the bearings. Having the true bearings of one side of a polygon, the true bearings of the others may be obtained by algebraic addition of the angles; or by using the declination vernier so as to read the true bearing direct on the fore sights.

Practical Hints.—Point the north end of the compass box along the line and read the north end of the needle. Protect the pivot from needless wear by turning the needle in about the proper direction before releasing it. Always lift the needle before disturbing the compass. Habitually obtain duplicate needle readings on each sighting. Read the needle by estimation to the nearest five minutes, that is, to the one-sixth part of one-half degree, which is the usual subdivision of the compass box. Care should be taken to avoid parallax in reading the needle.

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ADJUSTMENTS AND TESTS.

Elementary Lines.—The *elementary lines* of the compass, shown in (a) of Fig. 10, are: (1) the line of sight; (2) the vertical axis; (3) the plate level lines.

The maker should see: (1) that the needle is strongly magnetized; (2) that the magnetic axis corresponds with the line joining the two ends; (3) that the metal in the compass box is non-magnetic; (4) that the line of sights passes through the center of graduation; (5) that the plates are perpendicular to the vertical axis; (6) that the zero of the vernier coincides with the line of sights.

The needle may be magnetized with a bar magnet or by putting it into the magnetic field of a dynamo. The metal of the compass box may be tested by reading the needle, then moving the vernier and noting if the needle has moved the same amount, this process being repeated at intervals around the full circle.

The Principle of Reversion.—In adjusting surveying instruments, the presence, direction and amount of the error are made evident by the *method of reversions* which doubles the apparent error. If there is no difference after reversion, there is no error.

Plate Levels.—*To make the plane of the plate level lines perpendicular to the vertical axis.*—Level up the instrument by means of the plate levels and reverse the compass box in azimuth, that is, turn it through a horizontal angle of 180°. Correct one-half the error, if any, by means of the adjusting screws at the end of the level tube, and bring the bubble to the center by the ball and socket joint. The reasons for this process are shown in (a) of Fig. 13.

Sights.—*To make the plane of sights normal to the plane of the plate level lines.*—With one sight removed and the instrument leveled, range in with the remaining sight two points as far apart vertically as possible, say on the side of a building. Reverse in azimuth and bring the bottom of the sight in range with the lower point; if the upper point is then in range, the sight is in adjustment. If not, correct one-half the error by putting paper under one side, or by filing off the other side. Repeat process for the other sight.

The Pivot.—*To adjust the pivot to the center of the graduated circle.*—Set the south end of the needle to read zero, and read the north end of the needle; reverse the compass box in azimuth, repeat the observations, and correct one-half the difference between the two readings of the north

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end of the needle by bending the pivot, using the special wrench for the purpose. Turn the compass box 90° and repeat See (b), Fig. 13.

The Needle.—*To straighten the needle.*—Having adjusted the pivot, set the north end of the needle to read zero and bend the needle so that the south end reads zero also. Turn the compass box and test for other graduations.

PROBLEMS WITH THE COMPASS.

PROBLEM B1. DECLINATION OF THE MAGNETIC NEEDLE.

(a) *Equipment.*—Surveyors' compass, flag pole, reading glass.

(b) *Problem.*—At a point on the true meridian determine the mean magnetic declination with the surveyors' compass.

(c) *Methods.*—(1) Set the compass over one point and a flag pole at another on the true meridian. (2) Lower the needle and sight at the flag pole carefully with the north end of the compass box to the front. (3) When the vibra-

DECLINATION OF NEEDLE WITH SURVEYOR'S COMPASS.

No.	Needle Reading	Mean	Time P.M.	Mean P.M.
1	N 3°30′ E.		2:05	
2	N 3°35′ E.		2:11	
3	N 3°30′ E.		2:15	
4	N 3°30′ E.		2:22	
5	N 3°35′ E.		2:27	
6	N 3°35′ E.		2:31	
7	N 3°35′ E.		2:35	
8	N 3°30′ E.		2:42	
9	N 3°35′ E.		2:48	
10	N 3°35′ E.	N 3°33′ E	2:54	2:30

Note: Assuming that the magnetic conditions are normal for the day, the correction for daily variation by Diagram of Daily Variation is 3 minutes West, which added to the mean gives N. 3°36′ E. as the most probable value of the declination for this particular instrument.

Oct. 12, '14. (2 Hours) Clear and Cool.
Used Gurley Compass No. 26. (Needle recently remagnetized), and Watch.
Set compass on true meridian with declination vernier set to read zero.
Sighted at flag pole set on meridian at a distance of 200 ft., and read needle by estimation to 5 minutes (one sixth part of one-half degree), carefully avoiding parallax and magnetic disturbances. Observed time to nearest minute.
Disturbed needle by lifting it from pivot and verified sighting; then when oscillations had ceased re-read the needle.
Continued the process until ten consecutive readings, having a maximum range of not more than ten minutes, were obtained.

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tions of the needle have ceased, move the vernier by means of the tangent screw so that the north end of the needle reads zero, and check the sighting of the compass. (4) Read the declination on the vernier to the nearest minute. (5) Lift the needle, verify the zero needle reading and the sighting, read the vernier and record; repeat the process until ten satisfactory consecutive values of the declination are obtained. Observe the time of each reading to the nearest minute. (6) Correct the mean of the ten values for daily variation by reference to the diagram, Fig. 11, using the mean time. Record and reduce the data as in the form. (Note that the values in the form were obtained by estimating the nearest five minutes. Which is better? Try both if time allows.)

PROBLEM B2. ANGLES OF TRIANGLE WITH COMPASS.

(a) *Equipment.*—Surveyors' compass, two flag poles, reading glass.

(b) *Problem.*—Measure the angles of a given triangle with the surveyors' compass.

(c) Methods.—(1) Set the compass over one of the vertices of the triangle and a flag pole behind each of the other two. (2) Lower the needle and sight at one of the flag poles carefully, with the north end of the box to the front. (3) When the vibrations have ceased, read the north end of the needle to the nearest five minutes by estimation. (4) Lift the needle, verify the sighting and also the reading. (5) Turn the compass box to the other point and determine the bearing, as before. The required angle is the difference between the two bearings. (6) Measure the other two angles in like manner. The error of closure must not exceed 5 minutes. Follow the form.

PROBLEM B3. TRAVERSE OF FIELD WITH COMPASS.

(a) *Equipment.*—Surveyors' compass, 2 flag poles, engineers' chain, set of chaining pins.

(b) *Problem.*—Determine the bearings of the sides of an assigned field with the surveyors' compass and measure the lengths of the sides with an engineers' chain.

(c) *Methods.*—(1) Set the compass over one of the corners of the field which is free from local attraction, and set off the declination with the vernier. (2) Take back sight on the last point to the left and fore sight to the next point

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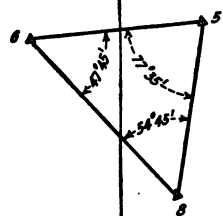
ANGLES OF TRIANGLE. 5-6-8 WITH SURVEYORS' COMPASS.

Station	Line	Observed Bearing	Needle Angle
5	5-6	S.83°25'W	
"	5-8	S.5°40'W	77°35'
8	8-5	N.5°40'E	
"	8-6	N.49°05'W	54°45'
6	6-8	S.49°00'E	
"	6-5	N.83°15'E	47°45'
			180°05'

Observers, R. Roe & J. Doe.
Oct. 13, '14. (2 Hours) Clear, Moderate.
Used Gurley Compass, Locker No. 26.
Each bearing was observed in duplicate, the needle being disturbed and the sighting verified between readings.

(Discrepancy not to exceed 5 minutes.)

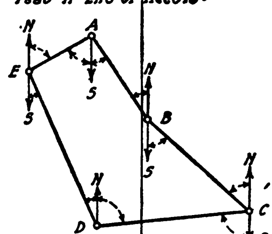


TRAVERSE OF FIELD A-B-C-D-E WITH COMPASS AND CHAIN.

Station	Line	Observed Bearing	Interior Angle	Adjusted Bearing	Distance Ft.
A	A-E	S.64°30'W	93°15'		
	A-B	S.32°45'E		S.32°45'E	336.5
B	B-A	N.33°05'W	190°20'		
	B-C	S.43°25'E		S.43°05'E	464.6
C	C-B	N.43°20'W	55°05'		
	C-D	S.81°35'W		S.81°50'W	483.3
D	D-C	N.81°35'E	103°55'		
	D-E	N.22°20'W		N.22°05'W	616.0
E	E-D	S.22°20'E	97°30'		
	E-A	N.60°10'E		N.60°25'E	241.6
			540°05'		

See calculation of latitudes and departures on pp.
See calculation of area on pp.

Observers: J. Doe & R. Roe.
Oct. 16, '14. (3 Hours) Clear & Windy
Used Gurley Compass, Locker No. 24.
Made needle read zero when pointing true north by setting off declination with vernier on declination arc of N.3°36'E.
Read bearings with N. End of Compass toward the Forward station and read N. End of Needle.



Allowable error of closure = 0°10'.
Chained each line once with Engineers' chain. Length of Chain = 100.08 Ft.

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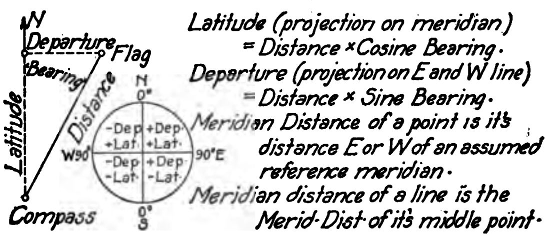
to the right, following the methods used in Problem B2. (3) Repeat this process for the remaining corners of the polygon taken in succession to the right. (4) Chain the sides of the field to the nearest 0.1 foot by estimation. (5) Compare the chain with standard. (6) From the observed bearings compute the interior angles of the field, and the true bearings of the sides. The angular error of closure must not exceed 10 minutes for a five-sided field. Record and reduce data as in the form.

PROBLEM B4. AREA OF FIELD WITH COMPASS.

(a) *Equipment.*—Five-place logarithms.

(b) *Problem.*—Compute the area of the assigned field by means of latitudes and departures.



(c) *Methods.*—(1) Prepare forms for calculations; transcribe data, and carefully verify copy. (2) Compute latitudes and departures by contracted multiplication, preserving results to the nearest 0.1 foot. (3) Make the same calculations by logarithms, as a check. (4) Determine the actual linear error of closure. (5) Determine the permissible error of closure (see chapter on errors of surveying). (6) If consistent, distribute the errors in proportion to the several latitudes and departures, respectively, repeating the additions as a check. (7) Transcribe field notes and adjusted latitudes and departures, and verify transcript. (8) Calculate the meridian distances of the several stations and lines. (9) Calculate the latitude coordinates. (10) Calculate the partial trapezoidal areas by multiplying the meridian distances of the lines by the respective latitudes, preserving consistent accuracy, and observing algebraic signs. (11) Determine the area by taking the algebraic sum of the partial areas. Reduce to acres, and correct for standard.

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Oct. 17, '14 Computer, J. Doe.
Data From pp. Transcript O.K.

COMPASS TRAVERSE OF FIELD A-B-C-D-E LATITUDES AND DEPARTURES.

Line	Adjusted Bearing	Observed Distance Ft.	Computation of Latitudes. Multiplication (Lat = Dist × Cos Bg.)	Logarithms	Computed Latitude Ft.	Lat. Cor. Ft.	Adjusted Latitude Ft.	Computation of Departures. Multiplication (Dep = Dist × Sin Bg.)	Logarithms	Computed Departure Ft.	Dep. Cor. Ft.	Adjusted Departure Ft.
AB	S 32° 45' E	336.5	336.5 40148.0 26920 1346 34 1 283.01	2.52698 9.92481 2.45180 (283.01)	S 283.0	-0.2	S 282.8	336.5 79045.0 16825 1346 30 2 182.03	2.52698 9.73318 2.26016 (182.04)	E 182.0	0.0	E 182.0
BC	S 43° 05' E	464.6	464.6 65037.0 32522 1394 14 2 339.32	2.66708 9.86353 2.53061 (339.32)	S 339.3	-0.2	S 339.1	464.6 20386.0 27876 3717 139 3 317.35	2.66708 9.83446 2.50154 (317.35)	E 317.4	+0.1	E 317.5
CD	S 81° 50' W	483.3	483.3 50241.0 4833 1933 97 2 68.65	2.68422 9.15245 2.83667 (68.65)	S 68.6	0.0	S 68.6	483.3 58989.0 43497 3866 435 39 3 478.40	2.68422 9.99557 2.67979 (478.40)	W 478.4	-0.2	W 478.2
DE	N 22° 05' W	616.0	616.0 46629.0 55440 1232 370 37 2 570.81	2.78958 9.96691 2.75649 (570.81)	N 570.8	+0.3	N 571.1	616.0 59573.0 18480 4312 308 55 3 231.58	2.78958 9.57514 2.36472 (231.59)	W 231.6	-0.1	W 231.5
EA	N 60° 25' E	241.6	241.6 96394.0 9664 2174 72 15 2 119.27	2.38310 9.69345 2.07655 (119.28)	N 119.3 N 690.1 S 690.9 S 0.8	+0.1 +0.4 -0.4 0.8	N 119.4 N 690.5 S 690.5 0.0	241.6 74968.0 19328 1450 217 14 1 210.10	2.38310 9.93934 2.32244 (210.11)	E 210.1 E 709.5 W 710.0 W 0.5	+0.1 +0.2 -0.3 0.5	E 210.2 E 709.7 W 709.7 0.0
	P =	2142.0										

Distribution of Error

Line	Lat.		Dep.	
AB	3	0.2	2	0.0
BC	3	0.2	3	0.1
CD	1	0.0	5	0.2
DE	6	0.3	2	0.1
EA	1	0.1	2	0.1
	14	0.8	14	0.5

Error of Closure. (See Diagram)

Actual Error $= \sqrt{L^2+D^2} = \sqrt{0.8^2+0.5^2} = 0.9$ Ft.

Permissable Error $= \frac{3ap}{10{,}000} = \frac{3 \times 5 \times 2142}{10000} = 3.2$ Ft.

Oct. 17, 14. Computer, J. Doe.
Data From pp. Transcript O.K.

COMPUTATION OF AREA OF FIELD A-B-C-D-E, COMPASS TRAVERSE.

Station	Line	Adjusted Bearing	Observed Distance Ft.	Adjusted Latitude N. Ft.	Latitude S. Ft.	Adjusted Departure E. Ft.	Departure W. Ft.	Meridian Distance Station Ft.	Meridian Distance Line Ft.	Areas N. Sq. Ft.	Areas S Sq. Ft.	Latitude Coördinate Ft.
A								E 210.2				N 119.4
	AB	S 32° 45' E	336.5		282.8	182.0		(Mean) =	E 301.2		85 179	
B								E 392.2				S 163.4
	BC	S 43° 05' E	464.6		339.1	317.5			E 551.0		186 844	
C								E 709.7				S 502.5
	CD	S 81° 50' W	483.3		68.6		478.2		E 470.6		32 283	
D								E 231.5				S 571.1
	DE	N 22° 05' W	616.0	571.1			231.5		E 115.8	66 133		
E								0.0				0.0
	EA	N 60° 25' E	241.6	119.4		210.2			E 105.0	12 549		
				690.5 =	690.5	709.7 =	709.7	1543.6 =	1543.6	78 682	304 306	

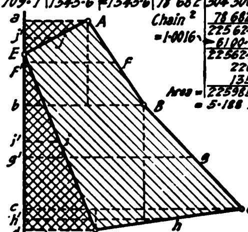
AUXILIARY CALCULATIONS

Sta.	Line	Mer. Dist.	Lat. Co-ord.
E		0.0	0.0
	EA	E 210.2	N 119.4
A		E 210.2	N 119.4
	AB	E 182.0	S 282.8
B		E 392.2	S 163.4
	BC	E 317.5	S 339.1
C		E 709.7	S 502.5
	CD	W 478.2	S 68.6
D		E 231.5	S 571.1
	DE	W 231.5	N 571.1
E		0.0	0.0

Area Multiplications

AB	CD	EA
301.2 8.282 60240 24096 602 241 85179	470.6 6.86 28236 3765 282 32283	105.1 4.911 10510 1051 946 42 12549

BC	DE
551.0 1.933 165300 16530 4959 55 186844	115.8 1.175 57900 8106 116 11 68133



	304 306
	78 682
Chain² = 1.0016	225 624
	6100.1
	225624
	226
	135
Area =	225985 Sq. Ft.
	= 5.188 Acres.

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Follow the form. (12) Make plat of field, using total rectangular coordinates, and checking by polar planimeter.

PROBLEM B5. ADJUSTMENT OF THE COMPASS.

(a) *Equipment.*—Surveyors' compass, adjusting pin, small screw driver.

(b) *Problem.*—Make the necessary tests and adjustments of the surveyors' compass.

(c) *Methods.*—Observe the following program: (1) test the magnetism of the needle; (2) test the metal of the compass box; (3) test and adjust the plate levels; (4) test the sights; (5) test the pivot; (6) test the needle.

PROBLEM B6. COMPARISON OF DIFFERENT MAKES AND TYPES OF COMPASSES.

(a) *Equipment.*—Department equipment, catalogs of representative makers of compasses.

(b) *Problem.*—Make a critical comparison of the several types of compasses.

(c) *Methods.*—Examine the department equipment and study the several catalogs carefully, noting the characteristic features, prices, etc. The following items, at least, should be included in the tabulated report: name of instrument, length of needle, length of alidade, vernier, tripod, weight, price, etc.

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CHAPTER IV.

THE LEVEL.

Description.—The engineers' level consists of a line of sight attached to a bubble vial and a vertical axis. Two types of level, the wye and dumpy, Fig. 14, are used by engineers. In the former the telescope rests in Y-shaped supports, from which it may be removed. In the dumpy level the telescope is fixed. The dumpy is a favorite with British

Engineers' Wye Level. Dumpy Level.
Fig. 14.

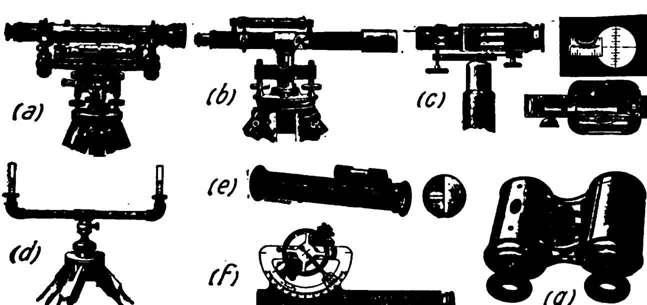

Fig. 15.—Types of Levels.

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and the wye level with American engineers. (The dumpy level with erecting eye-piece has been adopted as standard by the Division of Valuation, Interstate Commerce Commission.) The two types differ chiefly in the methods of adjustment. A third type, not shown in the cuts, is called the level of precision because of its use solely for work of extreme refinement.

In Fig. 15 are shown: (a) an architects' or builders' level of the wye type; (b) a road builders' level of the dumpy type; (c) a reconnaissance level with a decimal scale for reading horizontal distances direct; (d) a water level sometimes used in locating contours; (e) a Locke hand level; (f) a clinometer; (g) a binocular hand level.

THE TELESCOPE.

Principles.—The telescope used in the engineers' level and transit, shown in section in Figs. 16 and 22, consists of an *objective* or *object glass* which collects the light and forms an image in the plane of the cross-hairs, and an *ocular* or *eyepiece* which magnifies the image and cross-hairs. The cross-hairs are thus at the common focus of the objective and eyepiece. The principle of this type of telescope, both optically and mechanically, may be illustrated by the photographic camera if cross lines be ruled on the ground glass focusing plate and a microscope be used in viewing the image formed by the lens. Telescopes of the above class are called *measuring* telescopes, while those of the opera glass type are termed *seeing* telescopes. The latter have no real image formed between the object glass and eyepiece.

Line of Collimation.—The telescope of the level or transit may be represented by a line, called the *line of collimation*, which joins the optical center of the objective and the intersection of the cross-hairs. The optical center is a point such that a ray of light passing through it emerges from the lens parallel to its original direction. The line of collimation is independent of the eyepiece.

Objective.—The objective is a double convex or plano-convex lens. In all good telescopes the objective is compound, that is, made up of two lenses, with the view to correct two serious optical defects to which a simple lens is subject. These defects are called *chromatic aberration* and *spherical aberration*.

Chromatic aberration is the separation, by the objective, of white light into its component colors. A lens which is

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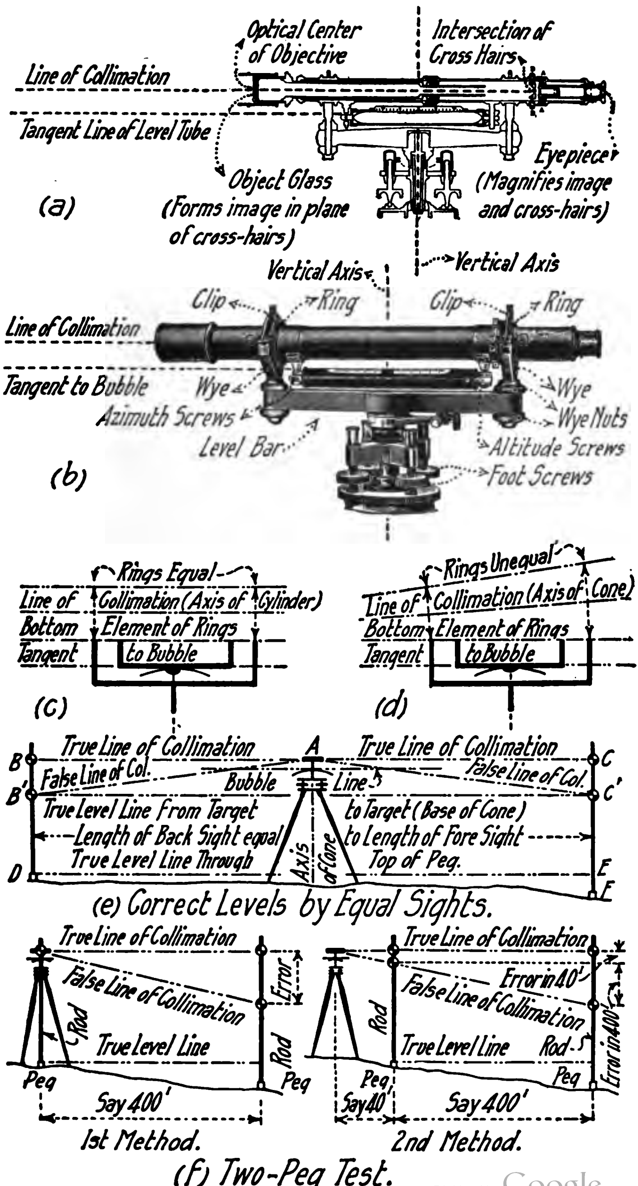

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Fig. 16.

free from this defect is called achromatic. A telescope is tested for the chromatic defect by focusing on a bright object, such as a piece of paper with the sun shining on it, and noting the colors on the edge of the object and especially at the edge of the field of view as the focus is slightly deranged. Yellow and purple are the characteristic colors indicating good qualities in the lens.

Spherical aberration is a defect which prevails to a serious extent in a simple lens having spherical surfaces. It is due to a difference in the focal distance for different concentric or annular spaces of the objective, so that the plane of focus for rays passing through the outer edges of the lens is different from that of the middle portion. A telescope is tested for this defect by focusing on a well defined object, such as a printed page, with the rays of light cut off alternately from the middle and the edge of the lens. This is best done by means of a circular piece of paper with a small round hole in it.

As a rule, the object glass in good levels and transits consists of a double convex lens of crown glass fitted to a concavo-convex or a plano-concave lens of flint glass, the former to the front. The defects described above are avoided through the different dispersive and refractive powers of the two kinds of glass, and by grinding the surfaces of the two lenses to the proper curvatures.

Eyepiece.—As in the camera, the image formed by the objective is inverted, so that if a simple microscope be used as an eyepiece, the observer sees objects inverted. Such an eyepiece is commonly used on the dumpy level, as shown in Fig. 14. This form of eyepiece consists of two plano-convex lenses with their convex sides facing each other. The form of eyepiece most used in American instruments is the erecting eyepiece in which two plano-convex lenses replace each of the two in the simpler form. The erecting eyepiece is much longer than the simple one, as may be seen at a glance in Fig. 14. While the simple eyepiece causes a little confusion at first, owing to the inversion of objects, it is much superior to the erecting eyepiece in the matter of clearness and illumination.

The chief inherent defect in the eyepiece is a *lack of flatness* of the field. A single lens usually causes a distortion or curving of straight lines in the image, especially towards the edge of the field. A telescope is tested for this defect by observing a series of parallel right lines, prefer-

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ably a series of concentric squares, which fill the entire field of view.

In the best achromatic eyepieces, one or more of the separate lenses may be compounded, the curvatures being such as to eliminate the color defect and give rectilinear qualities to the lens or combination of lenses.

Definition.—The definition of a telescope depends upon the finish and also the accuracy of the grinding of the curved surfaces of the lenses. It may be tested by reading the time on a watch or a finely printed page at some distance from the instrument.

Illumination.—Illumination and definition are apt to be confused. Poor definition causes indefinite details, while poor illumination causes faintness in the image. The latter may be tested about dusk, or in a room which can be gradually darkened, and can be best appreciated if two telescopes of different illuminating qualities be compared.

Aperture of Objective.—The aperture or effective diameter of the objective is determined by moving the end of a pencil slowly into the field and noting the point where it first appears to the eye when held say 8 or 10 inches back from the eyepiece. The process should be repeated in the reverse order. The annular space is deducted from the actual diameter to obtain the real aperture.

Size of Field.—The field of the telescope is determined by noting the angle between the extreme rays of light which enter the effective aperture of the objective. With the transit telescope, the limiting points may be marked on the side of a building and the angle measured directly with the plates; or with either level or transit the angle may be calculated from the measured spread in a given distance. For simplicity, a distance of 57.3 feet may be taken, and the result reduced to minutes.

Magnifying Power.—The magnifying power of a telescope is expressed in diameters, or as the multiplication of linear dimension. It is determined most readily by making an observation with both eyes open, one looking through the telescope and the other by natural vision. The comparison may be made by means of a leveling rod, or the courses of brick or weather-boarding on the side of a house may be used in like manner.

Parallax.—Parallax is the apparent movement of the cross-hairs on the object with a slight movement of the eye, and is due to imperfect focusing of the eyepiece on the cross-hairs before focusing the objective. The eyepiece

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should be focused *with the eye normal,* the cross-hairs being illuminated by holding the note book page or other white object a few inches in front of the objective.

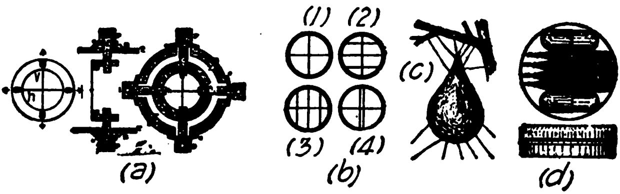

Fig. 17.

Cross-Hairs.—The cross-hairs are attached to a ring or reticule which is held by two pairs of capstan headed screws. The hairs usually consist of spider lines, although some makers use platinum wires for the purpose. To remove the reticule the eyepiece is taken out, one pair of screws is removed and a sharpened stick is inserted in a screw hole. The best spider lines are obtained from the spider's egg nest.

In Fig 17, (a) shows the usual arrangement of the cross-hair ring and the method of attaching the hairs; (b) shows the number and positions of hairs used, (1) being the most common, (2) the form for stadia work with the transit and also for estimating the lengths of sights with the level, (3) a form used by some makers with the level, and (4) a style found in English levels; (c) shows the egg pod or case of the large brown spider (about half size) which yields the best lines for engineering instruments; (d) illustrates a convenient vest pocket outfit for replacing cross-hairs in the field, consisting of a supply of spider lines and some adhesive paper (bank note repair paper) each in a capsule or tin tube, and several sharpened sticks for stretching the hairs. Cross-hairs stretched in this manner may last indefinitely, or they may be fastened on permanently with shellac at the first opportunity.

THE BUBBLE VIAL.

Principle.—The spirit level consists of a sealed glass tube nearly filled with ether or other liquid, and bent or ground so that the action of gravity on the liquid may indi-

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cate a level line by means of the bubble. The delicacy of the buble depends upon the radius of the curvature in a vertical plane, the greater the radius the more delicate the level. Thus, for example, a perfectly straight tube could not be used as a level.

Curvature of Bubble Vials.—Good bubble vials are now made by grinding or polishing the interior surface of a selected glass tube by revolution, as indicated in exaggerated form at (a) Fig. 18. As a general rule, only one side of the vial is actually used, it being customary to encase it in

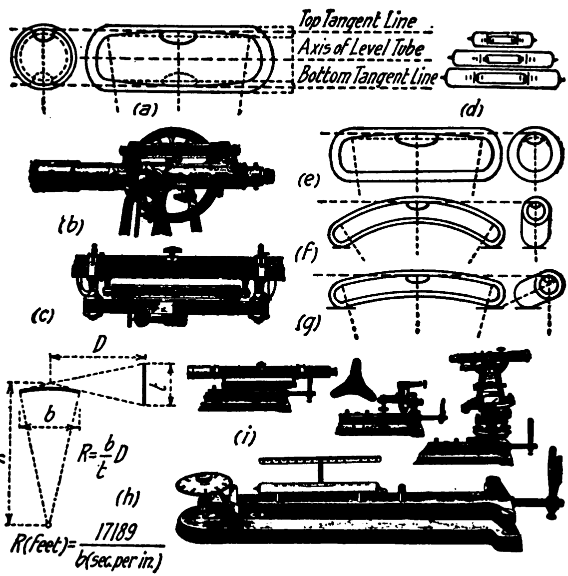

Fig. 18.

a brass tube having a slot or race on one side. However, both sides of the vial may be utilized, as in (b) and (c), Fig. 18, which show the *reversion level* adapted to the transit and wye level, respectively. Bubble vials of several sizes are shown in (d), Fig. 18. It was formerly customary to grind out only a portion of the upper side of the glass tube, as shown at (e). The cheap vial, consisting merely of a bent tube, used mostly in carpenters' and masons' levels, is

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shown at (f); and a method of increasing the precision of the bent tube by tilting it is indicated at (g), Fig. 18.

Delicacy.—The delicacy of the bubble vial is designated either by the radius, usually in feet, or by the central angle in seconds corresponding to one division or one inch of the bubble scale. Two methods are employed to determine the delicacy of level vials, (1) by the optical method, as at (h), Fig. 18, where the radius is calculated from an observed target movement at a given distance for an observed bubble movement, the two triangles being similar; and (2) by the level tester, as at (i), by means of which the angular movement is read from the micrometer head for a given movement of the bubble. The engineer usually employs the radial designation, while the maker expresses the delicacy in angular units. As shown at (h) and (i), Fig. 18, the radius in feet is equal to 17,189 divided by seconds per inch of bubble.

Bubble Line.—The relations of the bubble to the other parts of the instrument are best understood by representing the vial by a line. This line may be either the axis of the surface of revolution in (a), Fig. 18, or to provide for either of the three forms of vial shown, it may be taken as the tangent line at the middle or top point. This tangent line will be meant hereafter in referring to the bubble line.

LEVELING RODS.

Types.—There are two classes or types of leveling rods; (1) *target* rods, having a sliding target which is brought into the line of sight by signals from the leveler; and (2) *self-reading* or speaking rods which are read directly by the leveler.

In Fig. 19, (a) is the Philadelphia rod; (b) the New York rod; and (c) the Boston rod. The first is either a target or self-reading rod; the second is a target rod, but may be read from the instrument when the rod is "short"; the Boston rod is strictly a target rod. The Philadelphia rod is perhaps the favorite for most purposes, and the Boston rod is used least. A folding self-reading rod is shown at (d), Fig. 19; (e) is a woven pocket device which may be tacked to a strip of wood and used as a leveling rod; (f) is a railroad contouring rod with an adjustable base; (g) is a plain rod graduated to feet, for use with the water level.

Targets.—The targets shown on the Philadelphia and New York rods, (a) and (b), Fig. 19, are called quadrant targets. That on the Boston rod, (c), is a modified form of

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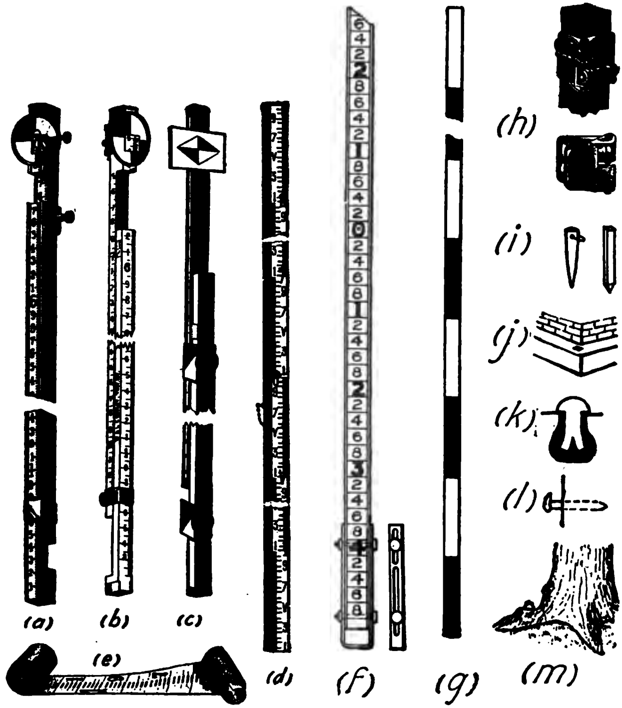

Fig. 19.

the diamond target. A special form, called the corner target, is bent to fit two sides of the rod to assist in plumbing it, and another target has two parallel planes for the same purpose. A detachable rod level is shown at (h). The target on rod (b), with the zero of the vernier 0.09 foot below the center of the target, frequently causes blunders.

USE OF THE LEVEL.

Use.—The engineers' level is used: (1) to determine differences of elevation; (2) to make profile surveys; (3) to locate contours; (4) to establish grade lines; (5) to cross section; (6) to run lines.

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Differential Leveling.—Differential leveling consists of finding the difference of elevation between two or more points. In the simplest case the difference of elevation between two points may be found from a single setting of the level, the leveling rod being used to determine the vertical distance from the plane of the instrument to each of the two points, and the difference between the rod readings taken. When the distance between the two points is too great, either vertically or horizontally, or both, to admit of this simple process, two or more settings of the level are taken so as to secure a connected series of rod readings, the algebraic sum of which gives the desired difference of elevation. This difference may be expressed either by the numerical result of the algebraic sum of the rod readings, or by assuming an elevation for the beginning point and calculating the elevation of the closing point by means of the observed rod readings.

A *back sight* is a rod reading taken to determine *the height of the instrument*. A *fore sight* is a rod reading taken to determine *the height of a point*. A *bench mark* is a point selected or established for *permanent reference* in leveling operations. A *turning point* is a *temporary* reference point used in moving the instrument ahead to a new setting. The same point is often both a turning point and bench mark. The *datum* is the plane or surface of reference from which the elevations are reckoned; it may be sea level, or an arbitrary local datum. A *level* line is a line parallel to the surface of a smooth body of water. A *horizontal* line is tangent to a level line at any point. The curvature varies as the square of the distance from the point of tangency, and is 0.001 foot in 204 feet, or 8 inches in one mile.

In Fig. 19, (i) shows a metal and also a wooden peg commonly used for turning points. Several forms of bench marks are shown in Fig. 19; (j) is a mark on the corner of a stone water-table; (k) a rivet leaded into a hole drilled in a stone slab; (l) a railroad spike driven into a wooden post or telegraph pole; (m) a projection cut on the root of a tree, preferably with a spike driven vertically into the top of the bench, and usually with a blaze above marked "*B. M. No.*—." All bench marks and also turning points should be clearly described in the notes.

Fig. 19a shows the essential details of differential leveling. In practice the calculations are made mentally.

Two chief essentials in correct differential leveling are: (1) that *the bubble be in exactly the same position* (usu-

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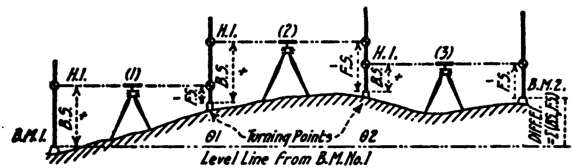
ally the middle) *on both back and fore sight;* and (2) that *the length of back sight and fore sight*, horizontally, *shall be balanced.* It is seen at (e), Fig. 16, that with the bubble always in the middle, the line of collimation generates a *horizontal plane* when in perfect adjustment, but a *cone* with *axis vertical* when out of adjustment; so that in taking equal distances in the opposite directions, the *base* of the *cone* is used, this base being parallel to the true colli-



Sta.	B.S.	H.I.	F.S.	Elev.	Dist.	Calculations	Description of B.M.s and θ's.
B.M.1				100.00	B.S. F.S.	100.00 B.M.1 +4.42 B.S.	City Datum B.M. Bolt, N.W. cor. water table, 1st Nat. Bank Bldg
T(1)	+4.42	104.42			340	104.42 T1 −1.16 F.S.	
θ 1			−1.16	103.26	= 340	103.26 θ 1 +4.37 B.S.	Peg, N.E. cor. J. Green's lot.
T(2)	+4.37	107.63			270	107.63 T 2 −3.55 F.S.	
θ 2			−3.55	104.08	300	104.08 θ 2 +1.91 B.S.	Sidewalk, E. gate post, J. Doe.
T(3)	+1.91	105.99			330	105.99 T 3 −2.40 F.S.	
B.M.2			−2.40	103.59	= 300	103.59 B.M.2	N.W. bolt, (nicked) water plug S.E. cor. High and East Sts.
	+10.70 − 7.11 + 3.59		−7.11	100.00 + 3.59	940 = 940		(Balanced B.S. and F.S. Dist. Checked Elev. by Σ(B.S., F.S.))

Fig. 19a.—Details of Differential Leveling.

mation plane. In the best leveling practice *the instrument is adjusted as perfectly as possible* and then used so that *the residual errors balance each other.*

The three common styles of leveling rods may be read to 0.001 foot by vernier or by estimation on a scale to 0.005 foot. However, for most kinds of leveling, it is an absurd refinement to read the rod closer than 0.01 foot, especially with the usual maximum length of sight of 350 to 400 feet, and with the more or less sluggish bubbles supplied in the general run of leveling instruments. Furthermore, the horizontal hair usually covers 0.01 foot or so of the target at the maximum length of sight, that is, the target can move that amount without being noticed by the observer.

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Profile Leveling.—Profile leveling consists of finding the relative elevations of a series of representative points along a surveyed line, for the purpose of constructing a profile or vertical section. The skeleton of profile leveling, that is, the precise bench marks and turning points with the successive heights of instrument, is identical with differential leveling, already described. Having determined the height of instrument by taking a back sight on a bench mark of known or assumed elevation, rod readings are taken at proper intervals along the measured and staked line. These readings are fore sights, but they are usually termed *intermediate sights* to distinguish them from the more precise rod readings taken on turning points and bench marks. On railroad surveys intermediate sights are taken usually to the nearest 0.1 foot on the ground; but in other cases, such as tile and sewer surveys, intermediates are often read to the nearest 0.01 foot on small pegs driven beside the station stakes flush with the surface of the ground. In railroad work, the benches, turning points, and intermediates of special importance are commonly read to 0.01 foot, although some engineers persist in the questionable practice of taking the nearest 0.001. In drainage surveys the nearest 0.01 foot is usually taken on bench marks, although more carefully than on the intermediate peg points, and the nearest 0.1 foot is read on ground points.

The errors of profile leveling are balanced on turning points by equal back and fore sights, as in differential leveling. If the instrument is seriously out of adjustment, an error is made in the case of odd bench marks with unbalanced sights, and also on all intermediate sights. However, the error is usually unimportant when ground readings are taken to the nearest 0.1 foot. In important leveling, such as canal and drainage work, it is customary to run a line of check levels to prove benches, before construction begins.

The profile is plotted to an exaggerated scale vertically on a special paper, called profile paper. Three kinds, known as plates A, B and C, are in general use. The most common is plate A, which is ruled in $\frac{1}{4}$-inch squares with a further subdivision to $\frac{1}{20}$ inch vertically. In railroad profiles the scales most used are 400 feet to the inch horizontally and 20 feet vertically. A still greater exaggeration is generally used in drainage profiles.

Reciprocal Leveling.—The application of differential leveling to the determination of the difference of elevation between two bench marks separated by a wide river or gorge

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is termed reciprocal leveling. A setting of the level is taken on each side of the river, and the mean of the two results is taken. The necessary unbalancing of distances in one setting is balanced up in the other. Each back or fore sight should be the mean of a series of careful observations. In best practice, simultaneous readings are taken with two levels.

Contour Leveling.—Contour leveling is an application of the methods of profile leveling to the location of contour lines, that is, lines having the same elevation. Two methods are employed: either (1) actually establishing points on the adopted contour planes on the ground and then locating these points; or (2) taking random elevations at representative points and interpolating the contour lines from the plotted data. The latter is the more common. The chief purpose of contour leveling is to make a contour map, and the process is essentially a part of topographic surveying, where it will be more fully considered.

Grade Lines.—The establishment of grade lines is usually the concluding part of profile leveling. After making the profile, the grade line is established by stretching a fine thread through the ruling points, taking into account the controlling conditions, such as maximum gradient or earthwork quantities on a railroad profile, the carrying capacity or the scour in the case of a ditch, etc. After laying the grade line on the profile, notes are made of the data, and the actual grade line is established. Two methods are used: (1) the height of instrument is determined as usual, and stakes are driven at measured intervals with their tops to match calculated rod readings; and (2) a limited number of ruling points are established by the first method or otherwise, and the remaining stakes are "shot in" by constructing a line parallel to the ruling line used. The latter is more rapid, since a constant rod reading is used; however, the method is unreliable unless the foresight be checked frequently on a fixed target.

Cross-Sectioning.—Cross-sectioning consists of staking out the limits of the transverse section of an excavation or embankment for the purpose of construction, and usually includes the collection of data for the calculation of the quantities. This may be done either with the engineers' level, rod and tape line, or with special rods called cross-section rods. The notes are taken as rectangular coordinates, usually with reference to the center of the finished

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roadbed. The slope stakes are set where the side slope lines pierce the surface of the ground.

Running Lines.—Lines are sometimes run with the engineers' level, provision being made in most good levels for the attachment of a plumb bob. A line may be prolonged by sighting in two points ahead. A clamp and tangent movement are necessary. Some builders' levels have a needle and also a roughly divided horizontal circle for use in staking out buildings.

Practical Hints.—The following practical suggestions apply more or less directly to all kinds of leveling, and also in a general sense to transit work.

Speed.—Cultivate the habit of briskness in all the details of the work. While undue haste lowers the standard of the results, an effort should be made to gain speed steadily without sacrificing precision. Gain time for the more important details by moving rapidly from point to point. On rapid surveys both leveler and rodman often move in a trot. Neither rodman nor leveler should delay the other needlessly.

Care of Instruments.—Do not carry the level on the shoulder in climbing fences. Clamp the telescope slightly when hanging down Keep the tripod legs at the proper tightness, and avoid looseness in the tripod shoes. Avoid undue exposure to the elements, and guard the level from injury. Do not leave the instrument standing on the tripod in a room over night.

Setting Up—In choosing a place to set the level up, consider visibility and elevation of back point and probable fore sight. Set up with plates about level. On side-hill ground place one leg up hill. In general, place two tripod shoes parallel to the general line of the levels.

Leveling Up.—A pair of foot screws should be shifted to the general direction of the back or fore sight before leveling up. Set the foot screws up just to a snug bearing and no tighter. If either pair of screws binds, loosen the other pair a little The bubble moves with the left thumb. Level up more precisely in the direction of the sight than transverse to it, but do not neglect the latter. Inspect the bubble squarely to avoid parallax, and also to prevent such blunders as reading the bubble five spaces off center.

Observations.—Adjust the eyepiece for parallax with the eye unstrained. It is much easier on the eye to observe with both eyes open. Read at the intersection of the crosshairs, since the horizontal hair may be inclined. Set the

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target approximately, check the bubble, and repeat the process several times before approving the sight. Be certain that the bubble is exactly in the middle at the instant of approving the target. If the level has horizontal stadia lines, beware of reading the wrong hair (the reticule may be rotated one-quarter so as to have the extra hairs vertical, or a filament may be attached to the middle horizontal hair to assist in identifying it). Avoid disturbance of the tripod by stepping about the instrument. Assist the rodman in plumbing the rod. Let signals be perfectly definite both as to direction and amount, using the left hand for "up" and the right for "down," or vice versa.

The leveler can work much more intelligently if he knows the space covered on the rod by one division of the bubble scale at the maximum length of sight, and also the space on the rod hidden by the cross hair.

Adjustments.—Keep the instrument in good adjustment and then use it as though it were out of adjustment.

Balancing Sights.—Balance the length of back sight and fore sight, and record the approximate distances. The distances in the two directions may be made equal roughly by equality of focus, but it is better on careful work to pace the distances or determine them by means of the stadia lines in the level. If necessary to unbalance the sights, they should be balanced up at the first opportunity, and in general they should be in balance when closing on important benches. When leveling up or down steep slopes, follow a zigzag course to avoid short sights. Take no sights longer than 350 or 400 feet.

Leveling Rod.—The rod should be carefully plumbed, to accomplish which the rodman should stand squarely behind the rod and support it symmetrically between the tips of the extended fingers of the two hands. In precise work wave the rod to and fro towards the observer and take the minimum reading of the target. With "short" rods avoid the somewhat common blunder of 0.09 foot when the vernier slot is below the center of the target. With "long" rods, see that the target has not slipped from its true setting before reading the rod. Read the rod at least twice, and avoid blunders of 1 foot, 0.1 foot, etc. *Careless rodmen sometimes invert the rod.* Each rod reading on turning points and bench marks should, when practicable, be read independently by both rodman and leveler.

Bench Marks and Turning Points.—Wooden pegs or other substantial points should be used to turn the instrument

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on. Select bench marks with reference to ease of identification, the balancing of sights, freedom from disturbance, etc. As a rule, each bench mark should be used as a turning point so that the final closure of the circuit may prove the bench. Mark the benches and turning points and describe them in the notes so plainly that a stranger may readily find them. Green rodmen sometimes hammer at turning point pegs with the rod. When leveling near a still body of water, its surface may be used to save time and check the work.

Record and Calculations.—Describe bench marks and turning points clearly. It is good practice to apply algebraic signs to the back and fore sight rod readings. The elevations should be calculated as fast as the rod readings are taken, and calculations on turning points should be made independently by leveler and rodman, and results compared at each point. The rodman may keep turning point notes in the form of a single column. The calculations should be further verified by adding up the columns of back sights and fore sights for each circuit, or page, or day's work, and the algebraic sum of the two compared with the difference between the initial and last calculated elevation.

Error of Closure.—A circuit of levels run with a good level by careful men, observing all the foregoing precautions, should check within 0.05 foot into the square root of the length of the circuit in miles (equivalent to 0.007 foot into the square root of the length of the circuit in 100-foot stations). In closing a circuit, the error should be carefully determined, as above indicated, and the value of the coefficient of precision found. (See discussion of errors of leveling and precision diagrams in Chapter IX, Errors of Surveying.)

ADJUSTMENT OF THE WYE LEVEL.

Elementary Lines.—The principal elementary lines of the wye level, as shown in Fig. 16, are: (1) the line of collimation; (2) the bubble line; (3) the vertical axis. For the purpose of adjustment there should be added to these: (4) the axis of the rings; (5) the bottom element of the rings. The following relations should exist between these lines; (a) the line of collimation and bubble line should be parallel; (b) the bubble line should be perpendicular to the vertical axis. The first of these relations involves two steps, viz., (1) to make the bubble line parallel to the bot-

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tom element of the rings, and (2) to make the line of collimation coincide with the axis of the rings. The other relation involves the wye adjustment, and is similar to the plate level adjustment described in the chapter on the compass.

Bubble.—*To make the bubble line parallel to the bottom element of the rings.*—Two steps are involved, (a) to place the bubble line *in the same plane* with the bottom element, and (b) to make the two lines *parallel.*

Azimuth Screws.—To make the bubble line in the same plane with the bottom element of the rings.—Clamp the level over a pair of foot screws, loosen the wye clips, and level up; rotate the telescope through a small angle, and if the bubble moves away from the middle, bring it back by means of the *azimuth* adjusting screws. Test by rotating in the opposite direction. Leave the screws snug.

Altitude Screws.—To make the bubble line and the bottom element of the rings parallel.—Make the element level with the foot screws and bring the bubble to the middle by means of the *altitude* adjusting screws. The element is made level by the method of reversions as follows: With the level clamped over a pair of foot screws, as above, lift the clips and level up precisely; cautiously lift the telescope out of the wyes, turn it end for end, and *very gently* replace it in the wyes; if the bubble moves, bring it half way back by means of the *foot screws.* Before disturbing adjusting screws make several reversals, and conclude the adjustment with screws snug. This end for end reversal is similar to that made with the carpenter's level, the straight edge of the level corresponding to the element of the rings. The lines involved are shown in Fig. 16.

Line of Collimation.—*To make the line of collimation coincide with the axis of the rings.*—Loosen clips, sight on a point, say a nail head or the level target, more distant than the longest sight used in leveling; rotate the telescope half way and note the movement of the hair, if any. The line of collimation generates a cone, the axis of which is that of the rings, and the apex of which is at the optical center of the objective. Correct one-half the observed error by means of the capstan headed screws which hold the cross-hair ring. Gradually perfect the adjustment until the intersection of the cross-hairs remains fixed on the same point when reversed by rotation with reference to either hair. The adjustment should be concluded with the screws at a snug bearing.

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After collimating the instrument for a long distance, the adjustment should be checked for a short distance, say 50 or 100 feet, so as to test the motion of the optical center of the objective.

Rings.—*The theory of the wye level demands perfect equality of the rings*, that is, the parallelism of the axis and element, as in (c), Fig 16. Should the rings be *unequal*, either from poor workmanship or uneven wear in service, they form a *cone* instead of a *cylinder*, and the axis is not parallel to the element, as in (d), Fig. 16. Under the latter conditions, the principle of the wye level fails, and an independent test is demanded. This is known as the two-peg test, the details of which are shown in (e) and (f), Fig. 16, and described in the adjustments of the dumpy level. If, after making the wye level adjustments above described, the two-peg test shows that the line of collimation and bubble line are not parallel, the rings are probably unequal and the instrument should thereafter be adjusted as a dumpy level. However, hasty conclusions should be guarded against.

In case the instrument has a reversion level, shown at (c), Fig. 18, the equality of the rings may be tested by first adjusting the top tangent line of the bubble vial parallel to the bottom element of the rings, and then after rotating the telescope half way round in the wyes, compare the bottom (now above) tangent line of the vial with the top (now below) element of the rings, all by the end for end reversion. However, the exact parallelism of the top and bottom tangent lines of the reversion level should first be proven by the two-peg method.

Wyes.—*To make bubble line perpendicular to the vertical axis.—Make the vertical axis vertical and bring the bubble to the middle by means of the wye nuts.* The vertical axis is made vertical by reversion thus: With clips pinned, level up, reverse over the same pair of screws, and bring the bubble half way back with the foot screws. When adjusted, the bubble will remain in the middle during a complete revolution. This adjustment is identical in principle with the plate level adjustment of the compass and transit, illustrated in (a), Fig. 13. The wye adjustment should follow the adjustment of the bubble line parallel to the element of the rings. The wye adjustment is a convenience, not a necessity.

Centering the Eyepiece.—After collimating the level, the cross-hairs should appear in the center of the field.

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The eyepiece is centered by moving its ring held by four screws. This adjustment is desirable, but not essential.

ADJUSTMENT OF THE DUMPY LEVEL.

Elementary Lines.—The principal elementary lines of the dumpy level are identical with those of the wye level (1) the line of collimation; (2) the bubble line; (3) the vertical axis. As in the wye level, the bubble line should be (1) perpendicular to the vertical axis, and (2) parallel to the line of collimation. However, owing to the difference in the construction of the two types of instrument, the auxiliary elementary lines are not recognized in the dumpy level. The transit with its attached level is identical in principle with the dumpy level.

Bubble.—*To make the bubble line perpendicular to the vertical axis.—Make the vertical axis vertical by the method of reversions, and adjust the bubble to the middle.* This adjustment is identical in principle with the plate level adjustment, shown in (a), Fig. 13. The bubble should remain in the middle through a complete revolution.

Line of Collimation.—*To make the line of collimation parallel to the bubble line.—Construct a level line, and adjust the cross-hairs to agree with it.* The level line is determined either by using the surface of a pond of water, or by driving two pegs at equal distances in opposite directions from the instrument, and taking careful rod readings on them with the bubble precisely in the middle, as shown at (e), Fig. 16. For simplicity, the two pegs may be driven to the same level, or two spikes may be driven at the same level in the sides of two fence posts, say 400 feet apart. Otherwise, determine the precise difference of elevation, as indicated in (e), Fig. 16. Then set the level almost over one of the pegs, level up, and as in the first method of (f), Fig. 16, set the target of the leveling rod at the line of collimation, as indicated by the center of the object glass or eyepiece (this can be done more precisely than most levels will set the target at 400 feet distance); now with the rod on the other peg, sight at the target (shifted to allow for the difference if the two pegs are not on the same level); adjust the cross-hair to the level line so constructed. If preferred, the second method shown in (f), Fig. 16, may be used; the level is set back of one peg, rod readings are taken on both pegs, allowance made for the difference in level of the two pegs, if any, the inclination of the line of

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collimation determined, correction made for the small triangle from the level to the first peg, and finally the level line constructed by means of the calculated rod readings. The second method is simplified and made practically equivalent to the first by setting the level at minimum focusing distance from the first peg. The small corrective triangle is thus practically eliminated. Strictly speaking the rod readings should be corrected for the earth's curvature (0.001 foot in about 200 feet, or say 0.004 foot in 400 feet distance). However, the effect of curvature is reduced by atmospheric refraction; and with errors of observation, sluggishness of bubble, etc., to contend with, the curvature correction should be ignored, especially when the rod is read to the nearest 0.01 foot.

(The foregoing process is known as the "two-peg adjustment." Although exceedingly simple, this adjustment is commonly regarded as a "bug-bear" by many American engineers. But for it, the dumpy level would have the extended use in this country which it merits. It is said that "the wye level is easy to adjust and usually needs adjustment." Many good levelers employ the "two-peg test" to prove the wye level adjustments. Time may be saved by establishing an adjusting base. The adjustments of a good dumpy level are very stable.)

Uprights.—In some dumpy levels the uprights which connect the telescope with the level bar are adjustable, similar to the wyes of the wye level. This adjustment is designed to bring the bubble line perpendicular to the vertical axis in case the bubble is first adjusted parallel to the line of collimation. However, the best order is that already described, viz., first adjust the bubble line perpendicular to the vertical axis, and then the line of collimation parallel to the bubble line, in which case the adjustable uprights are unnecessary.

PROBLEMS WITH THE LEVEL.

PROBLEM C1. DIFFERENTIAL LEVELING WITH THE HAND LEVEL (OR WATER LEVEL).

(a) *Equipment.*—Hand level (or water level), rod graduated to feet.

(b) *Problem.*—Run an assigned level circuit with the hand level (or water level), observing the nearest 0.1 foot by estimation, and closing back on the starting point.

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(c) *Methods.*—(1) Determine the correct position of the bubble of the hand level by sighting along a water table, or sill course of a building, or by the principles of the two-peg test. (If the water level is used, fill the tube so as to have a good exposure of the colored water in the glass uprights.) (2) Take sights of 100 feet or so (paced), estimating the rod reading to the nearest 0.1 foot; balance back and fore sights; assume the elevation of the starting point, and keep the notes in a single column by addition and subtraction, as in the 7th column, Fig. 19a. (3) Check back on the first point. Determine coefficient of precision. (The error of closure in feet should not exceed 0.5 $\sqrt{\text{distance in miles}}$.)

PROBLEM C2. DIFFERENTIAL LEVELING WITH ENGINEERS' LEVEL (OR TRANSIT WITH ATTACHED LEVEL).

(a) *Equipment.*—Engineers' level (or transit with attached level), leveling rod, hatchet, pegs, spikes.

(b) *Problem.*—Run the assigned level circuit, observing the nearest 0.01 foot, and closing back on the initial point.

(c) *Methods.*—Follow the practical suggestions given at the conclusion of the "Use of the Level," giving special attention to the following points: (1) eliminate parallax of the eyepiece; (2) balance back and fore sight distances; (3) have the bubble precisely in the middle at the instant of sighting; (4) both rodman and leveler read each rod and also make the calculations independently; (5) calculate elevations as rapidly as rod readings are obtained; (6) plumb the rod; (7) avoid blunders; (8) determine coefficient of precision; (9) no sights longer than 350 or 400 feet. Follow the first form shown to begin with,—the other after several circuits have been run.

PROBLEM C3. PROFILE LEVELING FOR A DRAIN.

(a) *Equipment.*—Engineers' leveling instrument, leveling rod, 100-foot steel tape, stakes, pegs, axe.

(b) *Problem.*—Make a survey, plat and profile, with estimate of cuts and quantities for a drain under assigned conditions.

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(c) *Methods.*—(1) Examine the ground, determine the head and outlet of the drain, and select the general route. (2) Stake out the line, set stakes every 50 feet, or oftener if required to get a good profile, and drive a ground peg flush, say 2 feet to the right (or left) of each stake; record data for mapping the line. (3) Starting with the assigned datum or bench mark, run levels over the line of the proposed drain, observing the nearest 0.01 foot both on turning points and ground pegs, the former somewhat more carefully; take rough ground levels, as required, to the nearest 0.1 foot; locate and determine the depth of intersecting drains or pipe lines, or other objects which may influence the grade line of the drain, and secure full data for placing the same on the profile; observe due care with the back and

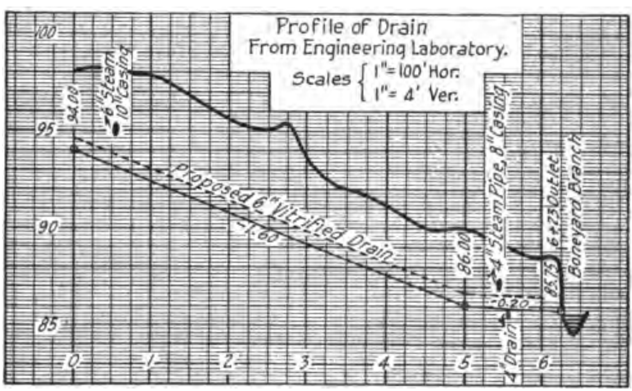

Fig. 19b.

fore sights, as in differential leveling, and conclude the leveling work with a line of check levels back to the initial bench mark; a permanent bench mark should be established at each end of the drain, and if the length is considerable, at one or more intermediate points as well. (4) Make plat and profile of the drain line; lay the grade line, taking into account all ruling points; calculate the cuts, both to the nearest 0.01 foot, and also to the nearest ¼-inch; mark the latter on the stakes for the information of the ditcher, using waterproof keel and plain numerals; make estimate of the quantity of drain pipe, and of the cost of the job. Follow the form and the profile in Fig. 19b.

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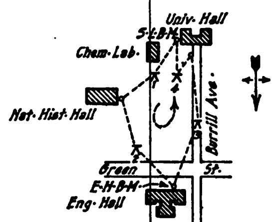
Leveler, J. Doe; Rodman, R. Roe.

LEVEL CIRCUIT, SEA LEVEL B.M. TO ENGG HALL B.M. I AND RETURN

Sta.	B.S.	H.I.	F.S.	Elev.	Sights B.S.	Sights F.S.	
							Oct. 20, '14. (2 Hours) Clear & Warm.
					B.S.	F.S.	B.&B. Dumpy Level, N.Y. Rod, Locker 15.
S.L.B.M.				720.00	180		Bolt, E. End, S. Coping, E. Entrance,
K (1)	+2.60	722.60				180	University Hall.
O 1			−7.56	715.04	200		W. End, N. Coping, W. Entrance,
K (2)	+2.00	717.04				210	Natural History Hall.
E.H.B.M.I			−3.42	713.62	280		Bolt, W. End, 1st Step, S. Entrance,
K (3)	+5.94	719.56				270	Engineering Hall.
O 2			−0.30	719.26	80		Cement walk, 20' N.E.; N. Entrance,
K (4)	+3.87	723.13				80	University Hall.
			−3.14	719.99			Bolt, starting point above described.
S.L.B.M.	+14.41				740	740	
			−14.42		L = 14.8		Permissable error = $0.007\sqrt{L} = 0.027$ Ft.
			+14.41	720.00			Actual error of Closure = 0.01 Ft.
			− 0.01	− 0.01			
			(Check)				



H. H. Sherwin, Leveler. C. E. Boyer, Rodman.

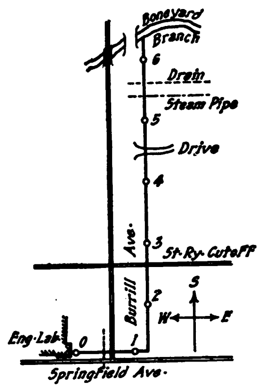
LEVELS FROM REYNOLDS TO LAFAYETTE, IND.

Sta.	B.S. +	H.I.	F.S. −	Elev.	Sights B.S.	Sights F.S.	
							Nov. 29, '14. Cloudy, Cool. B.&B. Dumpy Level.
B.M. 1	4.20	694.02	—	689.82	240		Penna. R.R. B.M., M.P. 27, driven rail (Levels
O 1	5.95	696.30	3.67	690.35	165	240	Peg. direct from tide gage at
O 2	5.70	698.18	3.82	692.48	240	240	" Sandy Hook, N.Y.)
O 3	4.93	698.74	4.37	693.81	240	165	" (South along Monon R.R.)
O 4	4.23	699.75	3.22	695.52	240	240	"
O 5	4.24	699.04	4.95	694.80	240	240	"
O 6	4.73	699.33	4.44	694.60	240	240	"
O 7	4.44	699.61	4.16	695.17	300	240	" opposite Catholic Church, Reynolds.
O 8	5.18	699.39	5.40	694.21	300	300	"
O 9	4.54	697.86	6.07	693.32	300	300	" E. side tr'k, between tel.' poles 2289–90.
O 10	4.81	698.12	4.55	693.31	300	300	"
O 11	4.82	698.86	4.08	694.04	300	300	" E. side tr'k, between tel." poles 2296–97.
B.M. 2.			3.76	695.10		200	B.M. on oak, tel.' pole 2299, 50' E. track.
O 12	4.10	698.00	4.96	693.90	300	300	Peg at tel.' pole 2300.
O 13	3.65	698.13	5.52	692.48	360	300	" " " " 2305.
O 14	2.71	692.92	5.92	690.21	360	360	"
O 15	3.21	690.03	6.10	686.82	360	360	" " " " 2313, E. side track.
B.M. 3			3.15	686.88		150	N.E. cor. parapet wall, Bridge No. 97.4
O 16	4.65	690.83	3.85	686.18	360	360	Peg.
B.M. 4.			5.29	685.54		100	Monon R.R. B.M. {Oak, 20' S. tel.' pole 2320
O 17	4.51	691.21	4.13	686.70	360	360	Peg {80' E. trk., El. = 688.12
O 18	5.16	691.37	5.00	686.21	300	360	"
O 19			5.13	686.24		300	" at tel.' pole 2330.
	85.76		89.34	689.82	= B.M. 1		
			85.76	−3.58			
			−3.58	Check			

Sta.	SURVEY FOR A DRAIN FROM Description
+36	Bed of stream.
+32	Suitable outlet for drain, 11' W. of W. face of stone arch bridge.
+23	Break of N. bank, Boneyard Branch.
6	S. edge of 7' drive.
+60.5	Crosses drain from Conservatory.
5+47.3	Steam pipe line to Conservatory.
4+52	Center 7' drive to Conservatory.
+69.6	Rails of main track, U.&C. St. Ry.
+64.8	(Cutoff through University Grounds.)
+58.6	Rails of side track, Urbana and
+53.8	Champaign Electric Ry.
2	Drain is 2' L. (E.) of stake.
1+21.5	Turns S. in W. parking, Burrill Ave.
+73	12" Ash tree 6' to R.
+70	Cement walk E. side Burrill Ave.
+64	
+54.6	Crosses Military Hall Steam Pipe Line.
0	A point 3' W. & 2' S. of N.W. Cor. of Eng. Lab. Line runs thence W. parallel to & 9' S. of S. line of Springfield Ave. Stakes are set 2' to right of proposed trench for drain, with leveling pegs flush with ground beside stakes.

ENGINEERING LABORATORY.

Head Chainman, J. Doe;
Rear Chainman, R. Roe.
Apr. 26, 1914. (2 Hours) Cloudy & Cool.
100 Ft. Steel Tape, No. 275, Locker No. 35.



Leveler, R. Roe. Rodman, J. Doe.

LEVEL NOTES FOR A DRAIN FROM ENGINEERING LABORATORY.

Sta.	B.S.	H.I.	F.S.	Elev.	Grade	Cut	
				100.00			Oct. 23, '14. (2 hrs) Clear and Cool.
⊼	+1.23	101.23					S. End stone sill, W. door, Eng. Lab. Station stakes are 2' R. proposed trench.
0			3.05	98.18	94.00	4.18	Peg driven flush with ground beside stake.
+50			3.22	98.01	93.20	4.81	"
+54.6			3.2	98.0	93.13	4.9	Ground, 6" Steam Pipe, 10" casing, top 2.6 deep.
+66			3.23	98.00	92.94	5.06	Cement walk, E. side Burrill Ave.
1			3.38	97.85	92.40	5.45	Peg.
+21.5			3.72	97.51	92.06	5.45	Drain turns S. in W. parking Burrill Ave.
+50			4.65	96.58	91.60	4.98	Peg
2			5.71	95.52	90.80	4.72	"
+50			6.31	94.92	90.00	4.92	"
• +59			-5.85	95.38	89.86	5.52	(Turning Point) N. Rail, Main Track, U.&C. Ry.
⊼	+0.13	95.51					
3			2.08	93.43	89.20	4.23	Peg
+50			3.45	92.06	88.40	3.66	" Fauth Wye Level.
4			4.34	91.17	87.60	3.57	" Phila. Rod, Lkr. 20.
+50			5.50	90.01	86.80	3.21	"
5			5.52	89.99	• 86.00	3.99	"
+48.3			6.1	89.4	85.91	3.5	Ground, 4" Steam Pipe, 8" Casing, top 2.0 deep.
+50			6.26	89.25	85.90	3.35	Peg.
+60.5			6.4	89.1	85.88	3.2	Ground, 4" vitrified drain, top 3.4 deep.
6			7.11	88.40	85.80	2.60	Peg.
+23			7.0	88.5	85.75	2.7	Break of N. bank, Boneyard Branch.
+36			10.9	84.6			Bed of Stream, water 10" deep.

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PROBLEM C4. RAILROAD PROFILE LEVELING.

(a) *Equipment.*—Engineers' leveling instrument, leveling rod, 100-foot steel tape, stakes, axe.

(b) *Problem.*—Run levels over a short section of line staked out after the manner of railroad surveys, for the purpose of constructing a profile.

(c) *Methods.*—Follow the general process outlined in the preceding problem, taking rod readings to the nearest 0.01 foot on turning points and bench marks, and also on important profiling points, when consistent; but take ground rod readings only to the nearest 0.1 foot. In calculating elevations, preserve the same degree of exactness in the result as observed in the rod reading, that is, when the rod readings are taken to the nearest 0.1 foot, use only the nearest 0.1 foot in the height of instrument to determine the elevations. When a hub or station stake is to be used as a turning point, the notes should show the ground rod and elevation to the nearest 0.1 foot on the line preceding the precise turning point record. Bench marks should be selected with reference to their freedom from disturbance during construction, and they should be located not more than 1500 or 2000 feet apart along the line. Check levels by the same parties should not differ more than 0.05 foot into the square root of the length of circuit in miles. Back and fore sights should be balanced, and no sight longer than 350 or 400 feet should be taken. In order to secure a representative profile, ground rods should be taken not only at every station stake, but also at every important change of slope between station points. Pluses may be determined either by pacing, or when short, by means of the leveling rod. The rodman should keep a record of the turning points. The notes should be checked and the other safeguards taken, as outlined in the practical hints under the "Use of the Level." Bottoms of deep gullies may be taken by means of the hand level, or with the engineers' level used like the hand level; or a "long" rod of 17 feet or more may be obtained by holding the 12-foot rod 5 feet or more from the ground.

The profile is best plotted by having another person read off the data. The horizontal scale on railroad profiles is usually 400 feet to the inch and the vertical scale 20 feet to the inch. Gradients are expressed to the nearest 0.01 per

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(Profile Level Notes, Ground Elevations to 0·1 Foot.)

S·	+	π	−	R·	E·	
209		718·33		5·0	713·3	In Brown St· (Unimproved)
210				4·7	713·6	" "
211				3·9	714·4	" "
B·M·26	6·79	$\overset{9}{723\cdot87}$	1·25		717·08	Water Plug, N·bolt, N·W·Cor·, Brown-Curtis·
212				7·6	716·3	Ground, Brown St·
213				6·4	717·5	" "
214				5·9	718·0	" "
+50				4·0	719·9	" "
215				6·1	717·8	" "
216				8·0	715·9	" "
217				8·5	715·4	" "
218				10·3	713·6	" "
219				12·2	711·7	In Corn Field
⊙Stake	9·22	$\overset{6}{721\cdot64}$	11·45		712·42	⊙ Stake, Sta· 219·
220				8·6	713·0	Corn Field·
221				4·4	717·2	"
222				2·7	718·9	"
223				2·9	718·7	"
224				2·3	719·3	"
225				3·4	718·2	Timber Pasture·
+90				12·4	709·2	Gully·
226				11·2	710·4	
+35				6·0	715·6	Break of bank, Plum River·
B·M·27	2·04	$\overset{5}{713\cdot52}$	10·16		711·48	B·M·, root, 24"elm, 72'R·, Sta· 226+65·
+80				6·0	707·5	
	+18.05	718·33 −4·81	−22·86 +18·05 −4·81	Check		Column Headings: S = Station· + = Back Sight π = Height Inst· − = Fore Sight· R = Rod (Intermediate) E = Elevation·

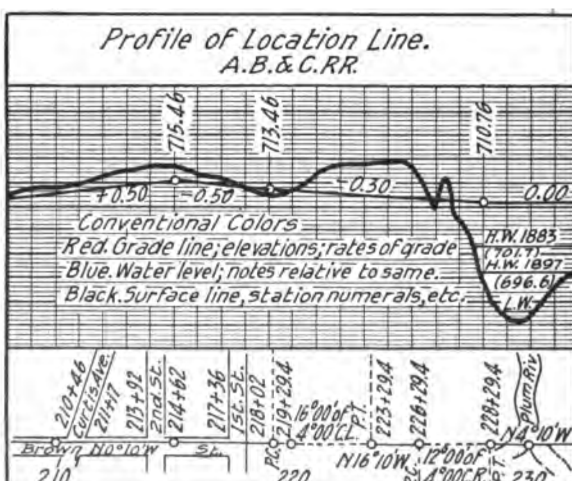

Fig. 19c.

cent. It is usual to give the alinement notes and prominent topography, as shown in Fig. 19c.

(The complete series of steps involved in railroad and similar leveling for location and construction purposes is: (1) setting the station stakes; (2) running the levels; (3) making the profile; (4) laying the grade line on profile; (5) calculating vertical curves; (6) cross-sectioning for earthwork; (7) calculating earthwork quantities; (8) setting grade stakes.)

PROBLEM C5. VERTICAL CURVE.

(a) *Equipment.*—Drafting instruments, profile paper.

(b) *Problem.*—Connect two grade lines by a parabolic curve, as assigned.

(c) *Methods.*—(1) Plot the given grade lines, station numbers, etc., on a sheet of profile paper. (2) Find the grade angle, i. e. the algebraic difference of the two rates of grade. (3) Determine the length of the vertical curve by dividing the grade angle by the assigned or adopted change of grade per station (notice the analogy to simple circular curves). (4) Calculate the apex correction. (5) Determine the corrections at the several stations or fractional stations (as assigned), and tabulate the stations and elevations. (6) Plot the vertical curve from the data so determined, as in Fig. 19d. (7) Also compute and plot the same curve by the method of chord gradients.

PROBLEM C6. ESTABLISHING A GRADE LINE.

(a) *Equipment.*—Leveling instrument, leveling rod, flag pole, 100-foot steel tape, stakes, axe.

(b) *Problem.*—Establish an assigned grade line, (1) by measured distances and calculate rod readings, and (2) by "shooting in" the same line, for comparison.

(c) *Methods.*—(1) Stake off the distance between ruling points, and drive stakes to the required grade, or if desirable, parallel to it, by dividing up the fall in proportion to the distance. (2) Set the level over one ruling point and determine the height from the point to the line of collimation by means of the leveling rod; set the flag pole behind the other ruling point and establish a target, consisting of a rubber band holding a strip of paper wrapped about the

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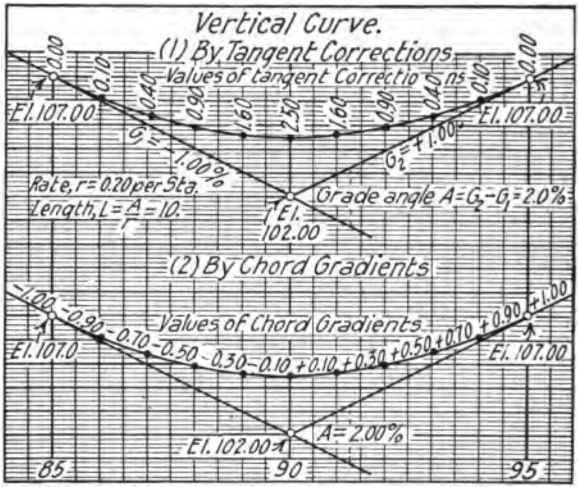

COMPARISON OF RESULTS						
Station.	Elevation of Grade Tangent.	By Tangent Corrections: Tangent Correction.	By Tangent Corrections: Curve Elevation.	By Chord Gradients. Chord Gradients. Diff.	By Chord Gradients. Chord Gradients. Gradient.	By Chord Gradients. Curve Elevation.
	Ft.	Ft.	Ft.	Per Cent.	Per Cent.	Ft.
84	108.00				(-1.00)	
85 (P.C.)	107.00	+0.00	107.00	+0.10	-0.90	107.00
86	106.00	+0.10	106.10	+0.20	-0.70	106.10
87	105.00	+0.40	105.40	+0.20	-0.50	105.40
88	104.00	+0.90	104.90	+0.20	-0.30	104.90
89	103.00	+1.60	104.60	+0.20	-0.10	104.60
90 (Apex)	102.00	+2.50	104.50	+0.20	+0.10	104.50
91	103.00	+1.60	104.60	+0.20	+0.30	104.60
92	104.00	+0.90	104.90	+0.20	+0.50	104.90
93	105.00	+0.40	105.40	+0.20	+0.70	105.40
94	106.00	+0.10	106.10	+0.20	+0.90	106.10
95 (P.T.)	107.00	+0.00	107.00	+0.10	(+1.00)	107.00
96	108.00			+2.00 = A		

Fig. 19d.

pole at a height equal to the rod reading; having thus constructed a line parallel to the desired grade line, direct the telescope on the fore sight target, and with the same rod reading, "shoot in" the same stakes. Make careful record of data and comparative results.

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PROBLEM C7. SETTING SLOPE STAKES.

(a) *Equipment.*—Leveling instrument, self-reading leveling rod, 50-foot metallic tape, stakes, axe, marking crayon. (Or, instead of levelling instrument and rod, use special cross-sectioning rods, if assigned.)

(b) *Problem.*—Set slope stakes for the construction of a railroad, canal, etc., as assigned.

(c) *Methods.*—(Follow the methods described in Chapter VIII, "Railroad Surverying," under the head of "Cross-Sectioning.")

PROBLEM C8. CALCULATION OF QUANTITIES.

(a) *Equipment.*—(No instrumental equipment unless planimeter is assigned.)

(b) *Problem.*—Compute the quantity of earthwork for an assigned set of cross-section notes.

(c) *Methods.*—(1) Transcribe the notes and carefully verify the copy. (2) Calculate the sectional area for each station and intermediate in the notes, and prove the results. (3) Calculate the volume by the "average end area" method, results to nearest 0.1 cubic yard, and check the same. (4) If so instructed, plot the notes on cross-section paper and determine the areas by means of the planimeter as a check. Record the results.

PROBLEM C9. STAKING OUT A BORROW PIT.

(a) *Equipment.*—Engineers' level or transit with attached bubble, leveling rod tape, stakes, axe.

(b) *Problem.*—Stake out a borrow pit and take notes required for calculation of earthwork quantities.

(c) *Methods.*—(1) Select a base line, preferably outside the limits of the proposed borrow pit, set substantial station stakes say 50 or 100 feet apart along this base; designate these stakes *A*, *B*, *C*, etc. (2) Establish auxiliary reference lines by erecting perpendiculars to the base line at the several stakes, driving temporary stakes for pegs at suitable distances on these lines. (3) Establish a permanent bench mark and run levels, as in profile leveling, along lines starting at *A*, *B*, *C*, etc., noting elevations both at pegs and at marked intermediate changes of slope. (4) In

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case actual construction is undertaken, repeat the levels along the same auxiliary lines from time to time and calculate the quantities. (5) Record complete data.

LEVELS FOR PROFILE AND QUANTITIES FOR PAVEMENT ON WRIGHT ST.

Leveler, J. Doe. Rodman, R. Roe. Chainmen, J. F. Keen and G. W. Swift

Station	B.S.	H.I.	F.S.	T.P.	B.M.	LEVELS ON THE L-Prop L-40 Ft.	L-S-Wlk L-37 Ft.	L-Gutter L-20 Ft.	Center	R-Gutter R-20 Ft.	R-S Wlk R-37 Ft.	R-Prop. R-43 Ft.
+43	North Property Line, Healy St.					707.4 / 4.1	707.9 / 4.2	707.3 / 4.8	707.4 / 4.7	707.4 / 4.1	707.2 / 4.9	707.9 / 5.2
4+06	Center Line, Healy St.					708.0 / 4.1	708.0 / 4.1	707.8 / 4.3	708.2 / 3.9	707.2 / 4.9	707.2 / 5.1	706.7 / 5.4
+76	South Property Line, Healy St.					707.5 / 4.6	707.5 / 4.6	707.6 / 4.5	708.6 / 3.5	707.1 / 5.0	706.7 / 5.4	706.6 / 5.5
3	North End of Bridge over Boneyard Creek					705.8 / 6.3	707.2 / 4.9	708.6 / 3.5	708.8 / 3.3	707.5 / 4.6	705.1 / 7.0	706.2 / 5.9
⊼	3.13	712.08										
●			7.89	708.95								
+62	South End of Bridge over Boneyard Creek					705.4 / 11.4	707.0 / 9.8	707.3 / 9.5	709.0 / 7.8	706.3 / 10.5	706.1 / 10.7	706.3 / 10.5
2						707.8 / 9.0	708.4 / 8.4	708.2 / 8.6	709.5 / 7.3	708.6 / 8.2	708.2 / 8.6	708.2 / 8.6
1						711.1 / 5.7	711.0 / 5.8	710.8 / 6.0	711.3 / 5.5	711.1 / 5.7	711.9 / 4.9	711.9 / 4.9
0	North Property Line, Green St.					713.9 / 2.9	714.0 / 2.8	713.6 / 3.2	714.0 / 2.8	713.9 / 2.9	714.9 / 1.9	714.2 / 2.6
⊼	0.88	716.84										
●			4.20	715.96								
⊼	1.07	720.16										
●			3.94	719.09								
⊼	3.03	723.03										
S.L.B.M.					720.00							

May 7, 1914 (3 hours) Warm and Windy.
Used Buff & Berger Dumpy Level, Locker No. 15.
Chained down center of street, lining in with transit poles, taking levels en route.

PROBLEM C10. LEVELS FOR PROFILE AND QUANTITIES FOR PAVING A STREET.

(a) *Equipment.*—Level, level rod, 3 flag poles, 100-foot steel tape, chaining pins, 50-foot metallic tape, hubs, axe.

(b) *Problem.*—Take level rod readings on the center line, right and left curb lines, right and left sidewalk lines, and right and left property lines to determine profiles and quantities for paving street. Plot profiles on Plate A profile paper to a scale of 100 feet to 1 inch horizontal and 10 feet to 1 inch vertical. Estimate the quantities of cut and fill, and paving materials.

(c) *Methods.*—(1) Locate the center line of the street and set flag poles on line about 400 feet apart by ranging in with the eye. (2) Drive a hub at one end of the street and call this point station zero. (3) Run a line of differential levels from the Standard B. M. to the zero end of the line.

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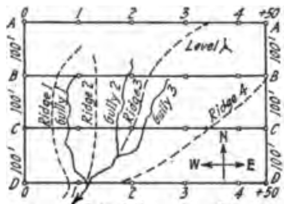
LEVELS FOR CONTOURS ON PROPOSED PARK SITE.

S.	+	⊼	−	R.	E.
B.M.	6.67	106.67			100.00
A0				11.4	95.3
A1				8.4	98.3
A1+45				8.3	98.4
A1+45	(46'R.)			6.9	99.8
A2				9.7	97.0
A3				5.6	101.1
A3+50	Ridge 3			3.7	103.0
A4				2.4	104.3
A4+50				0.5	106.2
B0				10.5	96.2
B0+75	Ridge 1			8.9	97.8
B1	Gully 1			8.9	97.8
B1+40	Ridge 2			8.1	98.6
B2	Gully 2			9.7	97.0
B2+40	Ridge 3			8.2	98.5
B2+70	Gully 3			8.8	97.9
B3				8.0	98.7
B4				5.3	101.4
B4+50				3.8	102.9
C0				11.7	95.0
C0+60	Ridge 1			11.2	95.5
C0+85	Gully 1			11.9	94.8
C1				11.3	95.4

(Continued on following pages)

J. Doe, Leveler. R. Roe, Rodman.
PROPOSED PARK SITE.
Oct. 16,'14. Clear, Warm. B.&B. Wye Level, Phila. Rod.
Boulder, Sta. A1+45, 46'R. on knoll.
Ground. (All levels from single setting.)



Dotted lines show ridges.

Set stakes only at Sta. 0 and 4+50 on each line for future reference. Used chaining pins for intermediate points.

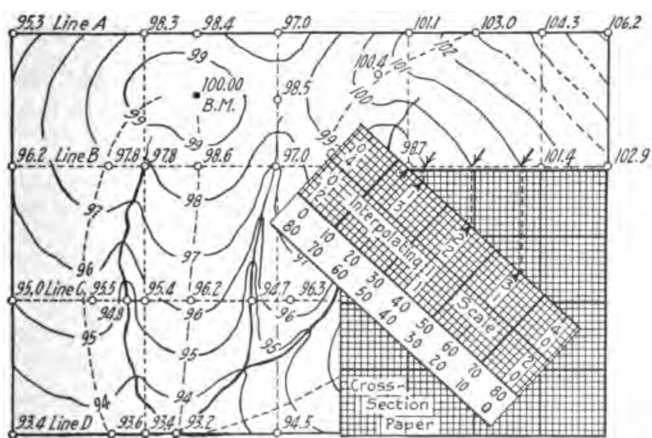


Fig. 19e.—Contour Plat and Device for the Rapid Interpolation of Contours.

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Read the rod to 0.01 foot. (4) Read the level rod to 0.1 foot on the ground at center hub. (5) Measure the distance out to the right curb line, right sidewalk and right property lines with the metallic tape and read the rod to 0.1 foot on the ground at station zero. (5) Measure the distance out to the center line to station 1. (8) Measure to the right and left from the chaining pin the required distances with the metallic tape and take rod readings as at station zero. (9) Repeat the process at each station and at abrupt changes intermediate. (10) Check the level circuit. (11) Make profile on Plate A paper, scales 100 feet to the inch horizontal and 10 feet vertical, indicating the several lines by conventional lines or colors. (12) Lay grade line as directed. (13) Show plat at bottom of profile. (14) Plot sections to a scale of 20 feet to the inch and determine areas. (15) Compute quantities of earthwork, paving, etc. Follow the form.

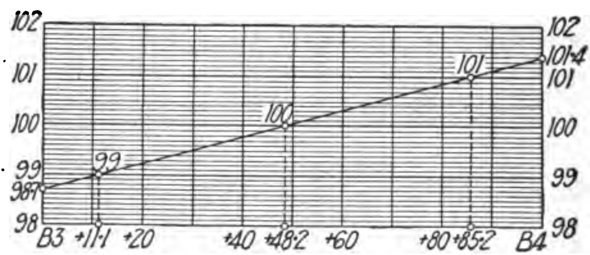

Fig. 19f.

PROBLEM C11. CONTOUR LEVELING.

(a) *Equipment.*—Engineers' leveling instrument, leveling rod, 100-foot steel tape, stakes, axe.

(b) *Problem.*—Make a rapid contour survey of an assigned tract of ground with the level and chain.

(c) *Methods.*—(1) Examine the tract and plan the system of reference lines for locating the points at which levels are to be taken; if the ground is comparatively regular, a simple subdivision into squares of 100 feet may suffice; but if much broken, special lines along gullies and ridges should be included in the survey plan. (2) Stake off the tract according to the plan, and make a record of the same. (3) Starting from an assigned bench, determine the elevations of the ground at the various stakes and at such other

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points as may be required to give a correct basis for accurate contouring. (4) Plot the data, and interpolate contours at a specified interval, employing both numerical calculations and geometrical methods, Fig. 19e. (5) Finish the plat, as required.

PROBLEM C12. USE OF CONTOUR MAP.

(a) *Equipment.*—Contour map, drafting instruments, etc.

(b) *Problem.*—From the given contour map: (1) construct profiles on the assigned lines; (2) project a line of specified grade through assigned points on the contour map; make profile, lay grade line and estimate earthwork quantities approximately; (3) calculate the earthwork quantities from the map for given grade planes and limitations of area. (The third step may, perhaps, best be taken with a different map from the first two.)

(c) *Methods.*—(1) Use profile paper for the profiles. (2) To project the line on the map, set the dividers at the horizontal distance in which the specified gradient will surmount the vertical interval between successive contour planes, Fig. 19f; then beginning at a specified point, locate points on the successive contour lines up or down on the given gradient, as required; sketch in the route roughly, and project a series of connected curved and tangent lines approximating to it; construct a profile along the new line; lay the required grade line on the profile, and estimate approximate earthwork quantities for specified dimensions and slopes of roadbed. (3) By means of end area method calculate the earthwork quantities required to establish the specified grade planes on the designated contoured area.

PROBLEM C13. RECIPROCAL LEVELING.

(a) *Equipment.*—Engineers' level, 2 leveling rods.

(b) *Problem.*—Determine the difference of elevation between two bench marks on opposite sides of a river (or wide ravine) by reciprocal leveling.

(c) *Methods.*—(1) Set the level up so that a rod reading may be taken on both benches at one setting. Station a rodman at each bench. (2) Take a back sight consisting of a series of say 5 or 10 careful consecutive rod readings. (3) Without delay take a like series of readings for a foresight. (4) Set the instrument on the opposite side of the

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river or ravine and repeat the above process. (5) Determine a difference of elevation by taking the difference between the mean back sight and fore sight for each setting, and finally take the mean of the two results. Observe rigid care in all details of the problem.

J. Doe & R. Roe, Observers.

DELICACY OF BUBBLE VIAL, B. & B. WYE LEVEL.

1st. Method, With Level Tester:

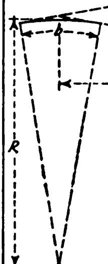
No.	Microm. Reading	Level A End	Scale* B End	Differences A End	Differences B End	Length Bubble
1	7	9.8	51.8			
2	17	14.2	47.2	4.4	4.6	61.4
3	27	18.5	42.7	4.3	4.5	61.2
4	37	23.0	38.5	4.5	4.2	61.5
5	47	27.0	34.4	4.0	4.1	61.4
6	57	31.5	30.0	4.5	4.4	61.5
7	67	35.8	25.7	4.3	4.3	61.5
8	77	40.2	21.0	4.4	4.7	61.2
8	77	40.0	21.2			61.2
7	67	35.5	25.5	4.5	4.3	61.0
6	57	31.1	30.2	4.4	4.7	61.3
5	47	26.7	34.6	4.4	4.4	61.3
4	37	22.3	38.8	4.4	4.2	61.1
3	27	18.2	42.8	4.1	4.0	61.0
2	17	14.0	47.1	4.2	4.3	61.1
1	7	9.5	51.5	4.5	4.4	61.0
				60.9	61.1	

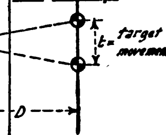
(* Level scale graduated to 20ths inch) Mean $= \frac{4.36}{20\text{ths}}$ inch.

Ten divisions of micrometer screw correspond to 19.2 seconds of arc, so that one division of scale $= \frac{19.2}{4.36} = 4.4$ seconds, and one division on bubble tube (tenths of an inch) $= 8.8$ seconds, or 88 seconds per inch.

Radius in Feet $= \frac{17182}{\text{secs per Inch}} = \frac{17182}{88} = 195$ Feet.

2nd Method, With Telescope.





$R : D = b : t$

$R = \frac{b}{t} D$.

$R = \frac{bD}{t}$

$= \frac{.083 \times 100}{0.042}$

$= 198$ Ft.

Target Movement (t) at 100 ft. (D) For Bubble Movement (b) of 1 inch (0.083 Ft)

No.	Rods (ft.)		t (ft)
1	4.631	4.588	0.043
2	4.586	4.546	0.040
3	4.547	4.591	0.044
4	4.593	4.634	0.041
		Mean =	0.042

PROBLEM C14. TEST OF DELICACY OF BUBBLE VIAL.

(a) *Equipment.*—Engineers' leveling instrument, leveling rod, tape, level tester.

(b) *Problem.*—Determine the radius of curvature of the assigned bubble vial. (1) by means of the optical test, and (2) by the level tester.

(c) *Methods.*—(1) Measure off a base line say 100 feet long, set level at one end and hold rod on a peg driven at the other end; note the target movement corresponding to a given bubble movement, both in the same linear unit; calculate the radius by the method shown at (h), Fig. 18. (2) Set the level tester on a solid base and place the instrument on it, as indicated at (i), Fig. 18; by means of the

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micrometer head and known relations of the level tester, determine the angular equivalent in seconds for one division and also one inch movement of the bubble, from which calculate the radius of curvature of the vial in feet. Follow the form.

PROBLEM C15. COMPARISON OF LEVEL TELESCOPES.

(a) *Equipment.*—Five (or other specified number) engineers' levels (both wye and dumpy), leveling rod, metallic tape.

(b) *Problem.*—Make a critical examination and comparison of the telescopes of the assigned instruments.

(c) *Methods.*—Carefully read the discussion of the telescope in the text. Then compare the telescopes with reference to: (1) magnifying power; (2) chromatic aberration; (3) spherical aberration; (4) definition; (5) illumination; (6) flatness of fields; (7) angular width of field; (8) effective aperture of objective. Make tabulated record of comparisons, giving in separate columns; (a) locker number; (b) kind of level; (c) name of maker; (d) magnifying power, and so on for the other points examined.

PROBLEM C16. TESTS OF THE WYE LEVEL.

(a) *Equipment.*—Wye level, leveling rod, tape.

(b) *Problem.*—Test the essential relations and adjustments of the wye level.

(c) *Methods.*—Carefully note the construction of the assigned level and the positions of the elementary lines. Then following the methods outlined in the text, test the following adjustments (but do not disturb the adjusting screws): (1) The bubble, both as to the azimuth and altitude movements; find the position of the bubble when parallel to the element of the rings. (2) The line of collimation; its deviation from the axis in 400 feet. (3) The wyes; finding the position of the bubble when the vertical axis is vertical. Keep a neat and systematic tabulated record of observed numerical data, with explanation of the several adjustments.

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PROBLEM C17. ADJUSTMENT OF THE WYE LEVEL.

(a) *Equipment.*—Wye level (reserved expressly for adjustment), leveling rod, tape, adjusting pin.
(b) *Problem.*—Make the full series of adjustments of the wye level.
(c) *Methods.*—Follow the methods detailed in the text according to the following program: (1) Adjust the bubble line (a) into the same plane with the bottom element of the rings, and (b) parallel to that element. (2) Adjust the line of collimation to coincide with the axis of the rings, first on a long distance; and then, to test the object glass slide, try it for a short distance; if necessary, shift the reticule in rotation to make the horizontal hair horizontal, and also center the eyepiece. (3) Adjust the bubble line perpendicular to the vertical axis by means of the wye nuts. (4) Test the rings of the wye level by the two-peg test; if the level has a reversion bubble, first test the parallelism of the top and bottom tangent lines, and then test the rings. Keep a clear and systematic record. In each case, state (a) the desired relation, (b) the test, and (c) the adjustment.

PROBLEM C18. SKETCHING THE WYE LEVEL.

(a) *Equipment.*—Wye level.
(b) *Problem.*—Make a first-class freehand sketch of the assigned wye level.
(c) *Methods.*—The sketch should be correct in proportion and clear in detail. The essential parts should be designated in neat and draftsmanlike form, and the elementary lines clearly indicated.

PROBLEM C19. TESTS OF THE DUMPY LEVEL.

(a) *Equipment.*—Dumpy level, leveling rod, tape.
(b) *Problem.*—Test the essential relations and adjustments of the dumpy level.
(c) *Methods.*—Carefully note the construction of the assigned level and the position of the elementary lines. Then, following the methods outlined in the text, test the following adjustments: (1) the bubble line, whether perpendicular to the vertical axis; and if not, what is the angular inclination of the vertical axis when the bubble is in the

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middle? (3) The line of collimation, whether parallel to the bubble line. Record the errors and observations systematically.

PROBLEM C20. ADJUSTMENT OF THE DUMPY LEVEL.

(a) *Equipment.*—Dumpy level (reserved expressly for adjustment), leveling rod, tape, pegs, axe, adjusting pin.

(b) *Problem.*—Make the essential adjustments of the assigned dumpy level.

(c) *Methods.*—(1) Adjust the bubble line perpendicular to the vertical axis. (2) Adjust the line of collimation parallel to the bubble line by the two-peg method. In describing the adjustments, the record should state (a) the desired relation, (b) the test, and (c) the adjustment.

PROBLEM C21. SKETCHING THE DUMPY LEVEL.

(See Problem C18.)

PROBLEM C22. STRETCHING CROSS-HAIRS.

(a) *Equipment.*—Engineers' level or transit (or cross-hair reticule), pocket cross-hair outfit, reading glass.

(b) *Problem.*—Renew the cross-hairs in a level or transit instrument by a method applicable to field use.

(c) *Methods.*—(If instrument is provided, follow the complete program outlined below; otherwise, merely stretch the lines on the reticule and test same.) (1) Remove the eyepiece, carefully preserving the screws from loss. (2) Remove one pair of the capstan headed reticule screws; turn the ring edgewise and insert a sharpened stick in the exposed screw hole, take out the other two screws and remove reticule from telescope tube. (3) Clean the cross-hair graduations, and support the reticule on a sharpened stick, or (if a transit) place it on the object glass with a piece of paper interposed to protect the lens. (4) Select from the capsule (see (d), Fig. 17) two spider lines 2 inches or more long, and fasten a stick to either end of each hair by means of glue from the adhesive paper. (5) Put the hairs in place, (with the bits of wood hanging loose), shifting them as desired with a pin point or knife blade. (6) Apply a bit of the moistened adhesive paper to the reticule over each hair,

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and after a few minutes cut or break the sticks loose. (7) Test the hairs by blowing on them full force. (8) If they stand this test, replace the reticule, and adjust the instrument. Make a record of the process.

PROBLEM C23. ERROR OF SETTING A LEVEL TARGET.

(a) *Equipment.*—Engineers' leveling instrument, leveling rod (preferably a New York or Boston rod), tape, pegs.

(b) *Problem.*—Determine the probable error of setting the level target at distances of 100 and 300 feet (or such other distances as may be assigned).

ERROR OF SETTING LEVEL TARGET.

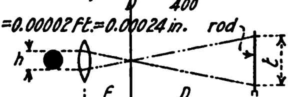
Distance 100 Feet.				Distance 300 Feet.			
Reading No.	Ft.	d Ft.	d^2	Reading No.	Ft.	d Ft.	d^2
1	3.169	0.000	0.000000	1	4.843	0.006	0.000036
2	3.170	.001	.000001	2	4.835	.002	.000004
3	3.170	.001	.000001	3	4.836	.001	.000001
4	3.170	.001	.000001	4	4.837	.000	.000000
5	3.169	.000	.000000	5	4.834	.003	.000009
6	3.168	.001	.000001	6	4.834	.003	.000009
7	3.171	.002	.000004	7	4.837	.000	.000000
8	3.168	.001	.000001	8	4.839	.002	.000004
9	3.169	.000	.000000	9	4.838	.001	.000001
10	3.168	.001	.000001	10	4.835	.002	.000004
n	3.169	0.0008	0.000010	n	4.837	0.002	0.000068
	Mean	Mean=f	Sum=Σ		Mean	Mean=f	Sum=Σ

Prob. Error Single Obs. $E_1 = 0.67\sqrt{\frac{\Sigma d^2}{n-1}} = 0.00070$	$E_1 = 0.0016$
Approx. Prob. Single Error Obs. $= 0.85F = 0.00068$	E_1 (approx.) $= 0.0017$
Prob. Error of Mean $E_m = \frac{E_1}{\sqrt{n}} = 0.00022$	$E_m = 0.0005$

Leveler, R. Roe. Young Level.
Rodman, J. Doe. Boston Rod, Lr. 12.

Nov. 1, 1914, (2 hours). Cloudy, cool, breezy.
Distances with 50-Ft. Metallic Tape.

Set instrument in sheltered place, measured off 100 Ft. and drove peg. Same at 300 Ft.
Placed pair of foot screws on general line of pegs and leveled up, leaving screws just snug.
Focused eyepiece on cross hairs very carefully, keeping eye in normal condition.
Set target ten times at each distance, carefully verifying the position of the bubble each time before approving sight.
Determined magnifying power of telescope by comparing 0.1 Ft. on rod natural size with one eye and magnified by telescope with other eye. Found Mag. Power to be 28 diameters.
Found radius of curvature of bubble
$R = \frac{b}{t} D = \frac{0.07}{0.048} \times 100 = 145.8'$
Diam. hor. hair, $h = \frac{tf}{D} = \frac{0.01 \times 0.8}{400}$
$= 0.00002$ Ft. $= 0.00024$ in.



(c) *Methods.*—(1) Determine the magnifying power of the telescope. (2) Determine the radius of curvature of the level vial by the field method. (3) Determine the space on the rod covered by the diameter of the hair. (4) Drive a peg at 100 feet from the level, level up, and secure ten satisfactory consecutive rod readings with rod held truly plumb on the peg; shift the target several inches between read-

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ings, and reset without bias; reject no readings; watch the bubble closely, but work briskly. (4) Repeat the series at 300 feet. (5) Determine for each distance the mean rod, the probable error of a single reading, and of the mean, as indicated in the form.

PROBLEM C24. MAKING A LEVELING ROD.

(a) *Equipment.*—Piece of straight dressed clear white pine of proper dimensions, steel tape graduated to 0.01 foot, carpenter's tri-square, paint, etc.

(b) *Problem.*—Make a self-reading leveling rod.

(c) *Methods.*—(To be devised by the student. See Fig. 27 for suggested graduations.)

PROBLEM C25. COMPARISON OF DIFFERENT MAKES AND TYPES OF ENGINEERS' LEVELS.

(a) *Equipment.*—Department equipment, catalogs of representative engineering instrument makers.

(b) *Problem.*—Make a critical comparison of the several types and makes of engineers' levels.

(c) *Methods.*—Examine the department equipment and study the several catalogs carefully, noting the usual and special features, prices, etc., and prepare a systematic summary or digest of the same. Prepare brief specifications for a leveling instrument, and also suggest the preferred make.

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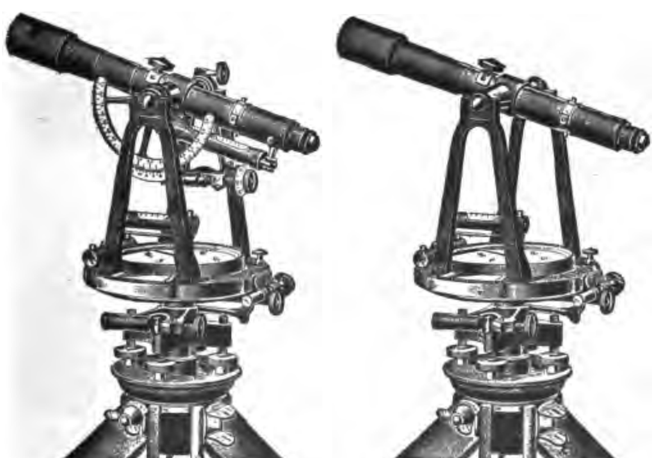
CHAPTER V.

THE TRANSIT.

Description.—The engineers' transit consists of an alidade, carrying the line of sight, attached to an inner vertical spindle (or upper motion) which turns in an outer annular spindle (or lower motion). The latter carries the horizontal graduated circle or limb, and is supported by the tripod head. The alidade includes the telescope, magnetic needle with its graduated circle, and the vernier; it may be revolved while the graduated limb remains stationary. The horizontal limb is graduated to degrees and half degrees and sometimes to twenty minutes, and is numbered preferably from zero to 360° in both directions.

The complete transit differs from the plain transit, Fig. 20, in having a vertical arc and level bubble attached to the telescope.



Complete Transit. Plain Transit.

Fig. 20.

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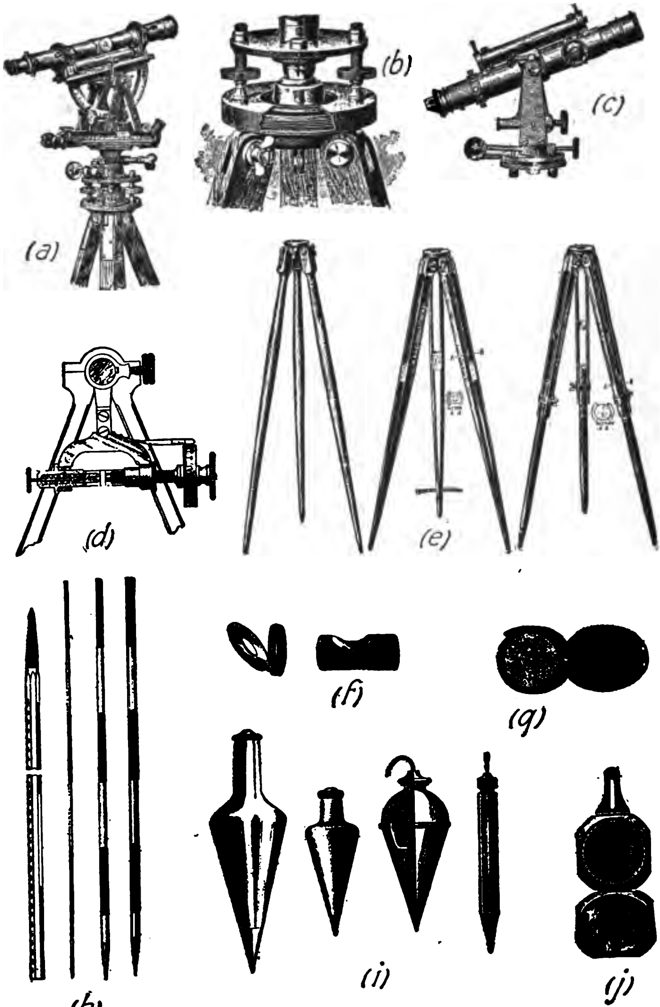

Fig. 21.

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In Fig. 21 are shown: (a) the English theodolite; (b) the shifting plates and foot screws of a transit; (c) the Saegmuller solar attachment to the transit; (d) the gradienter; (e) tripods; (f) reflectors; (g) reading glass; (h) flag poles; (i) plumb bobs; (j) the Brunton pocket transit.

The Vernier.—The vernier is an auxiliary scale used to read fractional parts of the main graduated scale or limb. The *least count* of a direct vernier is found by dividing the value of one division of the limb by the number of divisions on the vernier. With a limb graduated to half degrees and a direct vernier reading to single minutes 30 divisions on the vernier cover 29 divisions on the limb.

In *reading* a direct vernier observe the following rule: Read from the zero of the limb to the zero of the vernier, then along on the vernier until coincident lines are found. Add the reading of the vernier to the reading of the limb.

In *setting* the vernier to a given reading, as for example a zero reading for measuring an angle, the tangent movement should be given a quick short motion to secure the last refinement, since a slow movement is not noticed by the eye. Notice adjacent and end graduations.

In Fig. 23, (c) is a vernier reading to single minutes, (d) to half minutes (30″), and (e) to thirds of minutes (20″). The slant in the numerals on the limb corresponds with that on the vernier.

USE OF THE TRANSIT.

Use.—The complete transit is used: (1) to prolong lines; (2) to measure horizontal angles; (3) to measure vertical angles; (4) to run levels; (5) to establish grade lines. The plain transit is confined to the first two uses, unless it has a vertical clamp and tangent movement, when it may be used to "shoot in" grade lines.

Prolongation of Lines.—If the instrument is in adjustment a line can be prolonged by sighting at the rear station and reversing the telescope in altitude. It is, however, not safe to depend on the adjustments of the transit, and important lines should always be prolonged by the method of "double sights," as given in Problem D2. Lines may be prolonged with the plates by sighting at the rear station with the A vernier reading 180°, reversing the alidade in azimuth and locating stations ahead with the A vernier reading zero. A third method employs two points ahead of the instrument.

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Measurement of Horizontal Angles.—Horizontal angles are measured as described in Problem D1. If greater accuracy is required, angles may be measured by series or by repetition.

By Series.—In measuring an angle by series all the angles around the point are read to the right, both verniers being read to eliminate eccentricity. The instrument is then reversed in altitude and azimuth and all the angles around the point are read to the left. The readings are checked by sighting back on the first point in each case. These observations constitute one "set." The vernier is shifted between sets 360° divided by the number of sets. The arithmetical mean of the observed values is taken as the true value.

By Repetition.—Angles are measured by repetition as described in Problem D13. This method is especially suited to the accurate measurement of angles with an ordinary transit, and is to be preferred to the series method, which is a favorite where precise instruments are used. In the repetition method all the instrumental errors are eliminated and the error of reading is very much reduced. It is doubtful if it is ever consistent to make more than 5 or 6 repetitions.

Azimuth.—The azimuth of a line is the horizontal angle which it makes with a line of reference through one of its ends, the angles being measured to the right from 0° to 360°, as in (f) Fig. 23. It is usual to assume that the true meridian is the line of reference, the south point being taken as zero in common surveying.

Deflection.—The deflection of a line is the angle that it makes with the preceding line produced, and is called deflection right or left depending upon whether the angle is on the right or left side of the line produced, as in (h), Fig. 23.

Vertical Angles.—Vertical angles are referred to the horizon determined by the plane of the level under the telescope, and are angles of depression or elevation relative to that plane. In measuring vertical angles the instrument should be leveled by means of the level under the telescope and correction should be made for index error of the vernier. With a transit having a complete vertical circle, the true vertical angle may be obtained by measuring the angle with the telescope normal and reversed and taking the mean.

Traversing.—A traverse is a series of lines whose

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lengths and relative directions are known. Traverses are used in determining areas, locating highways, railroads, etc.

Azimuth Traverse.—In an azimuth traverse the azimuths of the lines are determined, usually passing around the field to the right. In *orienting* the transit at any station the A vernier is set to read the azimuth of the preceding course, the telescope is reversed, directed towards the preceding station and the lower motion clamped; the telescope is then reversed in altitude. The reading of the A vernier with telescope normal will then give the azimuth of any line sighted on. If there is any error in collimation the transit may be oriented by sighting back with the A vernier reading the back azimuth of the preceding course. In a *closed traverse* the last front azimuth should agree with the first back azimuth. The azimuth traverse is especially adapted to stadia and railroad work. Azimuths can be easily changed to bearings, if desired.

Deflection Traverse.—In a deflection traverse the deflection of each line is determined, usually passing around the field to the right. To avoid discrepancies due to error in collimation, the transit may be oriented by sighting at the preceding station with the A vernier set at 180°, the telescope being in its normal position, and the lower motion clamped. The reading of the A vernier will then give the deflection of any line sighted on.

Compass Bearings.—Compass bearings should always be read on an extended traverse as a check against such errors as using the wrong motion or an erroneous reading of the vernier. To guard against errors due to local attraction, back and front bearings should always be read, and the angle thus determined compared with the transit angle.

Leveling with the Transit.—The transit with an attached level is the complete equivalent for the engineers' level. The instrument is leveled up with the plate levels first, after which the position of the attached bubble is controlled by means of the vertical tangent movement.

Grade Lines.—Grade lines may be established with the transit either by means of known distances and calculated rod readings, or by "shooting in" a parallel line by means of the inclined telescope, as described under the use of the engineers' level. For the latter purpose the transit is rather more convenient than the level.

Setting up the Transit.—To set the transit over a point, spread the legs so that they will make an angle of about 30°, place them symmetrically about the point with two legs

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down hill. Bring one plate level parallel to two of the legs, force these legs firmly into the ground and bring the plumb bob over the point and the plates approximately level with the third leg, changing the position of the plumb bob with a radial motion and leveling the plates with a circular motion of the leg. Finish the centering with the shifting plates. In leveling up, the bubbles move with the left thumb. Use care to bring the foot screws to a proper bearing.

Parallax.—Before beginning the observations the eye-piece should be carefully focused on the cross-hairs so as to prevent parallax.

Back Sight With Transit.—*Always check the back sight before moving the transit* to see that the instrument has not been disturbed or that a wrong motion has not been used.

Instrumental Errors.—The transit should be kept in as perfect adjustment as possible, and should be used habitually as though it were out of adjustment, that is, so that the instrumental errors will balance. No opportunity should be lost to test adjustments.

ADJUSTMENTS OF THE TRANSIT.

Elementary Lines.—Fig. 22 shows the elementary lines of the transit, viz., (1) line of collimation; (2) horizontal axis; (3) vertical axis; (4) plate level lines; (5) attached level lines. These lines should have the following relations: (a) the plate levels should be perpendicular to the vertical axis; (b) the line of collimation should be perpendicular to the horizontal axis; (c) the horizontal axis should be perpendicular to the vertical axis; (d) the attached level line should be parallel to the line of collimation. The following additional relations should exist: (e) the vertical axes of the upper and lower motions should be coincident; (f) the optical center of the objective should be projected in the line of collimation; (g) the center of the graduated circle should be the center of rotation, i. e., there should be no eccentricity.

Plate Levels.—*To make the plate levels perpendicular to the vertical axis.—Make the vertical axis vertical and adjust the bubbles to the middle of their race.* The vertical axis is made vertical by leveling up, reversing in azimuth, and if the bubbles move, bring them half way back with the foot screws. The adjustment is the same as for the compass, and the reasons are shown in (a), Fig. 13.

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After adjusting the plate levels with reference to say the upper motion, test them with the lower motion to prove the coincidence of the vertical axes.

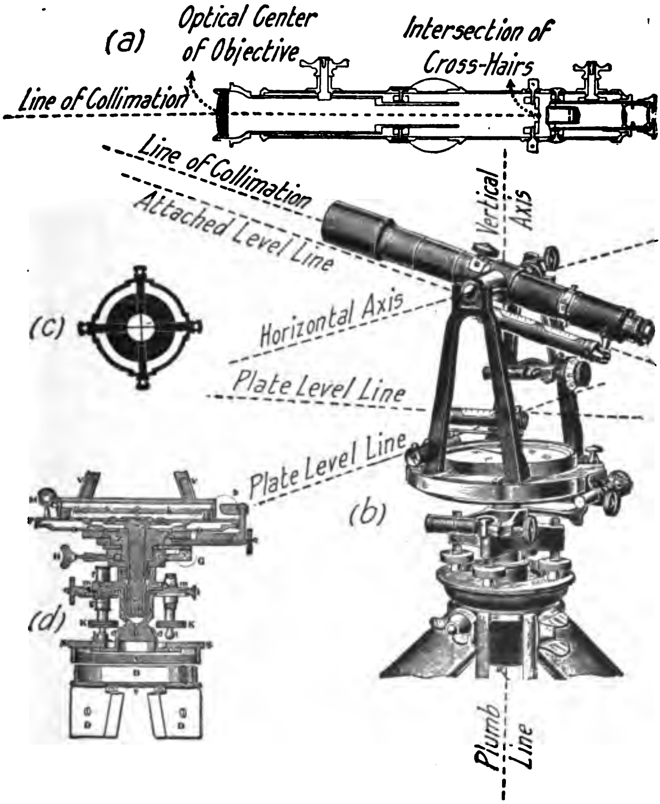

Fig. 22.

Line of Collimation.—*To make the line of collimation perpendicular to the horizontal axis.*—*Construct a straight line and adjust the vertical hair so that the instrument will reverse in altitude on it* The straight line may be established either by prolongation beyond a point in front, or

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preferably by the methods of double sighting, described in Problem D2. One-fourth the apparent error is corrected in second case as shown in (a), Fig. 23. In deciding which way to move the hair, notice that the optical center is the fulcrum. The transit should be collimated first for equal back and fore sights, say 100 feet or so, and then checked for a distant point in one direction and perhaps 50 feet in the other, so as to test the motion of the optical center of the objective. The points should all be as definite as possible. Chaining pins may be used, or V-marks may be made on the side of a stake driven securely. Each altitude reversal should be checked back and forth to make sure of the prolongations, and the telescope should be handled very carefully. If the cross-hair reticule is removed from the instrument or should be much disturbed, the foregoing adjustment is made approximately and the hair is made vertical by sighting on a plumb line, such as the corner of a building, or by noting whether the hair continuously covers the same point as the telescope is moved in altitude; the collimation adjustment is then made precisely.

Horizontal Axis.—*To make the horizontal axis perpendicular to the vertical axis.—Adjust the horiontal axis so that the line of collimation will follow a plumb line.* An actual plumb line may be used; or preferably a vertical line may be constructed by first sighting on a high point, then depressing the telescope and marking a low point; then reversing in altitude and azimuth (turning the horizontal axis end for end), sighting at the high point again and marking a second low point beside the first one. The mean of the two low points is vertically beneath the upper one. The transverse plate level is especially important in this process. One end of the horizontal axis is changed, as in (b), Fig. 23.

Attached Level.—*To make the attached level and the line of collimation parallel to each other.—Construct a level line and adjust the instrument to agree with it.* The level line may be obtained either by using the surface of a still body of water, as of a pond, or it may be constructed by equal back and fore sights, as indicated in (e), Fig. 16. Either the horizontal hair may be changed to bring the line of collimation parallel to the bubble line, or vice versa. The method is the same as used for the dumpy level.

If the bubble vial is a reversion level, as shown at (b), Fig. 18, the adjustment is much simpler. However, the

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two-peg test should be applied *at least once* to the reversion level to prove the parallelism of the top and bottom tangent lines of the bubble vial.

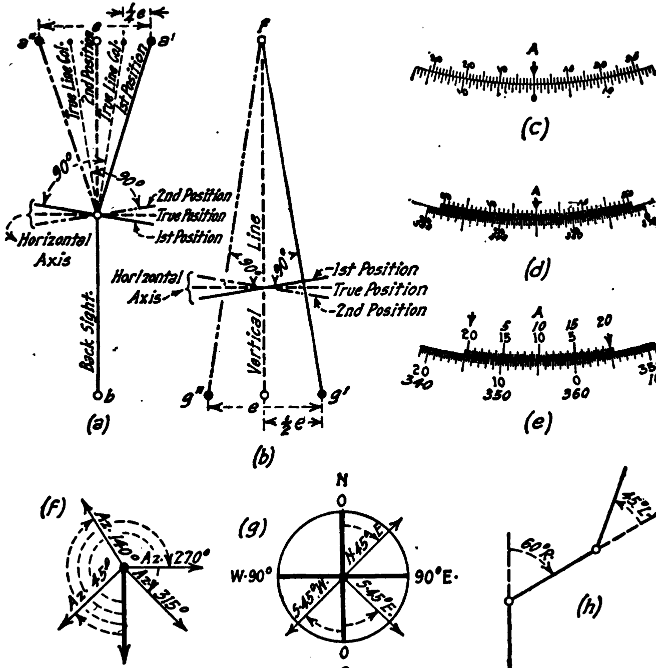

Fig. 23.

Vertical Arc.—After the last preceding adjustment, the vernier of the vertical circle should be made to read zero when the bubble is at the center of the tube. Bring the bubble to the center and shift the vernier to read zero. If the vernier is fixed, an index correction may be applied to all vertical angles; or the bubble may be made to agree with the vernier and the horizontal hair then adjusted by the two-peg method.

Eccentricity.—Read the two verniers at intervals around the circle; if the verniers have changed the same amount in

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each case the circle is well centered. If the two verniers have not changed the same amount, the mean of the angles passed over by the verniers is the actual angle through which the instrument has turned. The error cannot be adjusted.

Centering the Eyepiece.—If the intersection of the cross-hairs is not in the center of the field of view, move the inner ring of the eyepiece slide by means of the screws which hold it.

PROBLEMS WITH THE TRANSIT.

PROBLEM D1. ANGLES OF A TRIANGLE WITH TRANSIT.

(a) *Equipment.*—Transit, 2 flag poles, reading-glass.

(b) *Problem.*—Measure the angles of a given triangle with the transit.

(c) *Methods.*—(1) Set the transit over one of the vertices of the triangle and plumb a transit pole over each of the other two. (2) Set the *A* vernier to read zero, sight at the left hand point approximately, clamp the lower motion and make an exact bisection with the lower tangent movement. (3) Unclamp the upper motion, sight at the right hand point approximately and make an exact bisection with the upper tangent movement. (4) Read the *A* vernier to the nearest single minute. This reading is the angle sought. (5) With the *A* vernier set to read zero repeat the measurement, sighting first at the right hand station and then at the left. The recorded value of the angle is to be the mean of these two determinations which must not differ by more than one minute. (6) Measure the other angles in like manner. The error of closure must not exceed one minute. Follow the prescribed form.

PROBLEM D2. PROLONGATION OF A LINE WITH TRANSIT.

(a) *Equipment.*—Transit, 2 flag poles, axe, 6 hubs, 6 flat stakes, tacks.

(b) *Problem.*—Prolong a 300-foot base line successively with the transit by the method of "double sights" about 1500 feet, and check on a hub previously established.

(c) *Methods.*—(1) Drive two hubs, *A* and *F*, about 1500 feet apart. (2) Set the transit over tack in hub *A*, sight at

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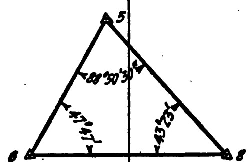
ANGLES OF TRIANGLE 5-6-8

Station	Value of Angle. 1st Meas.	2nd Meas.	Mean
5	88°50′	88°51′	88°50′30″
6	~~47°47′~~	~~47°50′~~	
"	47°47′	47°47′	47°47′00″
8	43°23′	43°23′	43°23′00″
			180°00′30″

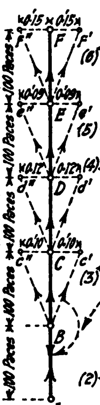
(Difference between measurements not to exceed 1′)
(Error not to exceed 1′)

Observers, J. Doe & R. Roe.
WITH ENGINEERS' TRANSIT.
Nov. 15, 1914, (2 hours). Warm and quiet.
Used Hellar & Brightly Transit No. 10.
The 1st measurement was made by sighting on Sta. 8 with the lower motion, the plates clamped at zero; then sighting on Sta. 6 with the upper motion, and reading the plates.
The second measurement was made by sighting on Sta. 6 and then on Sta. 8.
Used transit poles as targets, plumbing them very carefully over the monuments.
Sketch shows observed angles.



"DOUBLE SIGHTINGS"

PROLONGATION OF LINE



(6) Set up at E "double sighted to F. New tack F is 0.01 left of original tack. (Allowable error is 0.01 √No. Sightings For 300′ sights.)

(5) Set up at D, "double sighted" to E.

(4) Set up at C, "double sighted" to D.

(3) Set up at B, "double sighted to C; as Follows; (See Note→)
(a) Backsighted on A
(b) Plunged to c′
(c) Rotated to A
(d) Plunged to c″
(e) Bisected c′c″ to locate tack C.

(2) Set up at A; sighted on Flag at F; drove hub B; removed Flag F.

(1) Drove hubs A and F 1500′ apart (about)

NOTE. It is exceedingly important to plunge telescope delicately back and forth as a check on each altitude reversal. Take observations with speed and snap. ALWAYS check the back sight last. (See Chapter V.)

Observers { J. Doe / R. Roe } 11-16-14. (3 hrs.) Cool, Cloudy. Used K. & E. Transit No 4.
WITH ENGINEERS' TRANSIT.

INTERPOLATION OF POINT.

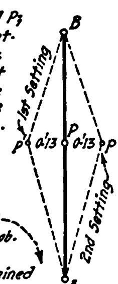
(5) Bisected pp′ at P. Set up at A and checked P; error, 0.02 to right.

(4) Reversed in azimuth; shifted transit so it would again plunge exactly on A and B. Drove hub p″ to bob.

(3) Set up and shifted transit laterally until it would plunge exactly on A and B. (See Note) Drove hub p′ to plumb bob.

(2) Set Flags on tacks at A and B, and determined point P. by lining in two poles successively by eye. (See p. 22.) Drove temporary peg.

(1) Drove hubs A and B about 600′ apart, assumed to have hill between them, both visible from desired hub P.



NOTE. Watched plate levels closely, especially transverse bubble.

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flag pole plumbed over tack in hub *F*, drive hub *B* about 300 feet from the transit and locate a tack in line very carefully. Remove the flag pole from hub *F*. (3) Set the transit over hub *B*, back sight on hub *A* and clamp the vertical axis. (4) Reverse the telescope, drive hub *C* at a distance of about 300 feet and mark line very carefully with a pencil. (5) Reverse the transit in azimuth, sight on hub *A*; reverse the telescope and locate a second point on hub *C*. Drive a tack midway between these two points. (6) Set the transit over the mean point on hub *C*, back sight on hub *B*, prolong 300 feet and set hub *D* by double sights. (7) Set over hub *D*, back sight on hub *C*, prolong, 300 feet and set hub *E*, as before. (8) Finally prolong from hub *E*, with back sight on *D*, and establish mean tack at terminal hub *F*. Record the collimation errors at *C*, *D*, *E*, and the final error at *F*. Follow the form.

PROBLEM D3. INTERSECTION OF LINES BY TRANSIT.

(a) *Equipment.*—Transit, 2 flag poles, plumb bob string, axe, 6 hubs, 6 flat stakes, tacks, marking crayon.

(b) *Problem.*—Determine the intersection of the bisecting lines of two angles of a triangle and check by bisecting the third angle.

(c) *Methods.*—(1) Drive and tack three hubs so as to form a triangle approximately equilateral and having sides about 400 feet long; properly witness the hubs with guard stakes. (2) Set the transit over one of the vertices of the triangle, and measure the angle as in Problem D1. (3) Set two hubs on the bisecting line, about 6 feet apart, so that the point of intersection of the bisecting lines will come between them, and mark the line by stretching a string between the hubs. Check by measuring each half angle independently. (4) Set the transit over one of the other vertices of the triangle, measure the angle and determine the bisecting line as at the first point. (5) Drive a hub at the intersection of the two bisecting lines and mark the exact point with a tack; check by measuring each half angle independently. (6) Set the transit over the third vertex and determine the angular and linear error of intersection. (7) As a final check measure the angles around the point of intersection of the bisectors. The angular error of closure of any triangle should not exceed one minute. Follow the form.

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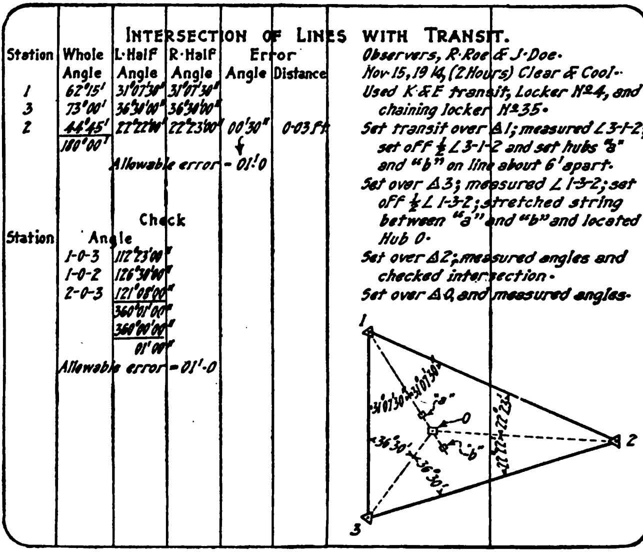


INTERSECTION OF LINES WITH TRANSIT.

Station	Whole Angle	L-Half Angle	R-Half Angle	Error Angle	Error Distance
1	62°15'	31°07'30"	31°07'30"		
3	73°00'	36°30'00"	36°30'00"		
2	44°45'	22°22'00"	22°23'00"	00'30"	0.03 ft
	180°00'				

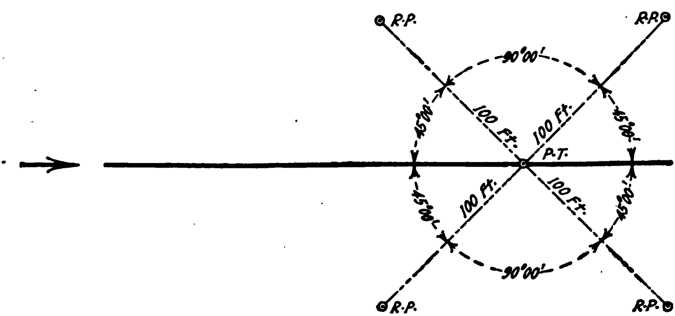
Allowable error = 01'.0

Check

Station	Angle
1-0-3	112°23'00"
1-0-2	126°30'00"
2-0-3	121°08'00"
	360°01'00"
	360°00'00"
	01'00"

Allowable error = 01'.0

Observers, R. Roe & J. Doe.
Nov. 15, 1914, (2 Hours) Clear & Cool.
Used K & E transit, Locker No. 4, and chaining locker No. 35.
Set transit over Δ1; measured ∠3-1-2; set off ½∠3-1-2 and set hubs "a" and "b" on line about 6' apart.
Set over Δ3; measured ∠1-3-2; set off ½∠1-3-2; stretched string between "a" and "b" and located Hub 0.
Set over Δ2; measured angles and checked intersection.
Set over Δ0, and measured angles.



PROBLEM D4. REFERENCING OUT A POINT.

(a) *Equipment.*—Transit, 2 flag poles, 100-foot steel tape, axe, 6 hubs, 6 flat stakes, marking crayon, tacks.

(b) *Problem.*—Reference out a point with a transit and tape.

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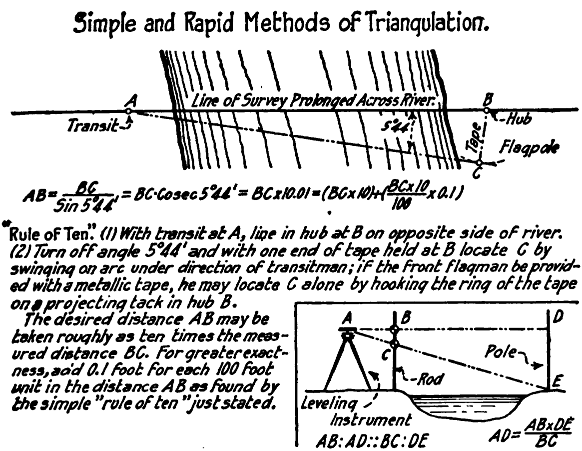
(c) *Methods.*—(1) Drive two hubs about 500 feet apart and mark them with guard stakes. (2) Set the transit over one of the hubs and reference it out as shown in the diagram. All hubs should be driven flush with the ground, and the exact points should be marked by means of tacks driven into the tops of the hubs. Record in proper form.

PROBLEM D5. TRIANGULATION ACROSS RIVER.

(a) *Equipment.*—Transit, 2 flag poles, 100-foot steel tape, axe, 4 hubs, 4 flat stakes, tacks.

(b) *Problem.*—Determine the distance across an imaginary river by triangulating with the transit and check by direct measurement.



(c) *Methods.*—(To be devised by the student. Use this and the next problem to learn the relative merits of several good methods. The "rule of ten" method in the sketch below is very rapid and also quite accurate.)

PROBLEM D6. PASSING OBSTACLE WITH TRANSIT.

(a) *Equipment.*—Transit, 100-feet steel tape, 2 flag poles, axe, hubs, flat stakes, tacks.

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Triangulation Across a River with Engineers' Transit

Station	Distance Ft.	Angle	Value
B	150.00	D-B-C	90°00'
C		B-C-D	50°30'
B-D	181.85		

Calculation of B-D.

B-D = B-C × tan 50°30'

Log. B-D = Log. 150.00 + Log. tan 50°30'

Log. B-D = 2.17609 + 10.08390

= 2.25999

B-D = 181.96 Ft.

B-D = B-C × tan 50°30'

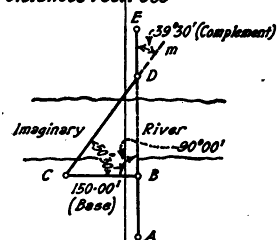
= 150.00 × 1.21310

= 181.96 Ft.

SUMMARY

Computed result	181.96	Ft.
Chained distance	181.85	Ft.
Difference	0.11	Ft.
1 : d	= 1:1650	
Permissible 1 : d	= 1:1000	

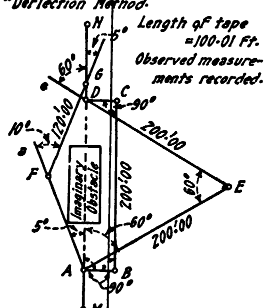
Observers: J. Doe & R. Roe.
Nov. 27, '14. (2 Hours) Cold & Clear.
Used Fauth Transit, Locker No. 8; and Chaining Locker No. 33.
With transit over B, set hub at D by "Method of Double Sights," with A as a backsight.
Set Hub at C, measuring B-C with care, and measured ∠ B-C-D.
Checked computed distance by chaining B-D.
Length of Tape, 99.98 Ft. Observed distances recorded.



Passing an Obstacle with Engineers' Transit

Station	Distance Ft.	Angle	Value	Error of Closure Dist. Ft.	Error of Closure Line Ft.
	"Equilateral Triangle Method"				
A	200.00	N-A-E	60°00'		
E	200.00	A-E-D	60°00'		
D	301.03	e-D-N	59°59'		
A-D	199.93			-0.07	0.09 R.
	"Right Angle Offset Method"				
A	20.00	N-A-B	90°00'		
B	200.00	A-B-C	90°00'		
C	20.00	B-C-D	90°00'		
D	300.88	C-D-N	90°01'	+0.08	0.10 L.
A-D	200.08				
	"Deflection Method"				
A	120.00	N-A-F	5°00'		
F	120.00	a-F-G	10°00'		
G	261.85	F-G-N	5°00'		
A-G	239.05			-0.03	0.03 R.

Observers: J. Doe & R. Roe.
Nov. 17, 1914, (2 Hours) Warm & cloudy.
Used Gurley Transit, Locker No. 5, and chaining Locker No. 32.
With the transit over M, set hubs at A and N, in the line MN. Set transit at A and prolonged the line MA by the "Equilateral Triangle Method," the "Right Angle Offset Method" and the "Deflection Method."
Length of tape = 100.01 Ft.
Observed measurements recorded.



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(b) *Problem.*—Prolong a line beyond an imaginary obstacle by three methods and check by direct measurement.

(c) *Methods.*—(To be devised by the student.)

PROBLEM D7. TRAVERSE OF FIELD WITH TRANSIT.

(a) *Equipment.*—Transit, 2 flag poles, 100-foot steel tape.

(b) *Problem.*—Determine the deflections of the sides of an assigned field with the transit, check angles by observing the magnetic bearings, and measure the lengths of the sides with a steel tape.

(c) *Methods.*—(1) Set the transit over one corner of the field, set the *A* vernier to read 180°, and sight at a flag pole plumbed over the point to the left with the telescope normal. Read and record the magnetic bearing. (2) Keep the telescope normal and sight at the next point to the right. The reading of the *A* vernier will be the deflection of the second line. (3) Read and record the magnetic bearing and compare the transit and magnetic deflections. (4) Repeat this process for the remaining corners of the polygon taken in succession to the right. Deflections will be based on duplicate readings agreeing within one minute. (5) Measure the sides to the nearest 0.01 foot with the tape. Compare the tape with the standard at the beginning and conclusion of the chaining. (6) From the observed deflections determine the bearings of the field assuming one side as a true meridian. The angular error of closure must not exceed one minute. Record and reduce data as in the prescribed form. Should a side of the field be obstructed, use one or more auxiliary points (see (c) of D8).

(Most engineers prefer "plunge reversals" to the above method of "plate reversals." To avoid the collimation error involved in a single plunge reversal, the principles of "double sights" must be used and the mean angle taken. To save time, some engineers try to keep the transit always in first-class adjustment, so as to omit one altitude reversal in the "plunge" method, and some turn the transit "end for end" (reverse in azimuth) every setting or so.)

PROBLEM D8. AREA OF FIELD WITH TRANSIT.

(a) *Equipment.*—Five-place table of logarithms.

(b) *Problem.*—Compute the area of the assigned field by means of latitudes and departures.

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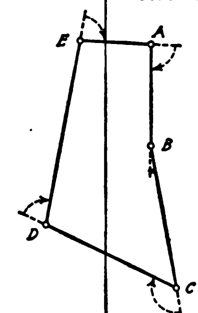
TRAVERSE OF FIELD A-B-C-D-E WITH ENGINEERS' TRANSIT, DEFLECTION METHOD

Station Inst.	Station Obj.	Distance Ft.	Deflection Angle	Magnetic Bearing	Check Angle	Calculated Bearing
A	E		86°31'R	N60°35'E		
	B	335·00		S32°45'E	86°40'R	S0°00'E
B	A		10°13'L	S33°05'E		
	C	464·98		S43°25'E	10°20'L	S10°13'E
C	B		124°53'R	S43°20'E		
	D	483·72		S81°35'W	124°55'R	N65°20'W
D	C		76°03'R	S81°35'W		
	E	616·53		N22°20'W	76°05'R	N10°43'E
E	D		82°46'R	N22°25'W		
	A	242·84		N60°[illegible]'E	82°35'R	S86°31'E
			370°13'		370°15'	
			10°13'		10°20'	
			360°00'	(Check)	359°55'	

Observers: J. Doe & R. Roe.
Nov 20-1914. (2 Hours) Warm & Misty.
Used Keuffel & Esser Transit, Locker #4.
Assumed that A-B was a true meridian.
Carefully checked each angle.
Bubble down on all sights.
Length of Tape = 100·01 Ft.
Reduced measurements recorded.
Allowable error of closure = 1'.



Calculation of Bearings

A-B	S0°00'E	D-E	N10°43'E	
B	10°13'L	E	82°46'R	
B-C	S10°13'E	E-A	S86°31'E	
C	124°53'R	A	86°31'R	(Check)
C-D	N65°20'W			
D	76°03'R			
D-E	N10°43'E			

Nov. 21, 1914 Computer, J. Doe.
Data from pp. Transcript O.K.

TRANSIT TRAVERSE, FIELD A-B-C-D-E LATITUDES AND DEPARTURES.

Line	Adjusted Bearing	Reduced Distance Ft.	Computation of Latitudes: Multiplication	Computation of Latitudes: Logarithms	Computed Latitude Ft.	Lat. Cor. Ft.	Adjusted Latitude Ft.	Computation of Departures: Multiplication	Computation of Departures: Logarithms	Computed Departure Ft.	Dep. Cor. Ft.	Adjusted Departure Ft.
AB	S0°00'	335·05			S335·05	−·10	S334·95			00·00	·00	00·00
BC	S10°13'E	464·98	464·98 [illegible] 457·606	2·66744 9·99306 2·66050 (457·61)	S457·61	−·10	S457·51	464·98 [illegible] 82·474	2·66744 9·24888 1·91632 (82·47)	E 82·47	+·03	E 82·50
CD	N65°25'W	483·72	483·72 [illegible] 201·237	2·68459 9·61911 2·30370 (201·24)	N201·24	+·05	N201·29	483·72 [illegible] 439·878	2·68459 9·95873 2·64330 (439·87)	W439·87	−·18	W439·69
DE	N10°43'E	616·53	616·53 [illegible] 605·777	2·78995 9·99236 2·78231 (605·78)	N605·78	+·15	N605·93	616·53 [illegible] 114·644	2·78995 9·26940 2·05935 (114·64)	E114·64	+·05	E114·69
EA	S86°31'E	242·84	242·84 [illegible] 14·756	2·38532 8·78361 1·16893 (14·76)	S 14·76	+·00	S 14·76	242·84 [illegible] 242·404	2·38532 9·99920 2·38452 (242·40)	E242·40	+·10	E242·50
	P =	2143·12			N807·02		N807·22			E439·51		E439·69
					S807·42		S807·22			W439·87		W439·69
					S 0·40	·40	0·00			W 0·36	·36	0·00

Error of Closure
Actual 0·55 Ft.
Permissible 0·6 Ft.
(See Diagrams)

Line	Ratio	Cor.
AB	2	·10
BC	2	·10
CD	1	·05
DE	3	·15
EA	0	·00
	8	·40

$\frac{\cdot 40}{8} = \cdot 05$

Line	Ratio	Cor.
AB	0	·00
BC	2	·03
CD	10	·18
DE	3	·05
EA	6	·10
	21	·36

$\frac{\cdot 36}{21} = \cdot 017$

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(c) *Methods.*—(Follow the instructions in the corresponding problem with the compass, Problem B4, preserving the same degree of precision in the computed latitudes and departures as in the field measurements. In case auxiliary stations are used on an obstructed side of the field, calculate the latitudes and departures of the polygon actually traversed in the field, and then to find the area drop the false corners in calculating the meridian distance and the latitude of the real side of the field.)

PROBLEM D9. STAKING OUT A BUILDING.

(a) *Equipment.*—Transit, 100-foot steel tape, 2 flag poles, axe, hubs, tacks, plan of building

(b) *Problem.*—On an assigned plot of ground stake out the assigned building.

(c) *Methods.*—(1) Orient one side of the enclosing rectangle with reference to a true meridian or a street line. (2) Locate and check up the corners of the rectangle by setting over each corner in turn, passing around to the right, back-sighting on the corner to the left, turning off 90° and locating the corner to the right. (3) Locate the corners of the building by setting stakes on the side lines of the building produced, using the rectangle as a base line. (4) Check all stakes by additional measurements. The rectangle should close to the nearest minute, the linear error should not exceed 1 : 50,000. Follow the form.

PROBLEM D10. HEIGHT OF TOWER WITH TRANSIT.

(a) *Equipment.*—Complete transit, 2 flag poles, leveling rod, 100-foot steel tape, axe, hubs, tacks.

(b) *Problem.*—Determine the height of an assigned tower with the transit and steel tape.

(c) *Methods.*—(1) Set the transit over a hub located a little further from the base than the height of the tower. (2) Level the instrument very carefully with the attached level and determine the index error of the vertical circle. (3) Bring the bubble of the attached level to the center and read a level rod held on the base of the tower (4) Sight at the top of the tower, read the vertical angle, correct for index error and record. (5) Reverse the telescope and locate a second point at least as far from the first as the height of the tower, check by "double sights." (6) Set

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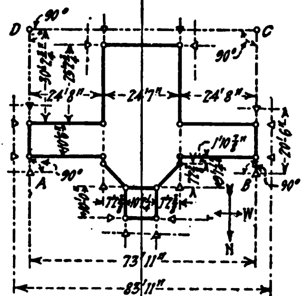
STAKING OUT BUILDING

Length of tape = 100.00, this tape being assumed as standard for the constuction of this building.
Located hub A to fit the site, and then established a true meridian through A by observation on Polaris.
Then constructed a checked rectangle ABCD as follows:
Set transit over hub A and set hub B and temporary hub D; set hub on line. Measured all distances twice.
Set transit over hub B, sighted at A and set temporary hub C; set hub on line.
Set transit over C, sighted on B and checked temporary hub D for angle and distance; error $\frac{1}{8}''$ for line, $\frac{1}{32}''$ for distance.
The remainder of the hubs were located with reference to the checked rectangle ABCD.

Surveyors, John Doe & Richard Roe.
WITH ENGINEERS' TRANSIT.
Nov. 28, 1914. (3 hours). Cool and clear.
Used Gurley Transit, Locker No. 6 and Locker No. 30.
Hubs are set on line 5' from corners.



Plan of Observatory.

HEIGHT OF TOWER

Station	Vertical Angle	D_1-D_2 Ft.	F.S. (Levels)
A	20°16'	150.00	4.50 = h_1
B	46°24'		3.82 = h_2

Calculation of Height.

(1) $D_1 = H_1 \text{ Cot. } M.$

(2) $D_2 = H_2 \text{ Cot. } N.$

(3) $H = H_1 + h_1 = H_2 + h_2.$

(4) $H_1 = H_2 - (h_1 - h_2)$; substituting (4) in (1) and subtracting (2) from (1),

(5) $D_1 - D_2 = H_2(\text{Cot. } M - \text{Cot. } N) - (h_1 - h_2)\text{Cot. } M$

(6) $H_2 = \dfrac{D_1 - D_2 + (h_1 - h_2)\text{ Cot. } M}{\text{Cot. } M - \text{Cot. } N.}$

Substituting,

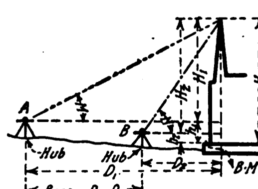
$$H_2 = \frac{150 + (4.50 - 3.82)\text{ Cot. } 20°16'}{\text{Cot. } 20°16' - \text{Cot. } 46°24'}$$

$= 86.47$ Ft.

$H = H_2 + h_2 = 86.42 + 3.82$

$= 90.29$ Ft.

Observers, J. Doe & R. Roe.
WITH ENGINEERS' TRANSIT.
Nov. 28, 1914, (2 hours). Warm & Cloudy.
Used Gurley Transit, Locker No. 5, and Chaining Locker No. 35.
Set transit over A and measured the vertical angle M, having first determined the index error of vertical circle.
Read level rod on base of tower. (h_1)
Set B in line with A and top of tower and measured D_1-D_2 as base line.
Set transit over B and found N and h_2.
Length of tape = 99.92 Ft.
Reduced measurements recorded.



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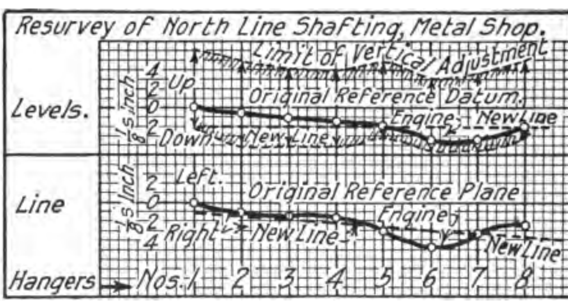
the transit over the second hub, sight at the top of the tower and read the vertical angle, as before. (7) Read the level rod on the base of the tower as before. Each angle and rod reading is to be based on duplicate readings. Follow the form.

PROBLEM D11. SURVEY OF LINE SHAFTING.

(a) *Equipment.*—Engineers' transit with attached bubble, leveling rod (or instead of these engineers' instruments, a 16-foot metal-bound straight-edge with an adjustable bubble of say 20-foot radius, a long braided fishing line, and 3 long metal suspenders made exactly alike, from which to suspend straight-edge from line of shafting), 2 good plumb bobs, 50-foot etched steel tape, copper tacks, hatchet.

(b) *Problem.*—Make a survey of a line of shafting in a machine shop, and establish a true alinement for it, both vertically and transversely.



(c) *Methods.*—(1) Establish a reference line for lateral deviations and carefully mark the same. (2) Select a suitable permanent bench mark to which the levels may be referred. (3) Determine the horizontal distance from the vertical reference plane to the line shafting at selected points, say at each hanger. (4) Determine the elevations of the same points by the methods of profile leveling. (5) Plot the data as suggested in the diagram. (6) Note the ruling points and permissible change both laterally and vertically at each hanger, and record the data. (7) Lay grade lines, and prepare data to shift the line shafting to a true position. (8) Make complete record of results.

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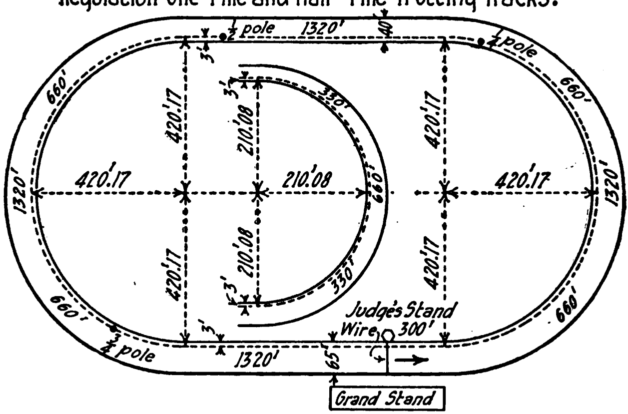
PROBLEM D12. SURVEY OF RACE TRACK.

(a) *Equipment.*—Outfit for transit party (instrument assigned, a long wire, say No. 20, spring balance, thermometer, etc.).

(b) *Problem.*—Make the survey for a race track, as instructed.

(c) *Methods.*—(1) Standardize steel tape, noting temperature and pull. (2) Make a careful examination of the tract of land with a view to secure the best location for the race

Regulation One-Mile and Half-Mile Trotting Tracks.



The standard distance is measured on a line 3 feet from the hub-board. The inner edge of the track is thus $2\pi \cdot 3 = 18.85$ feet shorter than the standard distance. The track is "banked" on curves from 1:12 to 1:15, and, to provide drainage, should be sloped one foot on the straight stretches. The ends of curves are sometimes flattened.

track as regards visibility, drainage, economy of construction and maintenance, etc. (3) After fixing the ruling points, establish the principal axis of the track by locating the centers of the two semi-circles and the intersections of the axis with the curves; also establish the ends of the curves, preferably on the true measured line (3 feet from the hub plank for a sulky track, and 18 inches from the inner edge for a bicycle track). (4) Run in each quadrant,

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either by the deflection angle method, or, if trees or other obstructions do not prevent, by using the wire as a radius with observed pull; set points 16 feet apart unless instructed otherwise. (5) After locating the true line, check up the total distance very carefully. (6) Make plat and complete record of survey.

PROBLEM D13. ANGLES OF TRIANGLE BY REPETITION.

(a) *Equipment.*—Transit, reading glass, 2 chaining pins, 2 tripods with plumb bobs (if necessary).

(b) *Problem.*—Measure the angles of a prescribed triangle with transit by repetition.

ANGLES OF TRIANGLE 5-6-8 BY REPETITION. Observers: John Doe & Richard Roe. Buff & Berger Transit 49.

Station	Bubble	Direction	Object	Vern. A.	Vern. B.	Mean	Difference	Angle	Mean Angle	Remarks
Δ6	Down	Right	Δ5	180°00'00"	0°00'00"	00'00"	Nov. 30, '14	(2 Hours)	Cool & Quiet.	
			Δ8	227°47'20"	47°47'20"	47'20"	47°47'20"			Single
				58°56'20"	238°56'20"	56'20"	238°56'20"	47°47'16"		5 Reps.
	Up	Left	Δ8	0°01'00"	180°00'00"	00'00"				
			Δ5	47°47'00"	227°47'00"	47'00"	47°47'00"			..ngle
				238°56'40"	58°56'40"	56'40"	238°56'40"	47°47'20"	47°47'18"	5 Reps.
Δ8	D	R	Δ6	0°00'00"	180°00'00"	00'00"	Dec. 1, '99.	(3 Hours)	Warm & Quiet.	
			Δ5	43°22'20"	223°22'20"	22'20"	43°22'20"			Single.
				216°52'00"	36°52'00"	52'00"	216°52'00"	43°22'24"		5 Reps.
	U	L	Δ5	180°00'00"	0°00'00"	00'00"				
			Δ6	223°22'20"	43°22'20"	22'20"	43°22'20"			Single
				36°51'40"	216°51'40"	51'40"	216°51'40"	43°22'20"	43°22'22"	5 Reps.
Δ5	D	R	Δ8	0°00'00"	180°00'00"	00'00"				
			Δ6	88°50'20"	268°50'20"	50'20"	88°50'20"			Single
				84°12'40"	264°12'40"	12'40"	444°12'40"	88°50'32'		5 Reps.
	U	L	Δ6	180°00'00"	0°00'00"	00'00"				
			Δ8	268°50'20"	88°50'20"	50'20"	88°50'20"			Single
				264°12'20"	84°12'20"	12'20"	444°12'20"	88°50'28"	88°50'30"	5 Reps.
							Error not to exceed 15"		180°00'10"	

(c) *Methods.*—(1) Set the transit over one of the vertices of the triangle and set chaining pins in the tops of the monuments at the other two. (2) Set the *A* vernier to read zero. (3) Sight at the left hand station with the bubble down, and clamp the lower motion. (4) Unclamp the upper motion, sight at the right hand station, read both verniers and record. (5) Unclamp the lower motion, sight at the

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left hand station, and check the verniers to see that they have not moved. (6) Unclamp the upper motion and sight at the right hand station but do not read verniers. Repeat until five repetitions of the angle are secured, and read both verniers to eliminate errors of eccentricity. (7) Divide the arithmetical mean of the two vernier readings by five and compare with the value obtained by single measurement. (8) Reverse the instrument in altitude, and set the *A* vernier to read zero. (9) Sight at the right hand station with the bubble up, and clamp the lower motion. (10) Unclamp the upper motion, sight at the left hand station, read both verniers and record. (11) Unclamp the lower motion, sight at the right hand station, and check the verniers to see that they have not moved. (12) Unclamp the upper motion and sight at the left hand station, but do not read the verniers. Repeat until five repetitions of the angle are secured, and read both verniers to eliminate errors of eccentricity. (13) Divide the mean of the two vernier readings by five and compare with the value obtained by single measurement. (14) Take the mean of the two sets as the most probable value. (15) Measure the other angles in the same manner. The angular error of closure should not exceed 15″. Follow the form.

PROBLEM D14. DETERMINATION OF TRUE MERIDIAN BY OBSERVATION ON POLARIS AT ELONGATION.

(a) *Equipment.*—Complete transit, reading glass, hub, 2 flat stakes, board 2″x 4″x 3′, 4 8d nails, axe, 2 lanterns, good watch set and regulated to keep railroad time.

(b) *Problem.*—Determine a true meridian by an observation on Polaris at elongation.

(c) *Methods.*—(1) Calculate the time of elongation of Polaris, and regulate and set a good reliable watch to keep railroad time (mean solar time). Calculate the time of elongation of Polaris from Table II.

Set the transit over a hub about 40 minutes before the time of elongation. Level the instrument very carefully, and set the vernier of the vertical circle to read the latitude of the place. (2) Focus the objective on a bright star; sight at Polaris which will be found by following the pointers of the Great Dipper, at an elevation equal to the latitude of the place. (3) With a reflector or a piece of white paper reflect light into the telescope so that the cross-hairs and the

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image of Polaris will be visible at the same time. (4) Depress the telescope and establish a target at a distance of about 500 feet; place the plank on the ground and nail it firmly to flat stakes, driving one at each end. (5) Level up again and follow Polaris with the telescope by means of the tangent movement; at elongation it will appear to traverse the vertical hair for several minutes. (6) Depress the tele-

DETERMINATION OF TRUE MERIDIAN BY OB'S ON POLARIS AT ELONGATION.

No. Obs.	Bubble	RR Time Obs.	Azimuth of Polaris Observed	Corect'n	Elong't'n	Error Azimuth
1	Down	$2^h 17^m$	1° 28'.4	0° 0'.0	1° 28'.4	−0'.7
2	"	2 30	1° 28'.9	0° 0'.1	1° 29'.0	−0'.1
3	Up	2 40	1° 29'.9	0° 0'.5	1° 30'.4	+1'.3
4	"	2 48	1° 29'.9	0° 1'.0	1° 30'.9	+1'.8
				Mean	1° 29'.8	+0'.7
			(Allowable	error =		1'.0)

Calculation of Railroad Time of Elongation.

Latitude 40° 06'; Longitude 88° 15'

Astron. Time U.C. Polaris, Dec. 1, 1915	$8^h 50.8^m$
Reduction for 3 days is 3×3.94^m	− 23.6
Astron. Time U.C. Polaris, Dec. 7, 1915	8 27.2
Correction for Railroad Time	− 7.0
R.R. Time U.C. Polaris, Dec. 7, 1915	8 20.2
Reduction for Western Elongation	+ 5 55.0
Railroad Time " "	$2^h 15.0^m$ A.M.

Calculation of Azimuth of Polaris at Elong'n

Azimuth Polaris, Elong't'n, Jan. 1, 1915	1° 29'.9
Correction for Dec. 7, 1915	− 0.8
Azimuth Polaris, Elong't'n, Dec. 7, 1915	1° 29'.1

Time of Elongation of Polaris.

For Western Elongation add $5^h 55^m$ to time U.C. Polaris. For Eastern Elongation subtract $5^h 55^m$ from time U.C.

Observers, J. Doe & R. Roe.

Dec. 7, 1915 (2 Hours), Clear and warm.

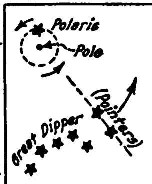
Buff & Berger Transit No. 9, 2 Lanterns, hubs, 2 flat stakes, plank 18"×4"×2", 4 8d nails, axe, watch set to keep Railroad time.

Set transit over hub at 1:40 A.M., sighted at Polaris, depressed the telescope and established target about 500 ft. from instrument. The plank was placed at right angles to line and nailed to a stake driven at each end.

Made first observation at western elongation.

Reversed instrument in altitude and azimuth between 2nd and 3rd readings.

Reduced observations 2, 3 & 4 by the following formula:- corr." = 0.058 t^2 where t = time from elongation in minutes; the correction being seconds of arc. (For Latitude 40°, 30 min. from elongation).



Polaris at Upper Culmination

scope, sight at a pencil held on the target and mark the point very carefully. (7) As a check make three observations within half an hour after elongation, noting the time of sighting on the star. Reverse the instrument in altitude and azimuth after the first check observation. (8) Reduce the check observations to observations at elongation by the following rule: Multiply the square of the time since elongation in minutes by 0.058, and the product will be the correction to the azimuth of Polaris in seconds of arc, for latitude 40°. (9) The next morning lay off the azimuth of Polaris for each observation to the east or west, depending upon whether the observation was made at western or east-

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ern elongation. (10) Check the observed meridian with the standard meridian. The error of the mean of the four observations should not exceed one minute. Record and reduce the data as in the form.

PROBLEM D15. DETERMINATION OF TRUE MERIDIAN BY OBSERVATION ON POLARIS AT ANY TIME.

(a) *Equipment.*—The same as in Problem D14.

(b) *Problem.*—Determine a true meridian by observing Polaris at any time.

(c) *Methods.*—Make the observations as described in Problem D14, noting the time of observation to the nearest minute, and reversing the instrument in altitude and azimuth between the 3rd and 4th observations. The transit should be leveled up very carefully with the attached bubble, particular attention being given to the horizontal plate level at right angles to the line of sight. (2) Reduce the observations by means of the tables.

A star comes to the meridian 4 minutes (nearly) earlier each day than it did the preceding day. The sideral day is therefore shorter than the solar day, the time from upper culmination to upper culmination being 23 hours 56.1 minutes mean solar time. The time from Upper Culmination to Lower Culmination is 11 hours 58 minutes.

Astronomical time, or Local Mean Solar time, is the time that would be kept by the mean sun and is obtained from Standard, or railroad time, by adding or subtracting 4 minutes for each degree of longitude that the place of observation is east or west of the Standard Meridian. The Astronomical day begins at noon of the civil day of the same date, and is reckoned from zero to 24 hours.

The Hour Angle of Polaris is found by subtracting the correct Local Mean Solar time of Upper Culmination, Table II, from the Local Mean Solar time of observation.

The Time Argument used in entering Table IV is the Hour Angle of Polaris, or 23 hours 56.1 minutes minus the Hour Angle of Polaris. Table IV is used as follows: Find the "hours and minutes" of the time argument in the left hand column of either page of Table IV. On the horizontal line with the "time before or after upper culmination" (time argument), the azimuth of Polaris for a declination of Polaris of 88° 51′ will be found in the column under the given latitude. The correction to the azimuth for each

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TABLE I.

Azimuth of Polaris at Elongation for Any Year Between 1915 and 1924.

Latitude	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924
30°	1° 19.6′	1° 19.2′	1° 18.8′	1° 18.5′	1° 18.1′	1° 17.8′	1° 17.4′	1° 17.0′	1° 16.7′	1° 16.4′
31	20.4	20.0	19.7	19.3	18.9	18.6	18.2	17.9	17.5	17.2
32	21.2	20.9	20.5	20.1	19.8	19.4	19.1	18.7	18.3	18.0
33	22.1	21.8	21.4	21.0	20.7	20.3	19.9	19.6	19.2	18.8
34	23.1	22.7	22.4	22.0	21.6	21.2	20.9	20.5	20.1	19.8
35	1 24.1	1 23.7	1 23.3	1 23.0	1 22.6	1 22.2	1 21.8	1 21.5	1 21.1	1 20.7
36	25.2	24.8	24.4	24.0	23.6	23.3	22.9	22.5	22.1	21.7
37	26.3	25.9	25.3	25.1	24.7	24.3	24.0	23.6	23.2	22.8
38	27.4	27.0	26.6	26.2	25.9	25.5	25.1	24.7	24.3	23.9
39	28.6	28.2	27.8	27.5	27.1	26.7	26.3	25.8	25.5	25.1
40	1 29.9	1 29.5	1 29.1	1 28.7	1 28.3	1 27.9	1 27.5	1 27.1	1 26.7	1 26.3
41	31.3	30.9	30.4	30.0	29.6	29.1	28.8	28.4	28.0	27.6
42	32.7	32.3	31.9	31.5	31.0	30.6	30.2	29.8	29.4	29.0
43	34.2	33.8	33.4	32.9	32.5	32.1	31.8	31.2	30.8	30.4
44	35.8	35.3	34.9	34.5	34.1	33.6	33.2	32.8	32.4	31.9
45	1 37.4	1 37.0	1 36.6	1 36.1	1 35.7	1 35.3	1 34.8	1 34.4	1 34.0	1 33.5
46	39.2	38.7	38.3	37.8	37.4	37.0	36.5	36.1	35.6	35.2
47	41.0	40.6	40.1	39.7	39.2	38.8	38.3	37.9	37.4	37.0
48	43.0	42.5	42.0	41.6	41.1	40.7	40.2	39.8	39.3	38.8
49	45.0	44.5	44.1	43.6	43.1	42.7	42.2	41.7	41.3	40.8
50	1 47.2	1 46.7	1 46.2	1 45.7	1 45.3	1 44.8	1 44.3	1 43.8	1 43.4	1 42.9

Correction For Above Table

The above table was computed with the mean declination of Polaris for each year. A more accurate result will be had by applying to the tabular values the following corrections, which depend on the difference of the mean and the apparent place of the star. The deduced azimuth will, in general, be correct within 0′.3.

For middle of	Correction in minutes	For middle of	Correction in minutes
January . . .	−0.5	July.	+0.2
February . .	−0.4	August. . . .	+0.1
March.	−0 3	September .	−0.1
April.	0.0	October. . . .	−0.4
May.	+0.1	November .	−0.6
June.	+0.2	December. .	−0.8

minute of change in Declination of Polaris are given in the last two columns on each page. The changes for latitudes between 30° and 40° and between 40° and 50° may be interpolated. The Declination of Polaris at any date may be

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TABLE II.

Local Mean (Astronomical) Time of the Culmination and Elongation of Polaris in the Year 1915.

(Computed for latitude 40° and longitude 90° or 6^h west of Greenwich.)

Date	East elongation		Upper culmination		West elongation		Lower culmination	
1915	h	m	h	m	h	m	h	m
January 1...	*0*	*51.7*	6	46.9	12	42.1	18	44.9
15...	23	52.5	5	51.6	11	46.8	17	49.6
February 1..	22	45.3	4	44.5	10	39.7	16	42.5
15..	21	50.1	3	49.2	9	44.4	15	47 2
March 1	20	54.8	2	54.0	8	49.2	14	52.0
15	19	59.6	1	58.8	7	54.0	13	56.8
April 1......	18	52.7	*0*	*51.9*	6	47.1	12	49.9
15......	17	57.7	23	52.9	5	52.0	11	54.8
May 1......	16	54.8	22	50.0	4	49.2	10	52.0
15......	15	59.9	21	55.1	3	54.2	9	57.0
June 1......	14	53.3	20	48.5	2	47.6	8	50.4
15......	13	58.5	19	53.7	1	52.8	7	55.6
July 1......	12	55.9	18	51.1	*0*	*50.2*	6	53.0
15......	12	01.1	17	56.3	23	51.5	5	58.2
August 1....	10	54.5	16	49.7	22	44.9	4	51.7
15....	9	59.8	15	55.0	21	50.2	3	56 9
September 1.	8	53.2	14	48.4	20	43.6	2	50.3
15.	7	58.3	13	53.5	19	48.7	1	55.4
October 1...	6	55.5	12	50.7	18	45.9	*0*	*52.7*
15...	6	00.6	11	55.8	17	51.0	23	53.8
November 1.	4	53.7	10	48.9	16	44.1	22	46.9
15.	3	58.6	9	53.8	15	49.0	21	51.8
December 1.	2	55.6	8	50.8	14	46 0	20	48.8
15.	2	00.4	7	55.6	13	50.8	19	53.6

Correction For Years After 1915

Year	Correction	Year	Correction
1916	add 1.6 up to March 1; subtract 2.3 on and after March 1	1922	add 3.1
1917	subtract 0.7	1923	add 4.5
1918	add 0.9	1924	add 5.9 up to March 1; add 2.0 on and after March 1
1919	add 2.5	1925	add 3.3
1920	add 4.0 up to March 1; subtract 0.1 on and after March 1	1926	add 4.6
1921	add 1.6	1927	add 5.9
		1928	add 7.2 up to March 1; add 3.3 on and after March 1

found from Table III. For example the azimuth of Polaris with a time argument of 9 hours and 15 minutes in latitude 40°, on April 21, 1915, was as follows: From Table III the declination of Polaris on April 21 was very closely 88° 51.25′. From Table IV for declination 88° 51′ the azimuth of Polaris was 58.65′; the correction for 0.25′ was 0.83 × 0.25 = 0.21′, and the azimuth was 58.65′ — 0.21 = 58.44′. If the exact time argument is not found in the table, the azimuth may be found with sufficient accuracy by direct inter-

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polation. Azimuths for latitudes between values given in Table IV may be found by direct interpolation. The nearest whole degree of latitude is usually sufficiently ac-

TABLE III.

Declination of Polaris for 1915.

Date	Declination	Date	Declination
Jan. 1......	88° 51′.54	July 1	88° 51′.05
15......	51.58	15.	51 05
Feb. 1......	51.57	August 1.	51.09
15......	51.54	15......	51.13
March 1......	51.49	Sept. 1......	51.21
15......	51.43	15......	51.28
April 1......	51.35	Oct. 1......	51.38
15.	51.27	15	51.47
May 1......	51.20	Nov. 1.	51.58
15......	51.14	15	51.67
June 1......	51.08	Dec. 1......	51.75
15......	51.06	15......	51.81

To obtain the declination for the corresponding time for years after 1915, add 0.31 min. for each year to the corresponding declination for 1915.

DETERMINATION OF TRUE MERIDIAN BY OBS. ON POLARIS AT ANY TIME.

No. Obs.	Bubble	R.R. Time Obs. h. m.	Mean Solar Time of Obs. h. m.	Hour Angle of Polaris h. m.	Azimuth of Pol's W. of N. ° ′	Error of Obs. ° ′
1	Down	8 26	8 33	9 00	1° 02.7	−1.0
2	"	8 41	8 48	9 15	58.4	0.0
3	"	8 56	9 03	9 30	53.9	−1.0
4	Up	9 11	9 18	9 45	49.2	+2.0
5	"	9 18	9 25	9 52	46.5	+1.0
6	"	9 26	9 33	10 00	44.2	+2.0
					Error	= +0.5
					Allowable Error	= 1.0

Calculation

Latitude 40°06′ Longitude 88°15′ W.

Mean Solar Time Upper Culmination

Atronom. Time U.C. Polaris, Apr. 15, '15 = $23^h:52.9^m$

Reduction to Apr. 21, = 3.94 × 5 = − 19.7

Astronomical (Local Mean Solar) Time U.C. Polaris, Apr. 21, " $23^h:33.2^m$

Hour Angle of Polaris = Mean Solar Time of Obs. + $24^h 00^m$ − Mean Solar Time, U.C. Polaris ($23^h 26^m$)

Time argument = Hour Angle of Polaris (or $23^h 56.1^m$ − Hour Angle)

Astronomical Time (Local Mean Solar) is reckoned from 12 o'clock Noon on the Civil Day of the same date.

Observers: J. Doe & R. Roe.

April 21, 1915 (2 Hours) Clear & Warm.

Buff & Berger Transit No. 9, 2 lanterns, hubs, 2 flat stakes, plank 36″ × 4″ × 2″, 4-8 d. nails, watch set to keep Railroad or Standard Time, axe.

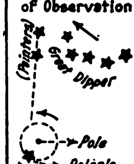
Set transit over hub at 8:15 P.M., sighted at Polaris, depressed telescope and established target 500 ft. from instrument, the plank was placed at right angles to line and nailed to a stake driven at each end.

Set vertical hair on star, noted time, depressed the telescope and marked line on target with pencil.

Apr. 22, 1915 (2 Hrs.) Reduced observations using Azimuth Tables.

Laid off Azimuth to the East and measured angle with the True Meridian.

Polaris at Time of Observation



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curate. The time used in making observations should be correct to the nearest minute, if accuracy is desired.

Table III was compiled from "Ephemeris for the Sun and Polaris and Tables of Azimuth of Polaris for the year 1915," published by the Department of Interior, General Land Office. Tables I, II and IV were compiled from "Principal Facts of the Earth's Magnetism," published by the U. S. Coast and Geodetic Survey, 1914.

The observations should be made as near elongation as possible, for the reason that Polaris is moving most rapidly in azimuth near culmination and errors in observing the time and using the table are then a maximum.

With careful work the range of 6 reduced observations should in no case exceed 1′ of arc. Record the data and make the calculations as in the form.

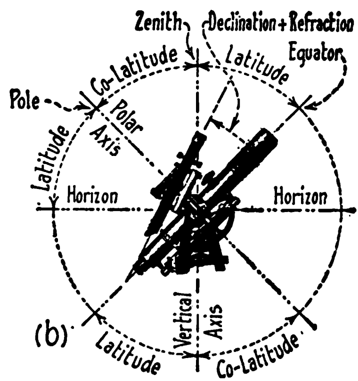

Fig. 23a.

PROBLEM D16. DETERMINATION OF TRUE MERIDIAN WITH SOLAR TRANSIT.

(a) *Equipment.*—Complete transit with solar attachment, reading glass, solar ephemeris, axe, hubs, tacks.

(b) *Problem.*—Determine a true meridian with a solar transit.

(c) *Methods.*—(There are various forms of solar attachments, to transits, among which are the Saegmuller, (a), and the Buff and Berger, (b), Fig. 23a; the former is the best known. The theory of all solar attachments in general use is the same, and is as follows: In order to bring

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TABLE IV. Azimuths of Polaris at Any Hour Angle.

Hour Angle before or after upper culmination	Azimuths of Polaris computed for Declination 88° 51′ Azimuths given in minutes						Correction for 1′ increase in declination of Polaris	
	Lat. 30°	Lat. 32°	Lat. 34°	Lat. 36°	Lat. 38°	Lat. 40°	Lat. 30°	Lat. 40°
h m	′	′	′	′	′	′	′	′
0 15	05.28	05.38	05.52	05.67	05.82	06.00	−0.08	−0.08
0 30	10.52	10.75	11.02	11.30	11.62	11.95	−0.15	−0.17
0 45	15.73	16.07	16.45	16.88	17.35	17.87	−0.23	−0.27
1 00	20.85	21.32	21.83	22.40	23.02	23.70	−0.30	−0.35
1 15	25.90	26.47	27.10	27.80	28.57	29.43	−0.38	−0.43
1 30	30.82	31.50	32.25	33.08	34.00	35.02	−0.45	−0.52
1 45	35.62	36.40	37.27	38.23	39.28	40.45	−0.52	−0.60
2 00	40.25	41.13	42.12	43.20	44.38	45.70	−0.58	−0.67
2 15	44.70	45.68	46.77	47.97	49.30	50.75	−0.65	−0.75
2 30	48.95	50.03	51.22	52.53	53.98	55.58	−0.72	−0 82
2 45	53.00	54.17	55.45	56.87	58.42	60.15	−0.77	−0.88
3 00	56.80	58.05	59.42	60.93	62.62	64.47	−0.83	−0.95
3 15	60.37	61.68	63.13	64.75	66.52	68.48	−0.88	−1.00
3 30	63.67	65.05	66.58	68.27	70.13	72.20	−0.93	−1.05
3 45	66.68	68.13	69.73	71.50	73.45	75.60	−0.97	−1.10
4 00	69.40	70.90	72.57	74.40	76.43	78.67	−1.02	−1.15
4 15	71.82	73.38	75.10	76.98	79.08	81.38	−1.05	−1.20
4 30	73.93	75.53	77.30	79.23	81.38	83.75	−1.07	−1.23
4 45	75.73	77.35	79.15	81.13	83.33	85.75	−1.10	−1.25
5 00	77.18	78.85	80.67	82.68	84.92	87.38	−1.13	−1.27
5 15	78.32	79.98	81.83	83.88	86.13	88.63	−1.15	−1.28
5 30	79.12	80.80	82.67	84.72	86.98	89.50	−1.15	−1.30
5 45	79.57	81.25	83.12	85.18	87.45	89.97	−1 15	−1.30
6 00	79.68	81.37	83.22	85.28	87 55	90 07	−1.17	−1.30
6 15	79.43	81.12	82.97	85.02	87.28	89.77	−1.15	−1.30
6 30	78.87	80.53	82.37	84.40	86.63	89.10	−1.13	−1.28
6 45	77 97	79.60	81.42	83.40	85.62	88.05	−1.12	−1 27
7 00	76.73	78.33	80.10	82.07	84.23	86.62	−1.10	−1 25
7 15	75.17	76.73	78.47	80.38	82.50	84.83	−1.08	−1.22
7 30	73.28	74.82	76.50	78.35	80.42	82 68	−1.07	−1.20
7 45	71.10	72.57	74.20	76.00	77.98	80.18	−1 03	−1.15
8 00	68.60	70.02	71.60	73.33	75 23	77.35	−1.00	−1.10
8 15	65.82	67.18	68.68	70.35	72.18	74.20	−0.95	−1.07
8 30	62.77	64.07	65.48	67.07	68.82	70.73	−0.90	−1.02
8 45	59.45	60.67	62.02	63.52	65 17	66.98	−0 85	−0.97
9 00	55.88	57.03	58.30	59.72	61.23	62.95	−0.80	−0.90
9 15	52.08	53.15	54.33	55.63	57.07	58.65	−0.75	−0.83
9 30	48.07	49.05	50.13	51.33	52.65	54.12	−0.70	−0.77
9 45	43.85	44.73	45.73	46.82	48 02	49.35	−0.63	−0.70
10 00	39.45	40.25	41.13	42.12	43.20	44.40	−0.57	−0.63
10 15	34.88	35.58	36.37	37.23	38.20	39.25	−0.50	−0.57
10 30	30.17	30.78	31.47	32.20	33.03	33.95	−0.43	−0.48
10 45	25.33	25.85	26.42	27.05	27.73	28.50	−0.37	−0.40
11 00	20.40	20.82	21.27	21.77	22.33	22.95	−0.30	−0.33
11 15	15.37	15.68	16.03	16.40	16.83	17 28	−0.22	−0.25
11 30	10.28	10.48	10.72	10.97	11 25	11.57	−0.15	−0.17
11 45	05.15	05.25	05.37	05.50	05.63	05.78	−0.07	−0.08
Elongation Azimuth	1° 19.68′	1° 21.37′	1° 23.23′	1° 25.30′	1° 27.57′	1° 30.08′	−1.15′	−1.30′
Hour Angle	h m s 5 57 21	h m s 5 57 08	h m s 5 56 54	h m s 5 56 39	h m s 5 56 24	h m s 5 56 08	s +2	s +3

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Hour Angle before or after upper culmination	Azimuths of Polaris computed for declination 88° 51′ Asimuths given in minutes						Correction for 1′ increase in declination of Polaris	
	Lat. 40°	Lat. 42°	Lat. 44°	Lat. 46°	Lat. 48°	Lat. 50°	Lat. 40°	Lat. 50°
h m	′	′	′	′	′	′	′	′
0 15	06.00	06.18	06.40	06.63	06.90	07.20	−0.08	−0.10
0 30	11.95	12.35	12.77	13.23	13.77	14.35	−0.17	−0.22
0 45	17.87	18.45	19.08	19.78	20.57	21.45	−0.27	−0 32
1 00	23.70	24.47	25.30	26.23	27.28	28.45	−0.35	−0.42
1 15	29.43	30.37	31.42	32.57	33.87	35.30	−0.43	−0.53
1 30	35.02	36.13	37.38	38.77	40.30	42.02	−0.52	−0.63
1 45	40.45	41.75	43.18	44.77	46.55	48.52	−0.60	−0.72
2 00	45.70	47 17	48.78	50.58	52.58	54.82	−0.67	−0.82
2 15	50.75	52.37	54.17	56.15	58.37	60.85	−0.75	−0.90
2 30	55.58	57.35	59.30	61.48	63.90	66.62	−0.82	−0.98
2 45	60.15	62.07	64.18	66.53	69.15	72.07	−0.88	−1.07
3 00	64.47	66.50	68.77	71.28	74.08	77.20	−0.95	−1.13
3 15	68.48	70.65	73.05	75.72	78.68	82.00	−1.00	−1.20
3 30	72.20	74.48	77.00	79.82	82.93	86.42	−1.05	−1.27
3 45	75.60	77 98	80.62	83.55	86.82	90.45	−1.10	−1.33
4 00	78.67	81.13	83.88	86.90	90.30	94.08	−1.15	−1.38
4 15	81.38	83.93	86.77	89.90	93.40	97.28	−1.20	−1.43
4 30	83.75	86.37	89.27	92.50	96.10	100.07	−1.23	−1.47
4 45	85.75	88.43	91.38	94.68	98.33	102.42	−1.25	−1.50
5 00	87.38	90.10	93.10	96.45	100.17	104.32	−1.27	−1.52
5 15	88.63	91.38	94.42	97.80	101.57	105.75	−1.28	−1.53
5 30	89.50	92.27	95.33	98.73	102.52	106.73	−1.30	−1.55
5 45	89.97	92.75	95.83	99.23	103.03	107.27	−1.30	−1.57
6 00	90.07	92.83	95.92	99.32	103.10	107.32	−1.30	−1.55
6 15	89.77	92.53	95 58	98.97	102.73	106.93	−1.30	−1.55
6 30	89.10	91.83	94.85	98.20	101.93	106.07	−1.28	−1.53
6 45	88.05	90.73	93.72	97.02	100 68	104.77	−1.27	−1.52
7 00	86.62	89.25	92.18	95 42	99.02	103.03	−1 25	−1.48
7 15	84.83	87.40	90.27	93.42	96.93	100.85	−1 22	−1.45
7 30	82.68	85.18	87.97	91.03	94.45	98.27	−1.20	−1.42
7 45	80.18	82.60	85.28	88.27	91.57	95.25	−1.15	−1.37
8 00	77.35	79.68	82.27	85.13	88.32	91.85	−1.10	−1.32
8 15	74.20	76.43	78.90	81.65	84.68	88.08	−1.07	−1.27
8 30	70.73	72.87	75.20	77.82	80.72	83.93	−1.02	−1.20
8 45	66.98	68.98	71.20	73.67	76.40	79.45	−0.97	−1.13
9 00	62.95	64.83	66.92	69.22	71.78	74.63	−0.90	−1.07
9 15	58.65	60.40	62.33	64.48	66.87	69.53	−0.83	−0.98
9 30	54.12	55.73	57.52	59.48	61.68	64.13	−0.77	−0.92
9 45	49.35	50.82	52.45	54.25	56.25	58.48	−0.70	−0.83
10 00	44.40	45.72	47.17	48.78	50.58	52.58	−0.63	−0.75
10 15	39.25	40.42	41.70	43.13	44.72	46.48	−0.57	−0.67
10 30	33.95	34.95	36.07	37.30	38.67	40.20	−0.48	−0.57
10 45	28.50	29.35	30.28	31.32	32.47	33.75	−0.40	−0.48
11 00	22.95	23.62	24.37	25.20	26.13	27.15	−0.33	−0.38
11 15	17.28	17.80	18.37	19.00	19.70	20.47	−0.25	−0.30
11 30	11.57	11.90	12.28	12.70	13.17	13.72	−0.17	−0.20
11 45	05.78	05.97	06.15	06.37	06.60	06.85	−0.08	−0.10
Elongation Azimuth	1° 30.08′	1° 32.85′	1° 35.93′	1° 39.33′	1° 43.13′	1° 47.35′	−1.30′	−1 55
Hour Angle	h m s 5 56 08	h m s 5 55 51	h m s 5 55 33	h m s 5 55 14	h m s 5 54 53	h m s 5 54 31	s +3	s +3

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the image of the sun into the center of the solar telescope when the line of collimation of the solar telescope makes an angle with the line of collimation of the main telescope equal to the sun's declination corrected for refraction, and the line of collimation of the main telescope is elevated at an angle equal to the co-latitude of the place of observation, it is rigidly necessary that the line of collimation of the main telescope lie in a true meridian as shown in (b), Fig. 23a.

The elementary lines of a solar attachment are: (1) The polar axis; (2) the line of collimation of the solar telescope; (3) the attached level line. These lines should have

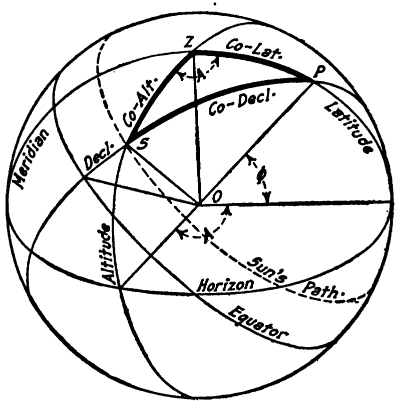

Fig. 23b.

the following relations: (1) The polar axis should be perpendicular to the line of collimation of the solar telescope and the horizontal axis of the main telescope; (2) the line of collimation of the solar telescope and the attached level line should be parallel. The methods of making these adjustments are obvious.

The declination of the sun (see Fig. 23b for explanation of astronomical terms) for the place of observation is found by adding, algebraically, the hourly change multiplied by the number of hours since Greenwich mean noon (6 A. M., 90th Meridian) to the declination of the sun, as given in the solar ephemeris for Greenwich mean noon for the given date. The setting (apparent declination) is found

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by taking the algebraic sum of the refraction correction and the declination of the sun obtained as above. The refraction is always plus; the declination is plus when the sun is north and minus when south of the celestial equator; and the hourly change in declination is plus when the sun is moving north and minus when moving south.

The "Pocket Solar Ephemeris and Refraction Tables for Use with Saegmuller's Solar Attachment," is given in "Handbook for Engineers" by George N. Saegmuller, published by Bausch & Lomb Optical Co., Rochester, N. Y. An "Ephemeris of the Sun and Polaris, and Tables of Azimuths of Polaris" is published by the General Land Office for each year. This Ephemeris may be obtained by addressing the Department of Interior, General Land Office, Washington, D. C., or may be purchased at a price of 5 cents per copy from the Government Printing Office, Washington, D. C. The true local mean solar time should always be used, and may be obtained from standard or railroad time by adding or subtracting four minutes for each degree that the place of observation is east or west of the standard meridian. The mean refraction of the sun for different altitudes is given in Table V.)

TABLE V.

Mean Refraction of the Sun.

Barometer 30 Inches, Temperature 50° Fahr.

(Refraction makes observed altitude too large.)

Altitude, Degrees	Refraction, Minutes	Altitude, Degrees	Refraction, Minutes	Altitude, Degrees	Refraction, Minutes
10	5.10	24	2.02	50	0.70
12	4 25	26	1.83	55	0.58
14	3.62	28	1.67	60	0.48
16	3 17	30	1.53	65	0.38
18	2 80	35	1.25	70	0.30
20	2 48	40	1.03	80	0.13
22	2.22	45	0.85	90	0.00

(1) Calculate the apparent declination (setting) of the sun for several different times, varying by 15 minutes, between 8 and 10 o'clock A. M. and 2 and 4 o'clock P. M. (2) Set the transit over the hub, level up very carefully with the attached bubble, and very carefully adjust the main transit and solar attachment. Determine the index error of the vertical circle, and either correct it or apply it to all vertical angles with its proper sign. (3) Level the transit

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DETERMINATION OF TRUE MERIDIAN WITH SOLAR TRANSIT.

No. Obs.	Time of Obs.	Declination	Refrac. Cor.	Setting (App. Dec.)	Azimuth
1	8h 30m	+19° 54'.8	+0'.7	+19° 55'.5	22° 11'
2	8:45	+19° 54'.9	+0'.6	+19° 55'.5	22° 12'
3	9:00	+19° 55'.1	+0'.6	+19° 55'.7	22° 10'
4	9:15	+19° 55'.2	+0'.5	+19° 55'.7	22° 13'
5	9:30	+19° 55'.3	+0'.5	+19° 55'.8	22° 11'
	P.M.				
6	2:30	+19° 58'.0	+0'.5	+19° 58'.5	22° 09'
7	2:45	+19° 58'.1	+0'.5	+19° 58'.6	22° 10'
8	3:00	+19° 58'.2	+0'.6	+19° 58'.8	22° 09'
9	3:15	+19° 58'.4	+0'.6	+19° 59'.0	22° 11'
10	3:30	+19° 58'.5	+0'.7	+19° 59'.2	22° 09'
					22° 10'.5

True Azimuth of Line = 22° 10'.0

Allowable error 01'. Error = 0° 00'.5

Calculation of Setting (App. Decl.) 8h 30m A.M.

Latitude 40° 06' N.; Longitude 88° 15' W.

App. Declination Greenwich Mean Noon. (6h 07m A.M. here), May 20, 1901, +19° 53'.5

Correction for 2h 23m = 2.4 × 0.53, + 01'.3

Declination of Sun at 8:30 A.M. = +19° 54'.8

Refraction Cor.—3h 30m before noon = + 0.7

App. Decl. at 8:30 A.M. = +19° 55'.5

Apparent Decl. (Setting) for the other times was calculated in like manner.

Observers, J. Doe & R. Roe.

May 20, 1901. (4 Hrs.) Clear, & warm.

Buff & Berger Transit No. 9, with Saegmuller Solar Attachment, hubs, axe, watch set to keep Railroad Time, Solar Ephemeris (Handbook for Engineers, By Geo. N. Saegmuller, Bausch & Lomb Optical Co. Rochester, N.Y.)

Tested Transit and Solar Attachment and found both in perfect adjustment.

Set transit over hub, leveled up very carefully with long bubble, found Index Error of Vert. Circle = Zero.

Set off −19° 55'.5 (−App. Decl.) on Vertical circle and leveled solar telescope by means of its attached bubble.

Set off +49° 54' (Co-Lat.) on Vertical circle—Telescope pointed S. both times.

Set A vernier at zero and sighted at Sta. 3 with lower motion.

Unclamped upper motion, moved transit on vertical axis and solar on its polar axis, and brought image of sun into center of solar at 8h 30m A.M. Mean Solar Time. (M.S. Time = R.R. time + 7.0m)

Read azimuth. Repeated until 10 values were determined. (5 A.M. & 5 P.M.)

very carefully with the attached bubble. Bring the line of collimation of the main telescope and the line of collimation of the solar telescope parallel by sighting on a distant point, and point the main telescope south. (4) Set off the apparent declination (setting) with opposite sign on the vertical circle, i. e., dip the telescope when the declination is plus (north), and elevate the telescope when the declination is minus (south). (5) Level the solar telescope by means of its attached bubble. (6) Set off a plus vertical angle on the vertical circle equal to the co-latitude of the place. (7) Set the *A* vernier at zero and sight at a point on the true meridian. (8) Unclamp the upper motion, turn the main telescope about its upper motion and the solar telescope about its polar axis until the image of the sun is brought to the center of the cross lines in the solar telescope at the time for which the declination was computed, and clamp the upper motion. The line of collimation of the main telescope will then be in the meridian. (9) Read the horizontal plates. The reading will be the azimuth of the line first sighted on. (10) Repeat, using the setting corresponding to the time of observation, until ten values are obtained. If possible make five determinations in the A. M.,

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and five in the P. M., about the same time from noon. The mean of these observations will eliminate instrumental errors. The most favorable time for making observations with a solar transit is from 8 to 10 A. M. and from 2 to 4 P. M. (11) Determine the true azimuth of the given line. The error of the determination of the meridian should not exceed one minute. Record as in the form.

SOLAR OBSERVATION.

Telescope	Horizontal Circle Readings		Vertical Circle Readings	Date & Time
	On Mark	On Sun		
Direct	180°50'00"	100°41'00"	45°59'00"	May 15, 1901
Rev'd	0°50'00"	280°47'30"	46°07'00"	9 A.M. Mean Solar
Mean	180°50'00"	100°44'15"	46°03'00"	Time

COMPUTATION

Declination at Greenwich Noon, 6 A·M·St'd Time 90th Md. = 18°45'56·1" N.
Hourly Change = 35·8". Change for 3 hrs = 3 × 35·8" = 1'47·4" N.
Declination at 9 A·M· = 18°47'43·5" N.

Average Vertical Angle by Observation 46°03'00"
Correction for Refraction 00'56"
True Altitude 46°02'04"

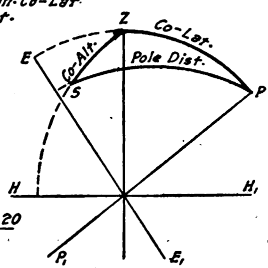
Latitude of Observatory, U. of I. 40°06'00"
Station 100' N. 00"
Latitude of Station 40°06'00"

$$\text{Cos} \cdot \tfrac{1}{2}\,PZS = \sqrt{\frac{\text{Sin} \cdot \tfrac{1}{2}S \times \text{Sin}(\tfrac{1}{2}S - \text{Pole Dist.})}{\text{Sin} \cdot \text{Co-Alt.} \times \text{Sin} \cdot \text{Co-Lat.}}}$$

where S = Pole Dist. + Co-Alt. + Co-Lat.

Pole Dist. = 71°12'16"
Co-Alt = 43°57'56"
Co-Lat. = 49°54'00"
S = 165°04'12"
½S = 82°32'06"
Pole Dist. = 71°12'16"
½S - Pole Dist. = 11°19'50"

Log·Sin· 82°32'06" = 9·99630
Log·Sin· 11°19'50" = 9·29329
Co-log·Sin· 43°57'56" = 0·15850
Co-log·Sin· 49°54'00" = 0·11638
2)19·56447 - 20
Log·Cos·½PZS = 9·78224
½PZS = 52°43'18"
PZS = 105°26'36"



Azimuth of Sun from the North = 105°26'36"
Angle between Sun and Mark = 80°05'45"
Observed Azimuth from North Station to Mark = 185°32'21"
True Azimuth from North Station to Mark = 185°32'00"
Error 21"

PROBLEM D17. DETERMINATION OF TRUE MERIDIAN BY DIRECT OBSERVATION ON THE SUN.

(a) *Equipment.*—Complete transit, reading glass, hub, axe, colored eyepiece or colored shade to fit over objective, good watch set to keep standard time, solar ephemeris.

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(b) *Problem.*—Determine a true meridian by a direct observation on the sun with a transit.

(c) *Methods.*—(1) Set the transit over a hub and level up very carefully with the attached bubble. (2) Test the adjustments of the transit very carefully, and determine the index error of the vertical circle. (3) Sight on a horizontal mark and read the horizontal plates. (4) Sight at the sun directly, by the aid of the colored eyepiece or colored glass shade, and bring his image tangent to the horizontal and vertical wires. (5) Read vertical circle and horizontal plates. (6) Reverse the telescope and make a second observation the same as the first except that the sun should be in the opposite quarter of the field of view. (7) The mean of the vertical and horizontal circle readings will give the apparent altitude and plate reading of the sun's center. (8) Observe the standard time of the observation and reduce to mean solar time by adding or subtracting 4 minutes for each degree that the place of observation is east or west of the standard meridian. (9) Calculate the angle PZS in the P Z S triangle as shown in the accompanying form. Refraction makes the sun appear too high and it should therefore be subtracted. (10) Determine the azimuth of the line from the hub to the mark and check the observed azimuth. (The data for this problem may be obtained from Saegmuller's "Solar Ephemeris and Refraction Tables," or from the "Ephemeris of the Sun and Polaris, and Tables of Azimuths of Polaris," by the General Land Office, mentioned in Problem D16. Mean refraction of the sun for different altitudes is given in Table V.) (11) Where considerable accuracy is desired, make a second observation when the sun is about the same distance on the opposite side of the meridian. The error of the determination should not exceed 1 minute.

PROBLEM D18. COMPARISON OF TRANSIT TELESCOPES.

(a) *Equipment.*—Five engineers' transits.

(b) *Problem.*—Make a critical comparison of the telescopes of five engineers' transits.

(c) *Methods.*—Follow the methods outlined in the comparison of level telescopes.

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PROBLEM D19. TEST OF A TRANSIT.

(a) *Equipment.*—Transit, reading glass, leveling rod, chaining pins, foot rule.

(b) *Problem.*—Test the following adjustments of an assigned transit: (1) Test the graduation for eccentricity. (2) Test the plate levels to see if they are perpendicular to the vertical axis. (3) Test the line of collimation to see if it is perpendicular to the horizontal axis. (4) Test the horizontal axis to see if it is perpendicular to the vertical axis. (5) Test the level under the telescope to see if the tangent to the tube at the center is parallel to the line of collimation. (6) Test the vertical circle to see if the vernier reads zero when the line of sight is horizontal.

(c) *Methods.*—Make the tests as described in the first part of this chapter but do not make any of the adjustments or tamper with any of the parts of the instrument. Check each test. Make a careful record of the methods and errors, including a statement of the manner of doing correct work with each adjustment out.

PROBLEM D20. ADJUSTMENT OF A TRANSIT.

(a) *Equipment.*—Transit, reading glass, leveling rod, chaining pins, adjusting pin, small screw driver.

(c) *Methods.*—Make the following tests and adjustments of an assigned transit that has been thrown out of adjustment by the instructor: (1) Test the graduation for eccentricity. (2) Adjust the plate levels perpendicular to the vertical axis. (3) Adjust the line of collimation perpendicular to the horizontal axis. (4) Adjust the horizontal axis perpendicular to the vertical axis. (5) Adjust the level under the telescope parallel to the line of collimation. (6) Adjust the zero of the vertical circle to read zero when the line of sight is horizontal. (7) Center the eyepiece.

(c) *Methods.*—Make the tests and adjustments as described in the first part of this chapter. Use extreme care in manipulating the screws and if any of the parts stick or work harshly, call the instructor's attention before proceeding. Repeat the tests and adjustments. Make a careful record of methods and errors.

PROBLEM D21. SKETCHING A TRANSIT.

(a) *Equipment.*—Engineers' transit.

(b) *Problem.*—Make a first-class sketch of an engineers' transit.

(c) *Methods.*—(See similar problem with the level.)

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PROBLEM D22. ERROR OF SETTING FLAG POLE WITH TRANSIT.

(a) *Equipment.*—Transit, iron flag pole, flat stake 1″x 2″x 15″, foot rule.

(b) *Problem.*—Determine the probable error of setting a flag pole with the transit at a distance of 300 feet. Repeat for 600 feet.

ERROR OF SETTING FLAG POLE

Distance Ft.	No. of Setting	Distance In.	d In.	d^2	
300	1	1·18	0·18	0·0324	
	2	1·38	·02	·0004	
	3	1·30	·06	·0036	
	4	1·53	·17	·0289	
	5	1·32	·04	·0016	
	6	1·38	·02	·0004	
	7	1·29	·07	·0049	
	8	1·46	·10	·0100	
	9	1·46	·10	·0100	
	10	1·30	·06	·0036	
	Mean	1·36		0·0958	$=\Sigma d^2$
600	1	1·14	0·25	0·0625	
	2	1·56	·17	·0289	
	3	1·14	·25	·0625	
	4	1·22	·17	·0289	
	5	1·76	·37	·1369	
	6	1·55	·16	·0256	
	7	1·23	·16	·0256	
	8	1·10	·29	·0841	
	9	1·55	·16	·0256	
	10	1·65	·26	·0676	
	Mean	1·39		0·5472	$=\Sigma d^2$

Observers, J·Doe & R·Roe·
WITH ENGINEERS' TRANSIT·
Dec·6, 1914. (2 hours) Cool and Quiet.
Used Buff & Berger Transit, Locker No·9,
Flat stake, 1″x 2″x 15,″ and Iron Flag pole·
Sighted at iron Flag pole set on stake which had been placed on ground at about 300 Ft. from the Transit, and clamped both plates; then measured the distance in inches from a line drawn across the board·
With both plates clamped, lined in the rod 10 times in all, the Flagman noting the distance from the line·
The pole was shifted each time·
Repeated test for 600 Ft.

Probable Error for 300 Ft.

$$E_1 = 0·6745\sqrt{\frac{\Sigma d^2}{n-1}} = 0·6745\sqrt{\frac{·0958}{9}} = 0·103 \text{ in}$$

$$E_m = \frac{E_1}{\sqrt{n}} = \frac{0·103}{\sqrt{10}} = 0·032 \text{ in.} = 0·0027 \text{ Ft.}$$

$$E_m \text{ (Angle)} = \tan^{-1}\frac{0·0027}{300} = 1''·8.$$

Probable Error for 600 Ft.

$$E_1 = 0·6745\sqrt{\frac{·5472}{9}} = 0·247 \text{ in.}$$

$$E_m = \frac{0·247}{\sqrt{10}} = 0·078 \text{ in.} = 0·0065 \text{ Ft.}$$

$$E_m \text{ (Angle)} = \tan^{-1}\frac{0·0065}{600} = 2''·2$$

(c) *Methods.*—(1) Set the transit up and sight at the flag pole plumbed near the middle of the stake at a distance of about 300 feet. (2) Measure the distance from the point of the flag pole to a mark on the stake. (3) Keep the vertical axis clamped, and move the pole to one side. (4) Set the pole with the transit, and measure the distance from the first line. (5) Repeat until at least ten consecutive satisfactory results are obtained. (6) Compute the probable error of a single observation and of the mean of all the observations (see chapter on errors of surveying), and reduce the mean error to its angular value. (7) Repeat for 600 feet. Determine distances by pacing. Follow the form.

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PROBLEM D23. REPORT ON DIFFERENT MAKES AND TYPES OF TRANSITS.

(a) *Equipment.*—Department equipment, catalogs of the principal makers of engineers' transits.

(b) *Problem.*—Make a critical comparison of the several types of transits made by the different makers.

(c) *Methods.*—(See similar problem with the level.)

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CHAPTER VI.

TOPOGRAPHIC SURVEYING.

Topographic Map.—A topographic map is one which shows with practical accuracy all the drainage, culture, and relief features that the scale of the map will permit. These features may be grouped under three heads as follows: (1) the culture, or features constructed by man, as cities, villages, roads; (2) the hypsography, or relief of surface forms, as hills, valleys, plains; (3) the hydrography, or water features, as ponds, streams, lakes. The culture is usually represented by conventional symbols. The surface forms are shown by contours (lines of equal height), (a), Fig. 24, or hachures, (b), Fig. 24. The water features are shown by soundings, conventional signs for bars, etc.

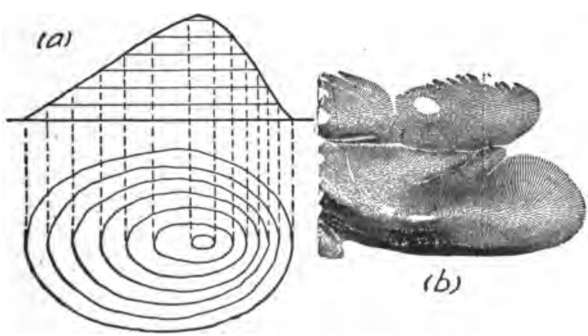

Fig. 24.

Topographic maps may be divided into two classes depending upon the scale of the map. Small scale topographic maps are made by the U. S. Coast and Geodetic Survey and the U. S. Geological Survey, and are drawn to a scale of 1 : 62,500, 1 : 125,000 or 1 : 250,000 with corresponding contour intervals of 5 to 50, 10 to 100, and 200 to 250 feet. These maps show the streams, highways, railroads, canals, etc., in

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outline but do not show any features of a temporary character. For topographic symbols, see Chapter XI.

Large scale topographic maps are drawn to a scale of 400 feet to 1 inch (1.4800), or greater, with contour intervals from 1 to 10 feet depending upon whether the ground is flat or hilly Roads, streets, dwellings, streams, etc., are drawn to scale. Features too small to be properly represented when drawn to scale are drawn out of proportion to the scale of the map.

Topographic Survey.—The object of a topographic survey is the production of a topographic map, and hence neither time nor money should be wastefully expended in obtaining field data more refined than the needs of the mapping demand. A topographic survey may be divided into three parts: (1) the reconnaissance; (2) the skeleton of the survey; (3) filling in the details.

Reconnaissance.—The reconnaissance is a rapid preliminary survey to determine the best methods to use in making the survey and the location of the principal points of control. A careful reconnaissance enables the topographer to choose methods that are certain to result in a better map and a distinct saving of time.

Skeleton.—There are three general methods of locating the skeleton of a topographic survey: (1) tie line survey with chain only, (2) traverse method with transit or compass; (3) triangulation system, (f), Fig. 30. The first method is used for the survey of small tracts. The second method, in which the distances are measured with the chain, tape, or stadia, is used on railroad and similar surveys. The third method, in which triangulation stations are connected with each other and with a carefully measured base line and base of verification, is used on surveys for small scale maps and on detailed or special surveys, such as surveys of cities and reservoir sites.

Filling in Details.—There are three general methods employed for filling in the details: (1) with transit or compass and chain; (2) with transit and stadia; (3) with plane table and stadia. The transit and stadia are used by the Mississippi and Missouri River Commissions. The plane table and stadia are used by the U. S. Coast and Geodetic and the U. S. Geological Surveys.

Topographic City Survey.—A topographic city survey is one of the best examples of a survey for a large scale map. It is usually based on a system of triangulation executed with precision and connected with carefully measured base

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lines. The details of the survey are usually taken up in the following order: (1) reconnaissance and location of triangulation stations; (2) measurement of base line and base of verification; (3) measurement of angles by repetition; (4) establishment of bench marks by running duplicate levels; (5) adjustment of angles of triangulation system; (6) computation of sides, azimuths and coordinates; (7) filling in details, usually with transit and stadia; (8) plotting of triangulation and other important points on the map by rectangular coordinates; (9) plotting the details and completing the map. The instructions given on the succeeding pages are for a survey of this type.

Hydrographic Survey.—Hydrographic surveying is divided into river and marine. The first includes the location of bars and obstructions to navigation, and the determination of the areas of cross-section, the amount of sediment carried, etc. The second includes the making of soundings, location of bars, ledges, buoys, etc. The depth of the water is determined by making soundings with a lead or rod, and the velocity is gaged by means of floats or a current meter, (d), Fig. 31.

Soundings are located: (1) by two angles read simultaneously from both ends of a line on the shore, (f), Fig. 31; (2) by keeping the boat in line with two flags on shore, and determining the position on the line by means of an angle read on the shore, or by a time interval; (3) by intersecting ranges, (g), Fig. 31; (4) by stretching a rope or wire across the stream; (5) by measuring with a sextant in the boat at the instant that the sounding is taken two angles to three known points on the shore, (c), Fig. 31; the point is located by solving the three point problem graphically with the three arm protractor, (e), Fig. 31; (6) by locating the position of the boat at the instant that the soundings are taken with transit and stadia. The first three methods are used on small river or lake surveys. The fourth method is used where soundings are taken at frequent intervals. The fifth method has been used almost exclusively in locating soundings in harbors, lakes, and large rivers. The sixth method is rapidly coming into general use and promises to be the favorite method.

THE STADIA.

Description.—The stadia is a device for measuring distances by reading an intercept on a graduated rod. The stadia-hairs, shown in (g), Fig. 27, are carried on the same

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reticule as the cross-hairs and are placed equidistant from the horizontal hair. The stadia-hairs are sometimes placed on a separate reticule and made adjustable. It is, however, considered better practice by most engineers to have the stadia-hairs fixed and use an interval factor, rather than try to space the hairs to suit a rod or to graduate a rod to suit an interval factor.

Stadia Rods.—Stadia rods are always of the self reading type. In Fig. 27, (a) and (b) are the kind used on the U. S. Coast Survey; (c) on the U. S. Lake Survey; (d) and (c) by the U. S. Engineers. A target for marking on the rod the height of the horizontal axis of the transit above the station occupied is shown in (f), Fig. 27.

Theory of the Stadia.—In Fig. 25, by the principles of optics, rays of light passing from points *A* and *B* on the rod through the objective so as to emerge parallel and pass through the stadia-hairs *a* and *b*, respectively, must inter-

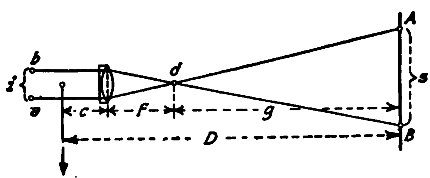

Fig. 25.

sect at the principal focal point *d* in front of the objective; therefore the rod intercept, *s* is proportional to the distance, *g* from the principal focal point in front of the objective.

Stadia Formula For Horizontal Line of Sight and Vertical Rod.—In Fig. 25, from similar triangles we have

$$s : g :: i : f \qquad (1)$$

From which
$$g = \frac{f}{i} s = k.s \qquad (2)$$

and
$$D = k.s + (c + f) \qquad (3)$$

Stadia Formula For Inclined Line of Sight and Vertical Rod.—In Fig. 26 we have

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$$BD = AE. \cos a \qquad \text{(approx.)} \qquad (4)$$

and $$D = k.\ s.\ \cos a + (c + f) \qquad (5)$$

but $$H = D.\ \cos a$$

$$= k.\ s.\ \cos^2 a + (c + f)\ \cos a \qquad (7)$$

$$= k.\ s - k.\ s.\ \sin^2 a + (c + f)\ \cos a \qquad (8)$$

also $$V = D.\ \sin a \qquad (9)$$

$$= k.\ s.\ \sin a.\ \cos a + (c + f)\ \sin a \qquad (10)$$

$$= \tfrac{1}{2}\ k.\ s.\ \sin 2\ a + (c + f)\ \sin a \qquad (11)$$

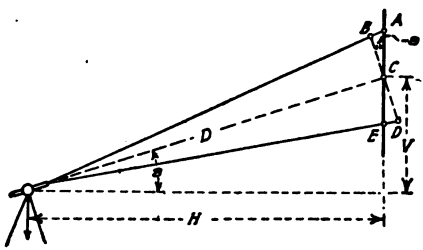

Fig. 26.

Use of the Stadia.—The transit is set up over a station of known elevation and with a given direction or azimuth to another visible station; the height of the line of collimation above the top of the station is determined either by holding the rod beside the instrument and setting the target, or preferably by graduating one leg of the tripod and using the plumb bob; then with the transit oriented on a given line, "shots" are taken to representative points, and record made of the rod intercept, vertical angle and azimuth. In reading the intercept the middle hair is first set roughly on the target, then one stadia-hair is set at the nearest foot-mark on the rod and the intercept read with the other stadia-hair, after which the precise vertical angle is taken, and the azimuth is read.

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Reducing the Notes.—The notes may be reduced by means of tables, diagrams, or a special slide rule. The slide rule is the most rapid. There are several forms of stadia slide rule that are very accurate and are convenient for field use.

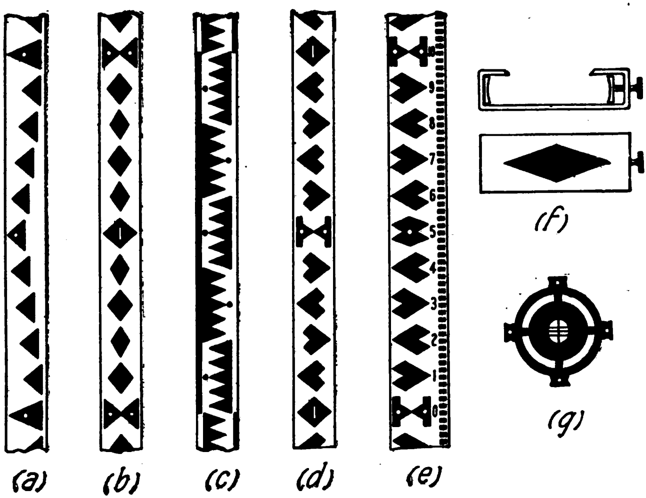

Fig. 27.

THE PLANE TABLE.

Description.—The plane table consists of an alidade, carrying a line of sight and a ruler with a fiducial edge. The alidade is free to move on a drawing board mounted on a tripod. The drawing board is leveled by means of plate levels. The line of sight should make a fixed horizontal angle with the fiducial edge of the ruler. The complete plane table is a transit in which the horizontal limb has been replaced by a drawing board.

There are three general types of plane tables: (1) the Coast Survey plane table, (a), Fig. 28; (2) the Johnson plane table, (b), Fig. 28; (3) the Gannet plane table, (d), Fig. 29.

Use of the Plane Table.—In making a survey with the plane table the angles are measured graphically and the

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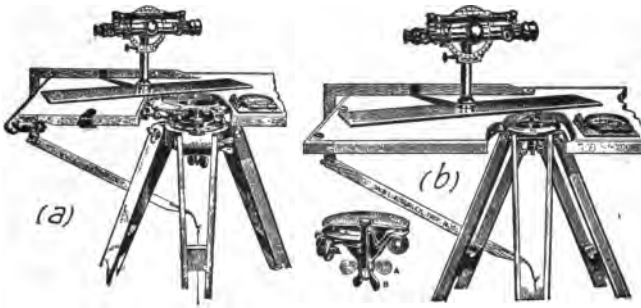
lines and points are plotted in the field. The principal methods of making a survey with a plane table are: (1) radiation; (2) traversing; (3) intersection; (4) resection.

Radiation.—In this method a convenient point on the



Complete Plane Tables.
Fig. 28.

paper is set over a selected point in the field, and the table clamped. The line of sight is then directed towards each point to be located in turn and a line is drawn along the

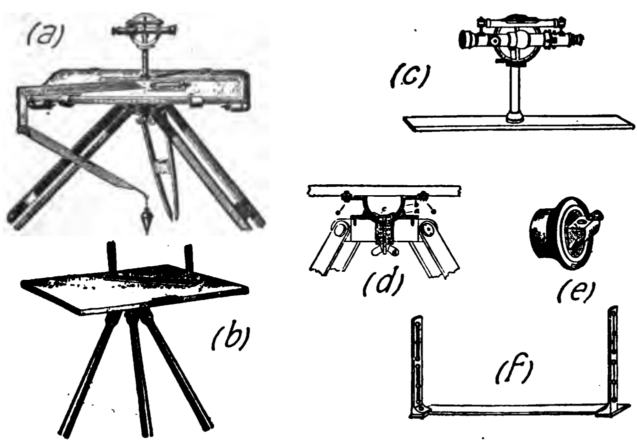


Fig. 29.

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fiducial edge of the ruler. The distances, which may be determined by measuring with chain, tape or stadia, are plotted to a convenient scale, (a), Fig. 30.

Traversing.—This method is practically the same as traversing with a transit, (b), Fig. 30. Care should be used in orienting the plane table to get the point on the paper over the corresponding point on the ground as nearly as the character of the work requires.

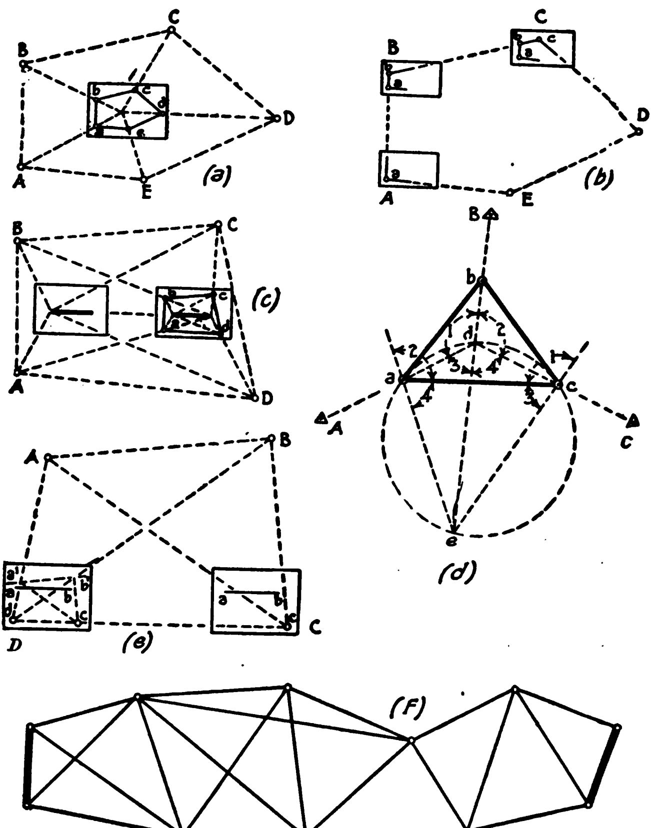

Fig. 30.

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Intersection.—In this method the points are located by intersecting lines drawn from the ends of a measured base line, (c), Fig. 30.

Resection.—In the resection method the plane table is set up at a random point and oriented with respect to either three or two given points, which gives rise to two methods known respectively as the three-point and two-point problems.

Three Point Problem.—Where three points are located on the map and are visible but inaccessible, the plane table is oriented by solving the "three point problem." There are several solutions, the best known of which are: (1) the mechanical solution; (2) the Coast Survey solution; (3) Bessel's solution; (4) algebraic solution. The problem is indeterminate if a circle can be passed through the four points.

In the mechanical solution the two angles subtended by the three points are plotted graphically on a piece of tracing paper, and the point is located by placing the tracing paper over the plotted points.

In Bessell's solution, (d), Fig. 30, *a*, *b*, *c* are three points on the map corresponding to the three points, *A*, *B*, *C* on the ground, and *D* is the random point at the instrument whose location, *d*, it is desired to find on the map. Construct the angle 1 with vertex at point *c* as follows: Sight along the line *ca* at the point *A*, and clamp the vertical axis. Then center the alidade on *c* and sight at *B* by moving the alidade, and draw a line along the edge of the ruler. Construct the angle 2 with vertex at *a* in the same manner. The line joining *b* and *e* will pass through the point *d* required. Orient the board by sighting at *B* with the line of sight along the line *e b*, and locate *d* by resection.

Two Point Problem.—To orient the board when only two points are plotted, proceed as follows: Select a fourth point, *c*, that is visible, and with these two points as the ends of a base line, (e), Fig. 30, laid off to a convenient scale, locate two points *a′* and *b′* on the map by intersection. The error of orienting the board will be the angle between the lines *a–b* and *a′–b′*. The table can now be oriented and the desired point located on the board by resection.

Adjustments.—The adjustments of the plane table are: (1) the plate levels; (2) the line of collimation; (3) the horizontal axis; (4) the attached level. These adjustments are practically the same as those for the transit.

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THE SEXTANT.

Description.—The sextant consists of an arc of 60°, with each half degree numbered as a whole degree, (a), Fig. 31, combined with mirrors so arranged that angles can be measured to 120°.

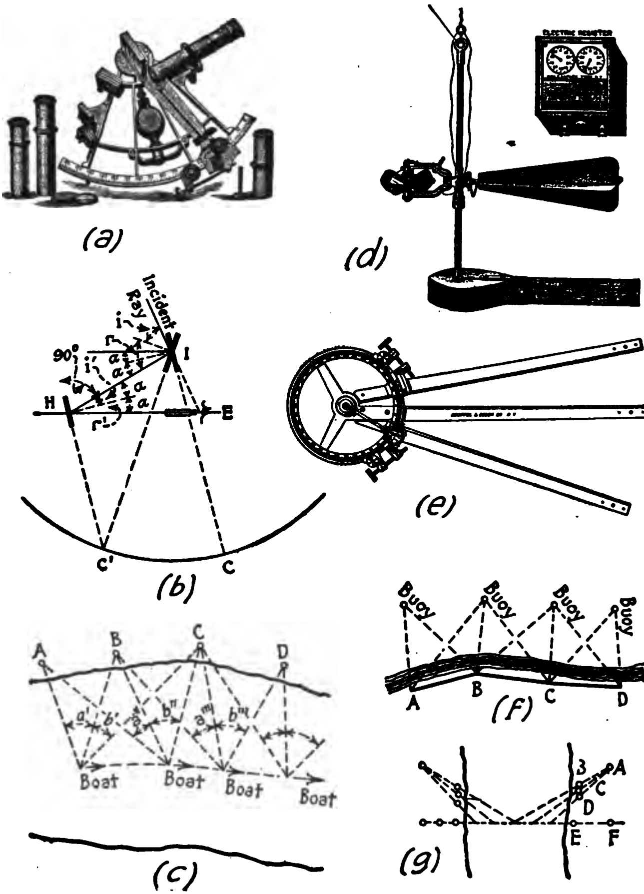

Fig. 31.

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Theory.—The principle upon which the sextant is constructed is that if a ray of light is reflected successively between two plane mirrors, the angle between the first and last direction of the ray is twice the angle of the mirrors.

In (b), Fig. 31, the angles of incidence and reflection are equal,

$$i = r \text{ and } i' = r', \text{ and}$$

$$E = (i + r) - (i' + r') = 2(r - r')$$

$$C' = (90° - i') - (90° - r) = (r - r')$$

and therefore $E = 2\,C'$

but $C' =$ angle CIC′, by geometry, since the mirrors are parallel for a zero reading.

Use of the Sextant.—To measure an angle between two objects with a sextant, bring its plane into the plane of the two objects; sight at the fainter object with the telescope and bring the two images into coincidence. The reading is the angle sought. The angle will not be the true horizontal angle between the objects unless the objects are in the same level with the observer. Since the true vertex of the measured angle shifts for different angles, the sextant should not be used for measuring small angles between objects near at hand.

Adjustments, *Index Glass.*—To make the index glass, I, perpendicular to the plane of the limb, bring the vernier to about the middle of the arc and examine the arc and its image in the index glass. If the glass is perpendicular to the plane of the limb, the image of the reflected and direct portions will form a continuous curve. Adjust the glass by means of the screws at the base.

Horizon Glass.—To make the horizon glass, *H*, parallel to the index glass, *I*, for a zero reading. With the vernier set to read zero, sight at a star and note if the two images are in exact coincidence. If not, adjust the horizon glass until they are. If the horizon glass cannot be adjusted, bring the images into coincidence by moving the arm and read the vernier. This reading is the index error which must be applied with its proper sign to all the angles measured.

Line of Collimation.—To make the line of collimation parallel to the limb. Place the sextant on a plane surface

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and sight at a point about 20 feet away. Place two objects of equal height on the extreme ends of the limb, and note whether both lines of sight are parallel. If not, adjust the telescope by means of the screws in the ring that carries it.

PROBLEMS IN TOPOGRAPHIC SURVEYING.

PROBLEM E1. DETERMINATION OF STADIA CONSTANTS OF TRANSIT WITH FIXED STADIA-HAIRS.

(a) *Equipment.*—Complete transit, stadia rod, steel tape, set chaining pins, foot rule.

(b) *Problem.*—Determine the stadia constants c, f and k for an assigned transit.

(c) *Methods.*—(1) Set up the transit and set ten chaining pins in line about 100 feet apart on level ground. (2) Plumb the stadia rod by the side of the first pin. (3) Set the lower hair on an even foot or half foot mark keeping the telescope nearly level, and read the upper stadia-hair. (4) Record the intercept. (5) Read the intercept on the rod at the remaining pins. (6) Measure the distance from the center of the transit to each pin with the steel tape. (7) Focus the objective on a distant object, measure f (the distance from the plane of the cross-hairs to the center of the objective), and c (the distance from the center of the objective to the center of the instrument). (8) Calculate the value of the stadia ratio, k, for each distance by substituting in the fundamental stadia formula. (9) Take the arithmetical mean of the ten determinations as the true value. (10) Compute the probable error of a single observation and of the mean of all the observations. The interval factor should be determined by the instrument man under the conditions of actual work. The determination should be checked at frequent intervals during the progress of the field work. Follow the prescribed form.

PROBLEM E2. STADIA REDUCTION TABLE.

(a) *Equipment.*—(No instrumental equipment required.)

(b) *Problem.*—Compute a stadia reduction table giving the horizontal distances from a point in front of the objective equal to the principal focal distance for the stadia intervals from 0.01 feet to 10 feet, for the transit used in Problem E1.

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DETERMINATION OF STADIA CONSTANTS — FIXED HAIRS.

No.	S Ft.	D Ft.	D-(c+f) (c+f=1.17)	k Ft.	d	d^2
1	1.81	180.41	179.24	99.02	0.02	0.0004
2	2.70	268.40	267.23	98.96	0.08	0.0064
3	3.58	355.32	354.15	98.92	0.12	0.0144
4	4.05	400.89	399.72	98.72	0.32	0.1024
5	4.86	482.80	481.63	99.11	0.07	0.0049
6	5.61	556.30	555.13	99.20	0.16	0.0256
7	6.50	643.58	642.41	98.84	0.20	0.0400
8	7.90	786.93	785.76	99.47	0.43	0.1849
9	9.15	914.40	913.23	98.91	0.13	0.0169
10	10.31	1024.71	1023.54	99.26	0.22	0.0484
			Mean	99.04	$\Sigma d^2 =$	0.4443

$$E_1 = 0.67\sqrt{\frac{\Sigma d^2}{n-1}} = 0.67\sqrt{\frac{0.4443}{9}} = 0.15 \text{ Ft.}$$

$$E_m = \frac{E_1}{\sqrt{n}} = 0.67\sqrt{\frac{\Sigma d^2}{n(n-1)}} = 0.05 \text{ Ft.}$$

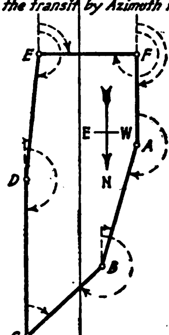
$c = 0.47$ Ft.
$f = 0.70$ Ft.
$c+f = 1.17$ Ft.

Observers, J. Doe & R. Roe.
Dec. 14, '14. (2 Hours) Cool & Cloudy.
Used Buff & Berger Transit, Locker 12, and Chaining Locker No. 38.
Set 10 chaining pins in line about 100 ft. apart on level ground.
With telescope of transit nearly level and determined intercept "s" at each pin by setting lower hair on a foot or half foot mark and reading upper hair.
Measured distance from center of transit to each pin with steel tape to nearest 0.01 Ft.
With object glass focused on a distant object, determined c and f by measuring distance from center of objective to center of the horizontal axis and the plane of the cross-wires respectively.
Determined the different values of k by substituting in the formula D=kS+c+f.

AZIMUTH TRAVERSE WITH TRANSIT AND STADIA.

Station Inst.	Obj.	Azimuth	Mag. Bearing	Distance Ft.	Vertical Angle	Elevation Ft.
A						718.00
	F	0°00'	N.4°00'W.	432	+0°20'	
	B	196°13'	S.12°10'W.	622	-0°40'	(-7.2)
B						710.8
	A	16°13'	N.12°10'E.	624	+0°38'	
	C	227°16'	S.43°15'W.	499	+0°50'	(+7.3)
C						718.1
	B	47°16'	N.43°10'E.	499	-0°50'	
	D	0°03'	N.4°05'W.	758	-1°10'	(-15.4)
D						702.7
	C	180°03'	S.4°00'E.	756	+1°12'	
	E	6°14'	N.2°20'E.	618	+0°56'	(+10.1)
E						712.8
	D	186°14'	S.2°20'W.	618	-0°56'	
	F	89°46'	N.85°15'E.	473	+0°54'	(+7.4)
F						720.2
	E	269°46'	S.85°15'W.	475	-0°54'	
	A	180°01'	S.4°05'E.	434	-0°20'	(-2.5)
A						717.7
		180°00				718.0
Error		= 0°01'			Error	= 0.3
Allowable error 2'.					Allowable Error	0.5 Ft.

Observers, J. Doe & R. Roe.
Dec. 15, 1914, (3 hours) Clear and Warm.
Used Buff & Berger Transit, Locker No. 12, and Stadia Board No. 6.
Stadia Constants: c+f=1.17 Ft., k=100.00
Sighted at target set at H.I., for Vert. Angle.
Oriented the transit by Azimuth reversals.



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(c) *Methods.*—(1) Prepare form for calculation. (2) Compute the horizontal distances by substituting the different values of *s* in the stadia formula. Compute *D′* for values of *s* varying from 0.01 foot to 0.1 foot varying by 0.01 foot; from 0.1 foot to 1 foot varying by 0.1 foot; and from 1 foot to 10 feet varying by 1 foot.

STADIA REDUCTION TABLE					
(c+f) = 1·20 feet.		k = 115·75		D = kS + (c+f) = D′ + (c+f)	
Stadia Reading S	Distance D′ = kS	Stadia Reading S	Distance D′ = kS	Stadia Reading S	Distance D′ = kS
0·01	1·2	0·1	11·6	1·0	115·8
·02	2·3	·2	23·2	2	231·5
·03	3·5	·3	34·7	3	347·2
·04	4·6	·4	46·3	4	463·0
·05	5·8	·5	57·9	5	578·8
·06	6·9	·6	69·4	6	694·5
·07	8·1	·7	81·0	7	810·2
·08	9·3	·8	92·6	8	926·0
·09	10·4	·9	104·2	9	1041·8
·10	11·6	1·0	115·8	10	1157·5

(To use the table, take the sum of the values of *D′* corresponding to the units, tenths and hundredths of *s* as given in the table. To the value of *D′* thus obtained add *c* plus *f*.)

PROBLEM E3. AZIMUTH TRAVERSE WITH TRANSIT AND STADIA.

(a) *Equipment.*—Complete transit, stadia rod, steel pocket tape.

(b) *Problem.*—Make a traverse of the perimeter of an assigned field with a transit and stadia.

(c) *Methods.*—(1) Set the transit over one corner of the field and set the *A* vernier to read the back azimuth of the preceding course. (2) Sight at a stadia rod held edgewise on the last station to the left with the telescope normal, and clamp the lower motion. (3) Read the intercept on the rod to the nearest 0.01 foot. (4) Sight at the target set at height of first station and read the vertical angle to the nearest minute. (The observer should measure the height of the horizontal axis above the station with the steel pocket tape, or one tripod leg may be graduated and the instrument height determined by swinging the plumb bob out against

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the leg.) (5) Unclamp the upper motion, sight at the next station to the right and clamp the upper motion. (6) Read the *A* vernier, (this will be the azimuth of the course). (7) Read the intercept on the rod. (8) Measure the vertical angle by sighting at the target set at the height of the horizontal axis as before. (9) Set the transit over the next station to the right and determine the intercepts and vertical angles as at the first station. (10) Determine the stadia intercepts and vertical angles at the remaining stations, passing around the field to the right. (11) Reduce the intercepts to horizontal distances before recording. (12) Compute the vertical differences in elevation using mean distances and vertical angles. (13) Compute latitudes and departures to the nearest foot using a traverse diagram or traverse table. Follow form B4. (14) Compute the permissible error of closure of the traverse by means of Baker's formula (see Chapter IX "Errors of Surveying"); using "*a*" equals one minute times square root of number of sides, and "*b*" equal 1:500. If consistent, distribute the errors in proportion to the several latitudes and departures, respectively. (15) Compute the area by means of latitudes and departures, and reduce to acres.

PROBLEM E4. SURVEY OF FIELD WITH PLANE TABLE BY RADIATION..

(a) *Equipment.*—Plane table, stadia rod, 2 flag poles, engineers' divided scale, drawing paper, 6H pencil.

(b) *Problem.*—Make a survey of an assigned field by radiation with the plane table.

(c) *Methods.*—(1) Set the plane table up at some convenient point in the field and select a point on the drawing board that will allow the entire field to be plotted on the paper. (2) Sight at one of the stations with the ruler centered on the point on the paper. (3) Draw a line along the fiducial edge of the ruler towards the point. (4) Measure the distance to the point with the stadia. (5) Lay off the distance on the paper to the prescribed scale. (6) Locate the remaining points in the same manner. (7) Complete the map in pencil. The map should have a neat title, scale, meridian, etc. (8) Trace the map on tracing linen. (9) Compute the area by the perpendicular method, scaling the dimensions from the map.

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PROBLEM E5. SURVEY OF A FIELD WITH PLANE TABLE BY TRAVERSING.

(a) *Equipment.*—Plane table, stadia rod, 2 flag poles, engineers' divided scale, drawing paper, 6H pencil.

(b) *Problem.*—Make a survey of an assigned field by traversing with the plane table.

(c) *Methods.*—Follow the same general methods as those given for traversing with the transit. Adjust the plane table before beginning the problem. Complete the map and compute the area as in Problem E4.

PROBLEM E6. SURVEY OF FIELD WITH PLANE TABLE BY INTERSECTION.

(a) *Equipment.*—Plane table, 2 flag poles, engineers' divided scale, drawing paper, 6H pencil.

(b) *Problem.*—Make a survey of an assigned field with the plane table by intersection.

(c) *Methods.*—(1) Select and measure a base line having both ends visible from all the stations in the field. (2) Set the plane table over one end of the base line, sight at the other end of the base line and at each one of the stations of the field. (3) Set the plane table over the other end of the base line, orient the instrument by sighting at the station first occupied and sight at all the stations in the field. (4) Complete map and compute area as in E4.

PROBLEM E7. THREE POINT PROBLEM WITH PLANE TABLE.

(a) *Equipment.*—Plane table, 2 flag poles, engineers' divided scale, 6H pencil.

(b) *Problem.*—Having three points plotted on the map, required to locate a fourth point on the map by solving the "three point problem" with the plane table.

(c) *Methods.*—(1) Use Bessell's solution. (2) Check by using the mechanical solution.

PROBLEM E8. ANGLES OF TRIANGLES WITH SEXTANT.

(a) *Equipment.*—Sextant, 2 flag poles.

(b) *Problem.*—Measure the angles of an assigned triangle with the sextant.

(c) *Methods.*—(To determine index error, sight at a dis-

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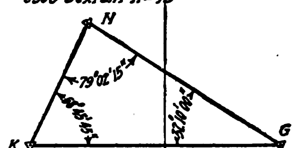
ANGLES OF TRIANGLE K-G-N WITH SEXTANT

Station	Sextant	Observed Angle	Index Error	Corrected Angle	Mean Angle
G	28°34'30"	Erect	3°36'	32°10'30"	
	28°33'30"	Inverted	"	32°09'30"	32°10'00"
K	65°10'00"	Erect	"	68°46'00"	
	65°09'30"	Inverted	"	68°45'30"	68°45'45"
N	75°26'00"	Erect	"	79°02'00"	
	75°26'30"	Inverted	"	79°02'30"	79°02'15"
					179°58'00"
	Actual error of closure				02'
	Allowable error of closure				03'

Observers: J. Doe & R. Roe.
Mch. 28, 1914. (2 Hours) Cool & Clear.
To determine index error, sighted at flag staff 1 mile away and made reflected image coincide with direct image. The reading of the vernier gave an index error of +3°36'.
Set flag poles back of K and N.
Held sextant over G, sighted at K and moved sextant arm until images of flag poles at K and N coincided.
Inverted sextant, sighted at N, and moved sextant arm to make images coincide.
Repeated at other stations.
Used Sextant No. 75.



tant object and bring the direct and reflected images into coincidence. The reading of the vernier will give the index error, which, with proper sign, must be applied to all angles measured.) (1) Set the flag poles behind the monuments at two of the vertices of the triangle and stand on the monument at the third. (2) Hold the plane of the sextant horizontal, sight at one flag pole directly with the telescope and bring the image of the other flag pole into coincidence by moving the arm. (3) Read the vernier, and correct the angle for index error. (4) Repeat the measurement with the sextant inverted. Take the mean of the two readings, which should not differ more than 2′, as the true value of the angle. (5) Measure the other angles in the same manner. The error of closure should not exceed 3′. Record the data in the form.

PROBLEM E9. DETERMINATION OF COEFFICIENTS OF A TAPE.

(a) *Equipment.*—Steel tape, spring balance, 2 thermometers, steel rule, 2 stout stakes, axe, 2 pieces sheet zinc 2 by 2 inches.

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(b) *Problem.*—Determine the coefficients of expansion, stretch, and sag of an assigned tape. Make three determinations of each, and take the arithmetical mean as the true value.

(*Standard Tapes.*—In laying off a standard or measuring a base line where a high degree of precision is required it is important that all measurements be referred to the same standard. The Bureau of Standards, Washington, D. C., will compare a tape with the government standard for a small fee. The tape tested is certified to be of a given length for a given temperature and pull. For example the standard tape marked "U. S. W. & M. 215" used in laying off the 100-ft. standard in Problem A23, was certified to be 99.9967 feet long at a temperature of 62° F. and a pull of 12 pounds, when tested on a plane surface. The coefficient of expansion of this tape was 0.0000061 per degree F. Tapes for measuring base lines with great precision have recently been made of Invar Steel.)

(c) *Methods.*—(1) *Correction for Expansion.*—Measure the length of the tape on a plane surface at two different temperatures but with a constant pull determined by a spring balance. Then substitute the lengths, l and L, and temperatures, t and T, in the formula

$$l - L = e(t - T)l$$

where e is the coefficient of expansion. Repeat the test and obtain three values of the coefficient e. As large a range of temperatures as possible should be secured. Take the arithmetical mean of the three determinations as the true value.

(2) *Correction for Stretch.*—Measure the length of the tape on a plane surface with two different pulls but at a constant temperature. Determine the pull with a spring balance. Then substitute the lengths, l and L, and the pulls p and P, in the formula

$$l - L = s(p - P)l$$

where s is the coefficient of stretch. Repeat the test and obtain three values of the coefficient s. The pulls should range from 10 to 40 pounds. Take the arithmetical mean of the three determinations as the true value.

(3) *Correction for Sag.*—Remove the handles from the tape and determine its weight very carefully. Divide the weight by the length to obtain the weight per foot, w.

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Drive two stout hubs a little less than 100 feet apart and fasten a piece of sheet zinc with a line ruled at right angles to the line on the top of each stake. With a pull of 10 pounds, as determined by the spring balance, measure the distance between the stakes. Calculate the correction for sag by substituting the lengths, l and L, pull p, and weight per foot w, in the formula.

$$l - L = \frac{l}{24}\left(\frac{w \cdot l}{p}\right)^2$$

Repeat the measurements using a pull of 20 and 30 pounds, respectively. Add the corrections for sag to each measurement and compare the results. The temperature should remain constant during the tests. To remove the possibility of an error due to temperature, observe the temperature at the time of each observation and correct the observed length for e pansion before substituting in the formula.

Report the methods, data, computations and results on a suitable form.

PROBLEM E10. MEASUREMENT OF BASE LINE.

(a) *Equipment.*—Standard tape, transit or level, stakes (number and size to be specified by instructor), axe, spring balance, 2 thermometers, lath stakes, 8-d nails, steel rule, pieces sheet zinc 2 by 2 inches.

(b) *Problem.*—Measure an assigned base line with a standard tape.

(c) *Methods.*—(1) Set the transit over one end of the base line, sight at the other end and determine the difference in elevation and grade. (2) Drive stout square stakes to grade, by "shooting" them in with the instrument in true line, a little less than a full tape length apart. The top of the lowest stake should not be less than 6 inches above the ground. (3) Fasten a piece of sheet zinc, with a fine line ruled at right angles to the direction of the base line, on the top of each stake. (4) Drive lath stakes in line about 20 feet apart. (5) Drive an 8-d nail through each lath stake at grade to support the tape. (6) Measure from stake to stake, the men working as follows: No. 1 plumbs up from the rear monument or holds the zero on the mark on the rear stake; No. 2 takes the spring balance and puts a pull of 16 pounds on the tape; No. 3 reads the tape and measures the fraction of a tenth with a steel rule to 0.001

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feet; No. 4 records the reading of the tape and reads the two thermometers placed at the quarter points of the tape. (7) Obtain at least three determinations of the length of the base line. (8) Correct each measurement of the base for standard, expansion, sag, stretch, and slope (see problem on coefficients of a tape). The three measurements should not differ more than 1: 100,000. Report methods, computations and results on a suitable form.

PROBLEM E11. CALCULATION OF TRIANGULATION SYSTEM.

(a) *Equipment.*—Seven-place table of logarithms.

(b) *Problem.*—Adjust and calculate an assigned triangulation system and plot the skeleton.

(c) *Methods.*—Observe the following program: (1) prepare forms for calculations and transcribe data; (2) adjust the angles of the triangulation system (see chapter on errors of surveying); (3) calculate the front and back azimuths of each line; (4) beginning with the base line compute the sides, to the nearest 0.001 foot; (5) calculate the latitudes and departures to the nearest 0.001 foot (6) calculate the coordinates of the triangulation stations to the nearest 0.001 foot. In computing the coordinates of the stations take the mean of the values found by taking the different routes from the base line as the true value. (7) Plot the skeleton of the triangulation system to the prescribed scale by means of the coordinates of the points. Check by lengths of sides. Use a steel straight edge.

PROBLEM E12. SKETCHING TOPOGRAPHY.

(a) *Equipment.*—Small drawing board or plane table, plat of assigned field, 4H pencil.

(b) *Problem.*—Sketch in the roads, walks, buildings and five-foot contours on the plat of the assigned field by eye having given the elevations of the ruling points.

(c) *Methods.*—(1) Transfer from the level notes to the plat the elevations of the ruling points of the field. (2) Locate the roads, buildings, etc., on the map as nearly as possible in their relative positions (the topographers' estimate of distance should be frequently checked by pacing). (3) Estimate the slopes and locate the contour points between the points of known elevation. (4) Join these points by smooth curved lines. (5) Finish the map in pencil, put-

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ting on a neat title, the scale of the map and a meridian. (6) Compare the finished map with a contour map furnished by the instructor.

PROBLEM E13. FILLING IN DETAILS WITH TRANSIT AND STADIA.

(a) *Equipment.*—Complete transit, 2 stadia rods, pocket tape.

(b) *Problem.*—Locate the topographic details of an assigned area with the transit and stadia.

Observer, O. B. Quck
Recorder, A. M. Sure
Stadiamen { J. Doe. R. Roe.

(TOPOGRAPHY BY TRANSIT-STADIA METHOD.)

Sta.	Ver. A.	Ver. B.	Obs. Dist.	Cor. Dist.	Vert. Ang.	Diff. El.	Elev.		Description of Point.
AT Δ7.		Elev. Δ7 = 633.5,	H. I. = 5.1		(A & B. Transit,		k = 100.00)		11-6-14 clear.
Δ5	152°51'	332°51'	523.2	523.2	− 0°11'	− 1.6	631.9	No.	El. Δ5 = 631.8 (Check)
Shot No. 1	89°25'		524	520	+ 5°13'	+ 47.3	680.8	1	Ridge No. 1.
2	85°18'		355	353	+ 5°48'	+ 35.5	669.0	2	" "
3	90°06'		293	290	+ 5°27'	+ 27.6	661.1	3	" "
4	106°35'		235	235	+ 1°23'	+ 5.8	639.3	4	" "
5	114°50'		245	245	+ 0°33'	+ 2.3	635.8	5	" "
6	132°33'		223	220	− 7°52'	− 30.4	603.1	6	" "
7	152°57'		228	202	− 18°55'	− 69.2	564.3	7	" "
8	75°04'		277	273	+ 5°20'	+ 25.5	659.0	8	Ridge No. 2
9	60°40'		245	245	+ 4°06'	+ 17.6	651.1	9	" "
10	46°55'		226	226	+ 1°32'	+ 6.0	639.5	10	" "
11	41°51'		218	218	+ 0°44'	+ 2.7	636.2	11	" "
12	34°00'		214	214	− 1°20'	− 5.0	628.5	12	" "
13	11°58'		217	210	− 9°20'	− 34.4	599.1	13	" "
14	4°10'		228	221	− 9°34'	− 37.5	596.0	14	" "
15	355°48'		250	238	− 11°45'	− 45.9	587.6	15	" "
16	352°40'		212	196	− 15°12'	− 53.5	580.0	16	Gully No. 1
17	5°06'		185	177	− 11°58	− 37.4	596.1	17	" "
18	33°28'		158	153	− 11°00'	− 29.7	603.8	18	" "
19	47°15'		146	142	− 8°21'	− 21.0	612.5	19	" "
20	67°58'		182	182	+ 1°10'	+ 3.7	637.2	20	" "
21	88°41'		145	145	+ 0°57'	+ 2.4	635.9	21	Gully No. 2
22	104°55'		124	119	− 12°02'	− 25.1	608.4	22	" "
23	155°52'		180	153	− 22°55'	− 69.2	564.3	23	" "

(c) *Methods.*—(1) Set transit up over assigned triangulation or other point. (2) Orient instrument, i. e., set plates to given azimuth and sight at given back sight. (3) Measure height of axis above station hub with tape or by graduations on tripod leg, and set target to correspond. (4) Take shot on given back sight and reduce results as a check before proceeding. (The program for each shot is: (a) set middle hair roughly on target, then set one stadia

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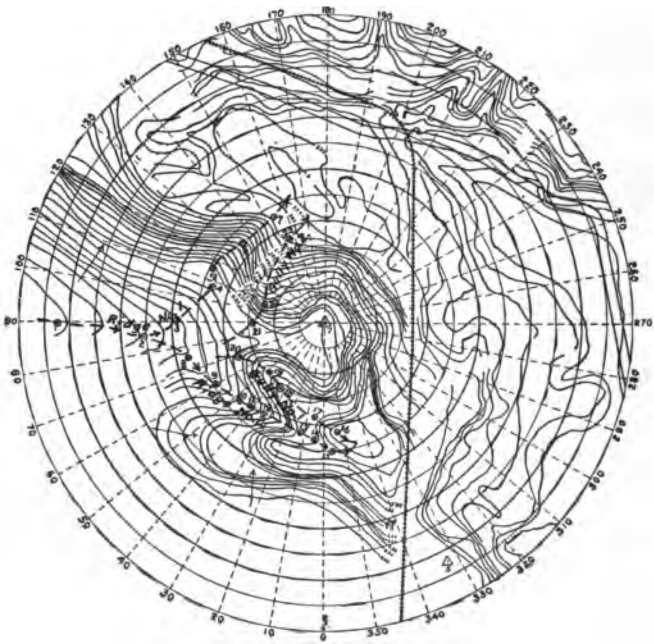
hair on nearest foot-mark and read intercept; (b) set middle hair precisely on target and signal rodman "all right"; (c) read vertical angle; (d) read azimuth.) (5) Take side shots to representative points, keeping in mind the scale of the proposed map. Select points according to a systematic plan, following along ridges, gullies, etc. Contour points should be taken with reference to change of



slope. (6) Reduce and plot the notes, and interpolate the contours, as in the accompanying diagram. (This topography sheet should be carefully preserved for use in Problem E15.) (7) After completing the survey at the assigned station, move the instrument ahead to a new stadia station, taking both fore and back sights. (8) Lose no opportunity to take check sights at other triangulation stations, traverse points, etc.

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PROBLEM E14. FILLING IN DETAILS WITH PLANE TABLE AND STADIA.

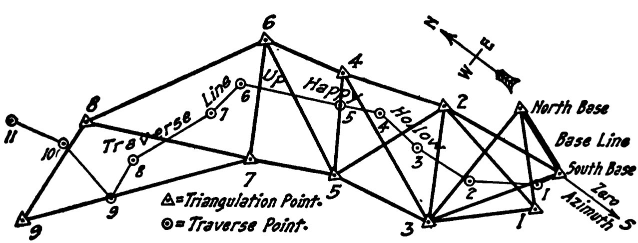
(a) *Equipment.*—Complete plane table (preferably with prismatic eyepiece), 2 stadia rods, engineers' divided scale, drawing paper, 6H pencil, pocket tape.

(b) *Problem.*—Locate the topographic details of an assigned area with the plane table and stadia.

(c) *Methods.*—Follow the same methods as in Problem E13 except that the notes are to be plotted on the drawing paper in place of being recorded in the field book. Mark the points by number and write the elevation of each point under the number in the form of a fraction. Locate the contour points by interpolation on the map and connect the points by smooth curves. Complete the map in pencil and make a tracing if required.

PROBLEM E15. TOPOGRAPHIC SURVEY.

(a) *Equipment.*—Complete transit, 2 stadia rods, stakes, hubs, spring balance, pocket tape, stadia slide rule, seven-place logarithm table, (extra tripods, stadia reduction table, stadia reduction diagrams, etc., as required).



(b) *Problem.*—Make a complete topographic survey of an assigned area and make a topographic map.

(c) *Methods.*—(1) Make a reconnaissance and locate the triangulation stations. Care should be used to select the triangulation stations so that the sights will be clear and the triangles well formed. A system composed of quadrilaterals or more complicated figures will give more conditions and checks than a simple string of triangles. A system composed of simple triangles is sufficient for this survey. (2) Mark the triangulation stations with gas pipe

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monuments about 4 feet long, the exact point being marked by a hole drilled in a bolt screwed into a cap on the top of the gas pipe. (3) Measure the base line and base of verification as described in Problem E10. (4) Measure the angles by repetition as described in Problem D13. (5) Calculate the skeleton as described in Problem E11. (6) Establish permanent bench marks and determine their elevations and the elevation of the stations of the triangulation system by running duplicate levels with the engineers' level, reading the rod to 0 001 foot. (7) Fill in the details with either the transit and stadia or the plane table and stadia, or both, as described in Problems E13 and E14. (8) Complete the map in pencil on manila paper, and after it has been approved by the instructor trace it on tracing linen. The title, meridian, scale, lettering and border should receive careful attention.

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CHAPTER VII.

LAND SURVEYING.

Kinds of Surveys.—Surveys of land are of two kinds: (a) original surveys; (b) resurveys.

Original Surveys.—An original survey is made for the purpose of establishing monuments, corners, lines, boundaries, dividing land, etc. The survey of a townsite and the government survey of a section are examples of original surveys.

Resurveys.—A resurvey is made for the purpose of identifying and locating corners, monuments, lines and boundaries that have been previously established. The resurvey of a city block, or a survey to relocate a section corner are examples of resurveys.

Functions of a Surveyor.—In an original survey it is the function of the surveyor to make a perfect survey, establish permanent monuments and true markings, and make a correct record of his work in the form of field notes and a plat.

In a resurvey it is the function of the surveyor to find where the monuments, courses, lines and boundaries originally were, and not where they ought to have been. Failing in this it is his business to reestablish them as nearly as possible in the place they were originally placed. No reestablished monument, no matter how carefully relocated, will have the same weight as the original monument if the latter can be found. In making resurveys the surveyor has no official power to decide disputed points. He can act only as an expert witness. If the interested parties do not agree to accept his decision the question must be settled in the courts.

Also see Problem F6, "Resurvey of a City Block."

Responsibility of the Surveyor for the Correctness of His Survey.—An engineer in the discharge of his professional duties requiring an exercise of judgment can be held liable only for failure to exercise reasonable care and skill, or for negligence or fraud. A surveyor is liable not only

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for negligence or fraud but for want of skill. A surveyor agrees to not only do his work carefully, honestly, diligently, but skillfully as well. The precision required in making any particular survey in order to satisfy the requirement for skill will depend upon the conditions; greater accuracy being required for making a survey of an expensive city lot than for a survey of a farm. Surveying is a trade and the precision required in any particular case to show proper skill is a matter to be decided by the court after evidence has been submitted.

Ownership of Surveyors' Notes.—Survey notes, data, maps, plats and records obtained by a surveyor while in the employ of a city, state, railroad or other corporation, or of a consulting or independent engineer belong to the employer. A city engineer or a county surveyor has no ownership rights in the notes, data, maps, plats and records which he prepares or obtains, or are prepared or obtained by him or by his assistants, in the exercise of the duties of his office as city engineer or county surveyor. Survey notes, data, maps, plats and records obtained by a consulting or independent engineer in preparing a report or plans for a client, belong to the consulting or independent engineer. The client, whether it be an individual, city, state, or corporation, is entitled only to the finished report or plans, and is not entitled to the notes and data used in the preparation of the report or the plans.

Rules for Resurveys.—The following rules may be safely observed in making resurveys.

(1) The description of boundaries in a deed are to be taken as most strongly against the grantor.

(2) A deed is to be construed so as to make it effectual rather than void.

(3) The certain parts of a description are to prevail over the uncertain.

(4) A conveyance by metes and bounds will convey all the land included within.

(5) Monuments determine boundaries and transfer all the land included.

(6) When a survey and a map disagree the survey prevails.

(7) Marked lines and courses control courses and distances.

(8) The usual order of calls in a deed is; natural objects, artificial objects, course, distance, quantity.

(9) A long established fence line is better evidence of

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actual boundaries than any survey made after the monuments of the original survey have disappeared.

(10) A resurvey made after the monuments have disappeared is to determine where the monuments were and not where they should have been.

(11) All distances measured between known monuments are to be pro rata or proportional distances.

If the above rules do not cover the case in question special court decisions on that particular point should be consulted.

THE UNITED STATES RECTANGULAR SYSTEM OF PUBLIC LAND SURVEYS.

Historical.—The United States rectangular system of subdividing lands was adopted by congress May 20, 1785. The first public land surveys were made in the eastern part of the present state of Ohio under the direction of Capt. Thomas Hutchins,* Geographer of the United States, and were known as the "Seven Ranges." The townships were six miles square, and were laid out in ranges extending northward from the Ohio river; the townships were numbered from south to north, the ranges from east to west. In these initial surveys only the exterior lines of the townships were run, but mile corners were established on the township lines, and sections one mile square were marked on the plat and numbered from 1 to 36, commencing with section 1 in the southeast corner and running from south to north in each tier to 36 in the northwest section.

The act of congress approved May 18, 1796, provided for the appointment of a surveyor general and changed the law relating to the surveys of public lands. Under this law the townships were subdivided into sections by running parallel lines two miles apart each way and setting a corner at the end of each mile. This law also provided that the sections be numbered beginning with section 1 in the northeast corner of the township, thence west and east alternately to 36 in the southeast corner. This is the method of numbering still in use, shown in Figs. 33 and 34.

* The earliest published reference to the rectangular system of land surveys is found in an appendix to "Bouquet's March," published in Philadelphia, 1764. Hutchins was engineer with this expedition to the forks of the Muskingum river, and wrote the appendix. (See reprint by Robt. Clarke, Cincinnati.)

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The act of congress approved May 10, 1800, required that townships be subdivided by running parallel lines through the same from east to west and from south to north at a distance of one mile from each other. Section corners and half section corners on the lines running from east to west were required to be set. The excess or deficiency was to be thrown into the north and west tiers of sections in the townships.

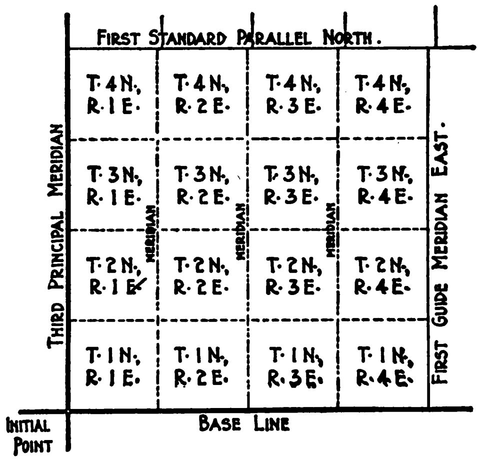

Fig. 32.

The act of congress approved February 11, 1805, required that interior section lines be run every mile; that corners be established every half mile on both township and section lines; that discrepancies be thrown on the north and west sides of the township. This act of congress further provided "that all corners marked in the original surveys shall be established as the proper corners of sections, or subdivisions of sections; and that corners of half and quarter sections not marked shall be placed as nearly as possible 'equidistant' from those two corners which stand on the same line. The boundary lines actually run and marked shall be established as the proper boundary lines of the sections or subdivisions for which they were intended; and the length of such lines as returned by the surveyor shall be held and considered as the true length thereof, and

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the boundary lines which shall not have been actually run and marked as aforesaid shall be ascertained by running straight lines from the established corners to the opposite corresponding corners." Under this law, which is still the established rule of procedure, each reported distance between established monuments is an independent unit of measure.

The revised instructions issued in 1855 required that the sections be subdivided as shown in Fig. 33. The full lines representing "true" lines, are parallel to the east exterior line of the township, and the dotted lines, representing "random" lines, close on corners previously established. The order of the survey of the interior section lines is indicated by the small numerals. Double corners on the north and west township lines, which were common in the earlier surveys, were thus avoided in the revised practice.

Laws Inconsistent.—It is obviously impossible to preserve a true rectangular system on a spherical surface, owing to the convergency of meridians.* To harmonize the methods of making surveys, the General Land Office has issued instructions for the survey of public lands from time to time.

DETAILS OF SURVEY.—The details of the survey are taken up in the following order: (1) selection of initial points; (2) establishment of the base line; (3) establishment of the principal meridian; (4) running standard parallels; (5) running the guide meridians; (6) running the township exteriors; (7) subdividing the township; (8) meandering lakes, rivers, streams, etc. See Figs. 32 and 33.

Initial Points.—Initial points from which to start the survey are established whenever necessary under special instructions prescribed by the Commissioner of the General Land Office.

Base Line.—The base line is extended east and west from the initial point on a parallel of latitude. The proper township, section and quarter corners are established and meander corners at the intersection of the line with all meanderable streams, lakes, or bayous. Two sets of chain-

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* The angular convergency, a, of two meridians is $m. \sin L$, where m is the angular difference of longitude of meridians and L is the mean latitude of the two positions. The linear convergency, c, for a length, t, is $t. \sin a$. Latitude 40°, the difference between the north and south sides of a township is 0.60 chains.

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men are employed and the mean of the two measurements is taken as the true value. When the transit is used, the base line—which is a small circle parallel to the equator—is run by making offsets from a tangent or secant line, the direction of the line being frequently checked by an observation of Polaris.

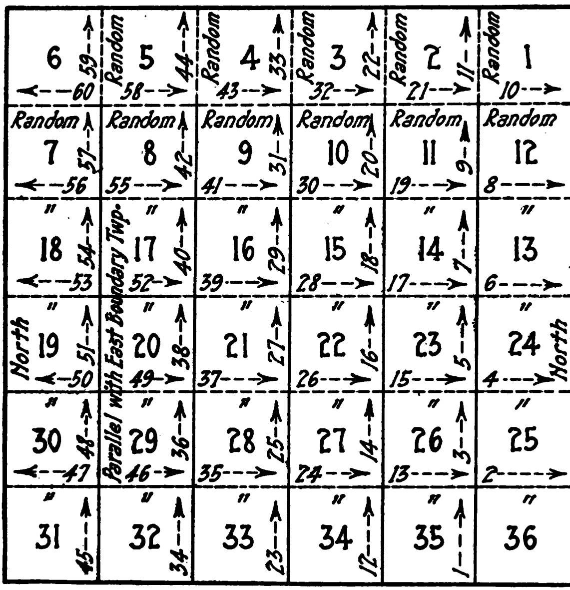

Fig. 33.

Principal Meridian.—The principal meridian is extended either north or south, or in both directions from the initial point on a true meridian. The same precautions are observed as in the measurement of the base line.

Standard Parallels.—Standard parallels, which are also called correction lines, are extended east and west from the principal meridian, at intervals of 24 miles north and south of the base line. They are surveyed like the base line.

Guide Meridians.—Guide meridians are extended north from the base line, and standard parallels, at intervals of 24 miles east and west from the principal meridian, in the

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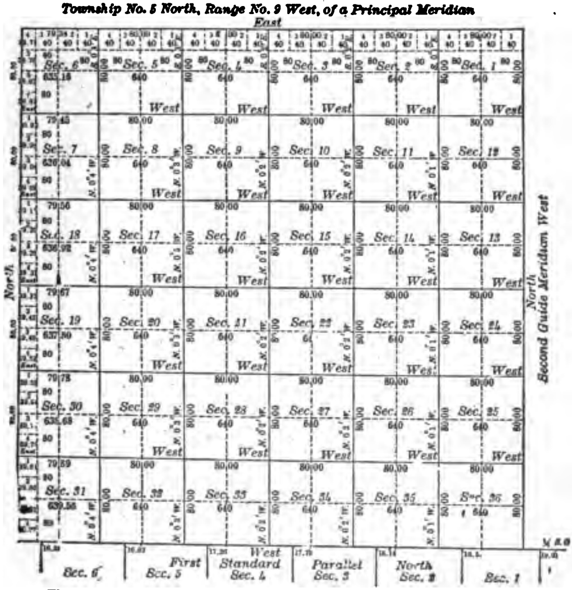
manner prescribed for running the principal meridian. When existing conditions require that guide meridians shall be run south from the base or correction lines, they are initiated at properly established closing corners on such lines.

Township Exteriors.—The township exteriors in a tract 24 miles square, bounded by standard lines, are surveyed successively through the block, beginning with the south-

Township No. 5 North, Range No. 9 West, of a Principal Meridian



The above plot represents a theoretical township with perfect subdivisions, contiguous to the north side of a Standard Parallel; in assumed Latitude 38 15′ N. and Longitude 100°00′ W. of Gr. Area 23024.16 A.

Fig. 34.

western township. The meridional boundaries are run first from south to north on true meridians with permanent corners at lawful distances; the latitudinal boundaries are run from east to west on random or trial lines and corrected back on true lines. Allowance for the convergency of meridians is made whenever necessary.

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Township Subdivisions.—A true meridian is established at the southeast corner of the township and the east and south boundaries of section 36 are retraced. Then beginning at the corner to sections 35 and 36 on the southern boundary, a line is run north parallel to the township line, corners are established at a distance of 40 and 80 chains; from the last named corner a random line is run eastward, parallel to the south boundary line of section 36, to its intersection with the east boundary of the township. A temporary corner is set at a distance of 40 chains, and a permanent corner is afterwards established midway be-

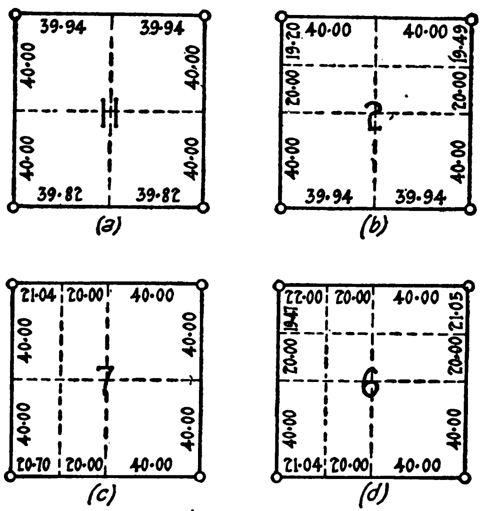

Fig. 35.

tween the two permanent corners. The other corners are located in a similar manner, as shown in Fig. 33. The lines closing on the north and west boundary lines of the township are made to close on the section corners already established. A theoretical township with perfect subdivisions is shown in Fig. 34.

Meandering.—Navigable rivers and other streams having a width of three chains and upwards are meandered on both banks, at the ordinary high water line by taking the general course and distances of their sinuosities. The

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meanders of all lakes, navigable bayous, and deep ponds of the area of twenty-five acres and upwards are surveyed as directed for navigable streams. Meander corners are established where meander lines cross base lines, township lines, or section lines.

Subdivision of Sections.—In Fig. 35, (a) gives the subdivision of an interior section, (b) of section 2 on the north side, (c) of section 7 in the west tier, and (d) of section 6 in the northwest corner.

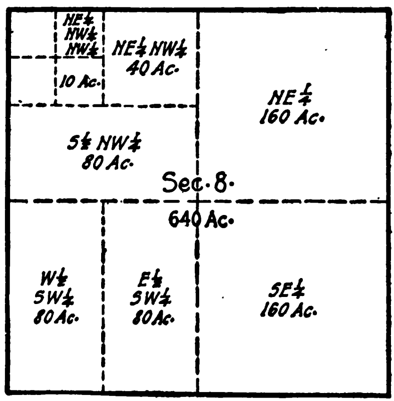

Fig. 36.

Description of Land.—Land is described in the rectangular system by giving its location in a civil township; for example, in Fig. 36, the northeast quarter, containing 160 acres, would be described as: N E ¼, Sec. 8, T 19 N, R 9 E, 3 P. M. The ten acre lot indicated in the northwest quarter would be described as: S E ¼, N W ¼, N W ¼, Sec 8, T 19 N, R 9 E, 3 P. M.

Corners.—The corner monuments may be as follows: (a) stone with pits and earthen mound; (b) stone with mound of stone; (c) stone with bearing trees; (e) post in mound of earth; (f) post in mound of stone; (g) post with bearing trees; (h) simple mount of earth or stone; (i) tree without bearing trees; (j) tree with bearing trees; (k) rock in place, etc. The trees on line are required to be blazed. The size, markings and proper corners to be used in any particular case and all other details are given in the

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"Manual of Surveying Instructions for the Survey of Public Lands of the United States," issued by the General Land Office, Washington, D. C.

The last edition of the "Manual of Surveying Instructions for the Survey of Public Lands" was issued in 1902 and may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C., price 75 cents per copy. A new edition of the Manual is promised for 1915. The circular on the "Restoration of Lost and Obliterated Corners" mentioned in the next paragraph gives instructions for making resurveys, and may be obtained free by addressing the Department of Interior, General Land Office, Washington, D. C.

Restoration of Lost or Obliterated Corners.*—"An obliterated corner is one where no visible evidence remains of the work of the original surveyor in establishing it. Its location may, however, have been preserved beyond all question by acts of landowners, and by the memory of those who knew and recollect the true position of the original monument. In such cases it is not a lost corner.

"A lost corner is one whose position can not be determined beyond reasonable doubt, either from original marks or reliable external evidence."

General Rules.—The following rules are derived from a brief synopsis of congressional legislation relating to surveys.

"(1) The boundaries of the public lands established and returned by the duly appointed government surveyors, when approved by the surveyor general and accepted by the government, are unchangeable.

"(2) The original township, section, and quarter-section corners established by the government surveyors must stand as the true corners which they were intended to represent, whether the corners be in place or not.

"(3) Quarter-quarter corners not established by the government surveyors shall be placed on the straight line joining the section and quarter-section corners and midway between them, except on the last half mile of section lines closing on the north and west boundaries of the townships, or on other lines between fractional sections.

"(4) All subdivisional lines of a section running between corners established in the original survey of a township

* Circular on the "Restoration of Lost and Obliterated Corners and Subdivision of sections," Department of Interior, General Land Office, Washington, D. C.

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must be straight lines, running from the proper corner in one section line to its corresponding corner in the opposite section line.

"(5) That in a fractional section where no opposite corresponding corner has been or can be established, any required subdivision line of such section must be run from the proper original corner in the boundary line due east and west, or north and south, as the case may be, to the water course, Indian reservation, or other boundary of such section, with due parallelism to section lines."

"From the foregoing it will be plain that extinct corners of the government surveys must be restored to their original locations, whenever it is possible to do so; and hence resort should always be first had to the marks of the survey in the field. The locus of the missing corner should be first identified on the ground by the aid of the mound, pits, line trees, bearing trees, etc., described in the field notes of the original survey.

"The identification of mounds, pits, buried memorials, witness trees, or other permanent objects noted in the field notes of survey, affords the best means of relocating the missing corner in its original position. If this can not be done, clear and convincing testimony of citizens as to the place it originally occupied should be taken, if such can be obtained. In any event, whether the locus of the corner be fixed by the one means or the other, such locus should always be tested and confirmed by measurements to known corners. No definite rule can be laid down as to what shall be sufficient evidence in such cases, and much must be left to the skill, fidelity, and good judgment of the surveyor in the performance of his work.

"Actions or decisions by county surveyors which may result in changes of boundaries of tracts of land and involve questions of ownership in connection therewith, are subject to review by the local courts in proceedings instituted in accordance with the local statutes governing such matters."

The pamphlet also contains much additional information of value.

Locations of Principal Meridians.—Principal meridians have been established as the needs of the surveys warranted. There are twenty-four principal meridians in all, the locations of which are given in the "Manual of Instructions," mentioned above.

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Abridging Field Notes.—The government surveyors use the method of abridging field notes shown in Fig. 38. Corners in the township boundary are referred to by letter; interior section corners are referred to by giving the numbers of the sections meeting at the corner; interior quarter section corners are referred to by giving the number on the section lines produced.

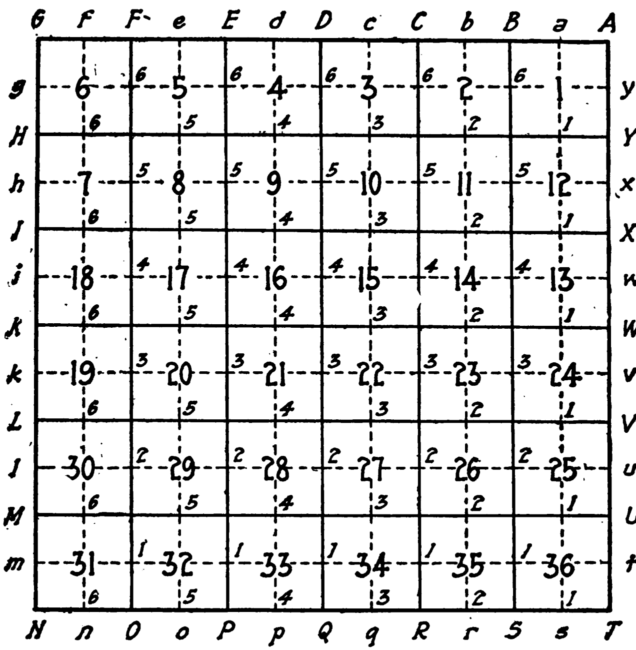

Fig. 38.

SURVEYS BY METES AND BOUNDS.

That portion of the United States settled before the adoption of the rectangular system was surveyed by the method of metes and bounds. For the most part these surveys were very irregular and often involved complex and conflicting conditions. The entire eastern portion of the United States, and the state of Kentucky, were surveyed in this manner,

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and further examples are found in the French surveys in the states of Michigan, Indiana, Illinois, Missouri, Louisiana, etc., and the Spanish surveys of Texas, California, etc. The general principles underlying the questions of ownership, priority of survey, the restoration of lost corners, etc., are identical whatever the system of survey used.

PROBLEMS IN LAND SURVEYING.

PROBLEM F1. INVESTIGATION OF A LAND CORNER.

(a) *Equipment.*—Digging outfit, tape, etc., as required.

(b) *Problem.*—Collect complete evidence relative to an assigned land corner, and after giving due weight to the same, make a decision as to the true corner.

(c) *Methods.*—(1) Make careful examination of the official field notes and records pertaining to the land corner in question and make extracts from the same for further reference. (2) Seek oral evidence from those acquainted with the history of the corner. (3) Make a survey of fence lines and other physical evidence, such as witness trees or their stumps, etc., near the corner under investigation. (4) Make

J. Doe, Surveyor.

INVESTIGATION OF S.W. CORNER, SECTION 8, T.19N., R.9E., 3D. P.M.

Original United States Field Notes, on file at Court House at Urbana, Ill., describe the S.W. Cor., Sec. 8, T.19 N., R.9 E., 3 P.M. as "Post in Mound," the corner being located on the Prairie remote from the heavy timber which surrounds the other three corners of the section. Original survey was made about 1822.

On Oct. 25, 1896, Col. S.T. Busey, when asked for information about the corner under investigation, stated that about 1850, when he was a boy, Mr. Campbell, who was then County Surveyor, was called on to re-establish the S.W. Cor., Sec. 8. At the time mentioned the section lines near the corner were occupied by rail fence. Col. Busey says that his Father (a pioneer settler) pointed out to the surveyor near the fence corner evidences of a mound which he believed marked the original U.S. Survey corner. Mr. Campbell, the surveyor dug carefully at the spot and found the decayed point of a sassafras stake which unquestionably marked the true position of the "Post in Mound" established some 25 years or more previous to Campbell's resurvey. Col. Busey states that he himself carried the boulder which was set in place by the County Surveyor to pepetuate the section corner, and that this monument was not disturbed until it was replaced by a much larger stone when the roads were opened up on the section lines.

This stone stood 18" or so above the level of the road for many years. About 1894 it was carefully lowered by the Street Commissioner under the direction of the City Engineer of Urbana. Resurveys made since the stone was lowered, indicate that its present position is identical with that previous to the change.

Conclusion. *In view of Col. Busey's valuable statement with the corroboration from other credible sources, and the entire absence of conflicting evidence of any character, it is concluded that the monument now and for many years so recognized is the true S.W. corner of Section 8, T.19 N., R.9 E., 3D. P.M.*

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careful examination of the site of the corner with the digging outfit; the digging should be done cautiously so as to avoid disturbance of existing stakes or other monuments. (5) If more than one monument be found, make due record of their character and positions, and make further inquiry respecting them. (6) If no monument of any sort be found at first, continue the search diligently and do not give up finding the true corner as long as there is a remote chance of locating it. In any event, avoid wanton disturbance of any object or evidence that may have a bearing on the same. Keep a clear and concise record.

PROBLEM F2. PERPETUATION OF A LAND CORNER.

(a) *Equipment.*—Digging outfit, a large boulder or other permanent monument, cold chisel, hatchet, plumb bob, string, stakes.

(b) *Problem.*—Replace a temporary land corner by a permanent monument.

(c) *Methods.*—(1) Uncover the identified temporary monument and carefully determine the true point with consist-

S. F. Kingsley, Head Chainman. F. Hodgman, Transitman.
G. Rowland, Rear " S. Comings, Flagman.

SURVEY OF SEC. 14, T. 2 S., R. 10 W. FOR J. R. COMINGS AND H. ROWLAND.

CHAINS	
	Commenced at the SE. cor. of Sec. 14. Found a piece of strap railroad iron driven for the cor. which Hugh Shafter says he knows to have been kept in the same place, unquestioned, as the cor. for over 30 years. Marked:
	maple, 8 ins. diam., S. 45° W., 77 lks. dist.
	burr oak 12 ins. diam., N. 43° W., 123 lks. dist.
	I set up a tall flag on the cor. and then ran W. on random, var. 2° 15′ E., setting temporary stakes every 10 chs. in line.
40.00	*¼ sec. cor. lost.*
80.24	*Intersected the W. line of Sec. 14, 42 lks. S. of the cor. Found rotten stake at correct point, N. 26° E., 104 lks. from stump of wh. oak, 24 ins. diam., bearing tree of U. S. Survey, having surveyor's mark distinct on it. Set a piece of steel T rail 28 ins. long for cor. Marked:*
	locust 16 ins. diam., S. 28° W., 116 lks. dist.
	burr oak 18 " " , N. 78° E., 152 " "
	(10:30 A.M.)
	Ran thence E. on corrected line at single sight with transit, from cor. to cor. Var. 2° 33′ E.
40.12	*Found cedar stake 3 ft. below surface of road crossing and 2½ lks. S of line. No other evidence of cor. to be found. Put a piece of T rail 24″ long on top of the stake for ¼ sec. cor., 55 lks. S. of S. rail of M. C. R. R. No tree near.*
60.18	*Planted granite boulder 20 × 12 × 6 ins., with cross + mark for ½ quar. sec. cor., in true line between qr. post and sec. cor. and marked:*
	maple, 12 ins. diam., S. 16° E., 55 lks. dist.
	burr oak, 16 " " N. 54° E., 118 " " .

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ent exactness. (2) Reference out the point by driving two pairs of stakes with strings stretched so as intersect squarely over the corner. (3) After carefully checking the referencing, dig out the old monument to a depth sufficient to receive the boulder and permit its top to set several inches beneath the natural surface if located in a road or where disturbance is probable. (4) Cut a plain cross mark on the top of the stone, and set it in place in the hole, packing the earth about it, testing the position of the mark by means of the reference stakes and strings and plumb bob; finally leave the boulder set firmly in the correct position. (5) Make reference measurements to suitable permanent points such as marks on curbing, gas pipes, witness trees, etc., selected with respect to good intersections, and make reliable record of the witness notes after checking the same. (Other forms of permanent monuments are: gas pipe; fish plate; section of T-rail; farm tile or vitrified pipe filled with cement mortar; post hole filled with mortar; special solid monument burned like farm tile; special casting similar to a gas main valve box, with hole in top to receive flag pole; etc.)

PROBLEM F3. REESTABLISHING A QUARTER-SECTION CORNER.

(a) *Equipment*—Transit party outfit, digging tools, etc.

(b) *Problem.*—Reestablish a quarter-section corner that has been obliterated or lost.

(c) *Methods.*—(1) Collect and record all the available evidence which may assist in the discovery and identification of the corner. Examine the field notes of the original survey, the surveyors' plat book and the county atlas on file at the court house, and make diligent inquiry for credible and competent information, either written or oral as to the location of the corner. (2) Make a careful search for the monument. Trace all the lines of the original survey, paying particular attention to bearing and sight trees. Dig in all the places indicated by the different lines and give up the search only after you have exhausted every possible clue. (3) If the corner cannot be found, reestablish it, giving due weight to all the evidence. The surveyor should remember that the corner should be reestablished where it originally was and not where it ought to be. After having located a stake at the supposed location of the original monument, reference it out and renew the search. (4)

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After the monument has been relocated, mark it in a permanent manner as indicated in Problem F2, by a stone with a cross cut in its top or with a gas pipe well driven into the ground. Reference it out to at least two permanent objects selected with a view to securing a first class intersection. Make a careful record and preserve consistent accuracy in the work.

PROBLEM F4. REESTABLISHING A SECTION CORNER.

(a) *Equipment.*—Transit party outfit, digging tools, etc.

(b) *Problem.*—Reestablish an obliterated or lost section corner.

(c) *Methods.*—Follow the various methods described in Problem F3, giving special attention to the search for the original corner; upon failing to find trace of it, run out lines with reference to the section, quarter, and quarter-quarter corners in the four directions, with linear measurements from the same and finally reach the most consistent decision with reference to such survey lines, ownership lines, fences, hedges, road centers, etc. (A fruitful cause of disturbance of section and other corners is careless use of road graders, or the failure to lower the corner sufficiently below the surface of the road.)

PROBLEM F5. RESURVEY OF A SECTION.

(a) *Equipment.*—Transit party outfit, digging tools, etc.

(b) *Problem.*—Make a resurvey of an assigned section.

(c) *Methods.*—(1) Make extracts from the field notes of the original survey and of all resurveys on file at the court house, and other notes that may be of value. Make diligent inquiry among the property owners for evidence as to the location of corners. (2) Retrace the lines, recording the location of old fences, timber markings and other evidences as to prior recognition of lines and corners. Use consistent accuracy. Record the original notes as given in the forms. Record the field notes in narrative style using the designation of corners as given in the resurvey plat in the form. Make a plat of the section in the manner prescribed by state law for a resurvey.

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Surveyor, J. Doe.

INVESTIGATION OF LAND CORNERS OF SEC. 8, T. 19 N., R. 9 E., 3RD. P.M.

Apr. 25, 1899.

COLLECTION OF EVIDENCE

Extracts from Surveyor's Plat Book of Resurveys of Champaign County.

Nov. 5, 1897, Found in the County Recorder's office at Urbana, Ill., the "Surveyor's Plat Book" containing plats of townships showing existing monuments and subdivisions of sections made by the County Surveyor, with certificates of various resurveys. Made the following extracts relating to Sec. 8, T. 19 N., R. 9 E., 3RD. P.M.:—

(From P. 156)

"Dec. 3, 1876, Surveyed at the request of F. Adams the east line of Sec. 6. Beginning at a stone previously planted at NE cor. of said section, and running thence S. to S.E. Cor. of same, where I found a stone previously set by John Thrasher and Lewis Sommers, divided the distance pro rata and set Cor. at NE Cor. of S.E.¼ of same."

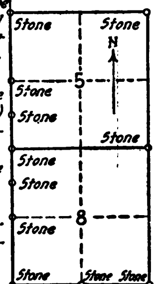
(Signed) Thos. B. Kyle
Co. Surveyor.

(From p. 157)

"Apr. 11, 1884. Surveyed by request of S.T. Busey the W. lines of Secs. 8 and 5. Beginning survey at S.W. Cor. Sec. 8 where a stone is planted and running thence N. to N.W. Cor. Sec. 5, Found an excess of 40 lks., corrected back, came on to a stone planted by Lewis Sommers at ¼ Sec. Cor. on line between Secs. 5 and 6. I also planted a stone at Sec. Cor. (5-6-7-8) and made the following witnesses to the corner, viz.: A double burr oak, 15" diam. bearing N. 60½° E, 102½ lks., also a Wh. Oak, 14" diam., bearing N. 35½° E, 188 lks. I also set a stone at the NW. Cor. of the SW¼ of the S.W¼, of Sec. 5.

(Signed) Thos. B. Kyle
Co. Surveyor.

(Portion of Plat on p. 155, showing existing monuments.)



Surveyor, J. Doe.

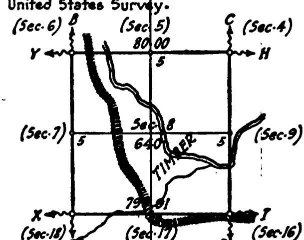
INVESTIGATION OF LAND CORNERS OF SEC. 8, T. 19 N., R. 9 E., 3RD P.M.

Apr. 25, 1899.

COLLECTION OF EVIDENCE (Continued)

Extracts from Field Notes of Original United States Survey.

Nov. 4, 1897, Found in the County Treasurer's Office at Urbana, Ill., the Plat Book containing Plats and Abstracts of Field Notes of Original United States Survey of Champaign County, and made the following extracts relating to Sec. 8, T. 19 N., R. 9 E., 3RD P.M.:—



DESCRIPTIONS OF ORIGINAL CORNERS (P. 30)

Corners Designation Sec. Cors.	Witness Trees Kind	Inches Diameter	Courses They Bear	Links Distant
4, 5, 8, 9	Ash	24	S. 58° E.	35
	B. Oak	14	N. 64° W	26
5, 6, 7, 8	W. Oak	28	N. 88° E.	230
	W. Oak	20	N. 36° E.	272
7, 8, 17, 18	Post in Mound			
8, 9, 16, 17	B. Walnut	24	N. 32° E.	44
	B. Walnut	24	S. 10° W.	42
¼ Sec. Cor.				
H to Y at 5	Elm	12	S. 67° W.	20
	Elm	8	N. 78° E.	30
I " X " 5	W. Oak	6	N. 58° E.	23
	W. Oak	6	S. 26° E.	20
R " C " 5	Ash	22	S. 7° E.	18
	Elm	8	N. 30° E.	13
S " B " 5	Post in Mound			

DESCRIPTIONS OF "OBJECTS ON THE LINES" (P. 75)

DESIGNATION	DISTANCES Chs. Lks.	DESCRIPTION
N. between 8 & 9	25.00	Brook leading N. thence along the channel of the same 13 chs. then leaving it running E'ly.
	50.19	Ash 12" diam.
E " 8 " 17	24.50	Brook 5 lks. rs. N. E'ly.
	39.00	Entered timber bs. N & S.
E. " 5 " 8	4.00	Entered timber bs. N & S.
	16.50	Brook 60 lks. rs. S'ly.

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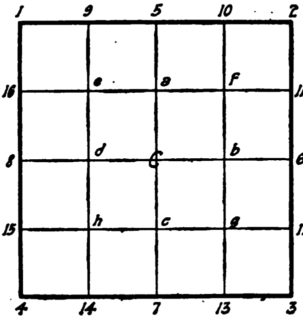
S. H. Smith, Head Chainman. L. B. Brown, Axman.
J. E. Wilson, Rear " G. W. Smith, Flagman.

RESURVEY OF SEC. 27, T. 12 N., R. 16 W., 3D. P. M. FOR THE ESTATE OF JOHN W. SMITH.
July 12, '82. Cloudy with showers.

CHAINS	
	Began at 7. Found stake in place and both bearing trees standing. Planted stone 25"x8"x6"; marked + for cor.
	Thence N. on random, var. 2° 30' E., setting temp. stakes every 10 chs.
80.22	Intersected sec. line 26 lks. W. of 5.
	At 5 found rotten stake at correct point, S. 28° W., 66 lks. from stump of wh. oak, bearing tree of U. S. Survey. Drove stake for cor. and put broken earthenware and glass around it. Mkd. wh. oak, 12" diam., N. 66° E., 42 lks., also wh. oak 18" diam., N. 34° W., 63 lks.
	From 5 ran E. on random, setting temp. stakes every 10 chs.
39.92	Intersected sec. line 12 lks. N. of 2.
	At 2 found earthen post in correct position and bearing trees of resurvey standing.
	Thence W. on corrected line.
9.98	Set stake on true line.
(Cont'd on next page)	

RESURVEY REFERENCE PLAT.



RESURVEY. SEC. 27, SMITH ESTATE (CONTINUED)

CHAINS	
19.96	(Line 5–2 cont'd.) At 10 set stake with stones around it and marked: pine, 12" diam., N. 46° W., 79 lks. red oak, 24" diam., S. 19½° W., 72 lks.
29.94	Set stake on true line.
	From 10 ran S. on random, var. 2° 19' E., and set temp. stakes at 20 and 40 chs.
	Then went to 6. Found post and bearing trees of resurvey standing.
	Ran thence W. on random, var. 2° 20' E.
20.02	Intersected random line from N. 6 lks. S. of temp. stake.
40.18	Intersected random ¼ line 8 lks. N. of temp. stake.
80.04	Intersected sec. line 10 lks. S. of 8.
	Cor. post dug out in road. Set iron plow beam for cor., S. 29° W., 76 lks., from bearing tree of U. S. Survey.
	Thence E. on corrected line.
39.99	At intersection of quarter lines set post.

PROBLEM F6. RESURVEY OF A CITY BLOCK.

(a) *Equipment.*—Transit, 100-foot steel tape, chaining pins, axe, hubs, stakes, 4 pieces one-inch gas pipe 2 feet long, notes of previous surveys, etc.

(b) *Problem.*—Make a resurvey of an assigned city block.

(c) *Methods.*—(1) Procure full notes of all the surveys and resurveys of the assigned block from the records at the court house and from any other source available. (2) Make a resurvey of the block, using the notes, and drive hubs for temporary corners. (3) Compute the latitudes and departures of the courses, and if consistent balance the survey. (4) If the corners of the block as located are consistent with the existing property and street lines, drive gas pipes as permanent corners. (5) Subdivide the block into lots as shown in the notes. (6) Make a plat of the block on manila paper to the prescribed scale, showing block and lot lines, distances and angles obtained in making the survey, the names of the owners of the property and the names of the streets. Prepare a surveyors' certificate as provided by law. Trace the map if required. (The accuracy attained should be based on the valuation and other local conditions. Before beginning the survey use every possible care to find the corners with reference to which the original survey was made. When lots are sold by number, the excess or deficiency should be divided pro rata. However, when lot lines have been long acquiesced in, it is doubtful if the courts will uphold the surveyor in interfering with the ancient lines of ownership. It then becomes necessary either to make a compromise survey that will be satisfactory to the owners, or to make a survey that is strictly according to the letter of the law, and submit the map and certificate to the courts for settlement. The surveyor should remember that he is simply an expert witness and that he had no final judicial powers.)

PROBLEM F7. RESURVEY BY METES AND BOUNDS.

(a) *Equipment.*—Transit party outfit, digging tools, etc.

(b) *Problem.*—Make a resurvey of an assigned tract whose original survey was made by metes and bounds.

(c) *Methods.*—(1) Collect full notes and data relating to the monuments, magnetic bearings, magnetic variation, date of survey, lengths of lines, etc. (2) Make a careful investigation of the lines and corners on the ground and

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make notes of any evidence there found. (3) Locate and identify with certainty as many as possible of the original monuments; where double or contested corners exist, locate each definitely for further reference; if corners are generally lacking or doubtful, concentrate attention on at least two which give most promise of definite relocation, and reestablish these corners as carefully as possible. (4) Having at least two corners, retrace by random line the perimeter of the tract, according to the original description, beginning at one and closing on the other corner; set temporary corner stakes at the several points; note the linear and angular error of closure of the random traverse on the last monument. (5) Calculate the latitudes and departures of the random survey, and determine the angular and linear relations between the random and the original survey; also fix the position of the several random stakes relative to the supposed true positions of the respective corners. (6) Set stakes in the true positions, as calculated, reference them out, and renew the search for the original monuments. (7) Finally reestablish each corner in the most consistent position, put permanent corners in place, and take witness notes for each, making complete notes of the proceedings. Follow the form.

PROBLEM F8. PARTITION OF LAND.

(a) *Equipment.*—Transit party and digging outfits, etc.

(b) *Problem.*—Make a partition of an assigned tract of land in accordance with instructions.

(c) *Methods.*—(1) Make the necessary resurveys of the assigned tract, identifying original monuments, and reestablishing lost corners as required. (2) Make a plat of the partition. (3) Subdivide the land and set permanent corners; carefully establish witnesses to the corners and secure witness notes. (4) Prepare and file plat and description as required by law.

PROBLEM F9. DESIGN AND SURVEY OF A TOWN SITE (OR ADDITION).

(a) *Equipment.*—Equipment for topographic survey for both field and office.

(b) *Problem.*—Make a preliminary topographic survey of the proposed town site (or addition), design the plat, and make the surveys for blocks, lots, etc.

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RESURVEY OF "MISSION RIDGE"

Consulted County Records and confirmed following Meander Notes for center line of highway as described in J. W. Martin's deed to J. D. Clark.

"N. 62° E., 14 ch.; N. 43½° E., 8 ch.; N. 5° W., 12 ch.; N. 72½° E., 10.25 ch.; S. 12° W., 6.43 ch."

Description referred to stones at beginning and ending points.

Found first stone projecting above road, but could not locate last corner.

Began at first monument and ran on random according to meander notes, with 2° 17′ E. as magnetic declination. Drove temporary stake at each deflection point and made careful search for monuments. Found no corners at intermediate points, but identified marked boulder as true corner at closing point 62 links due west of last stake of random.

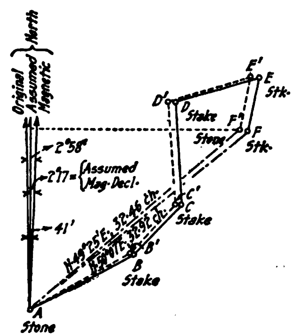
Made careful calculation of notes for shifting over from random to true corners. (See plat opposite and calculations on next pair of pages.)

J. Doe, Surveyor. Mar. 10, 1915.

PUBLIC ROAD FOR J. D. CLARK.

Transferred corners according to calculations and renewed search for original monuments, keeping close watch for decayed stakes, but without success.

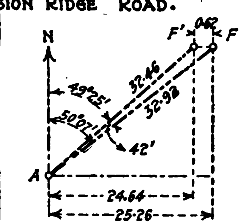
Set stone at each true corner.



CALCULATIONS, RESURVEY OF "MISSION RIDGE" ROAD.

Data transcribed from pp. Copy OK.

Sta. (Random Line)	Bearing	Dist. Ch.	Lat. Ch.	Dep. Ch.	Tot. Lat. Ch. (N)	Tot. Dep. Ch. (E)
A	N. 62° 00′ E.	14.00	N. 6.57	E. 12.36		
B	N. 43° 30′ E.	8.00	N. 5.80	E. 5.51	6.57	12.36
C	N. 5° 00′ W.	12.00	N. 11.95	W. 1.05	12.37	17.87
D	N. 72° 30′ E.	10.25	N. 3.08	E. 9.78	24.32	16.82
E	S. 12° 00′ W.	6.43	S. 6.29	W. 1.34	27.40	26.60
F			N. 27.40	E. 27.65	21.11	25.26
			S. 6.29	W. 2.39		
			N. 21.11	E. 25.26		
(Original Survey in Terms of Resurvey)						
A′	N. 61° 18′ E.	13.80	N. 6.33	E. 12.10		
B′	N. 42° 48′ E.	7.88	N. 5.78	E. 5.35	6.63	12.10
C′	N. 5° 42′ W.	11.83	N. 11.77	W. 1.17	12.41	17.45
D′	N. 71° 48′ E.	10.10	N. 3.15	E. 9.60	24.18	16.28
E′	S. 11° 18′ W.	6.34	S. 6.22	W. 1.24	27.33	25.88
F′			N. 27.33	E. 27.05	21.11	24.64
			S. 6.22	W. 2.41		
			N. 21.11	E. 24.64		



Notes for Shifting from Random to True Corners

Line	Difference Lat. Lks.	Difference Dep. Lks.	Dist. Lks.	Bearing
BB′	N. 6	W. 26	26.7	N. 77° 00′ W.
CC′	N. 4	W. 42	42.2	N. 84° 34′ W.
DD′	S. 14	W. 54	55.8	S. 75° 28′ W.
EE′	S. 7	W. 72	72.3	S. 84° 27′ W.
FF′	0	W. 62	62.0	W.

Note: The above solution is based on the assumption that the error of closure of 0.62 ch. at FF′ is due to difference of both chain and needle. Distances AF and AF′ and angles NAF and NAF′ were calculated, giving chain and needle corrections.

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(c) *Methods.*—(1) Make a careful resurvey of the entire tract. Reference the existing monuments and carefully relocate all missing corners. (2) After the monuments have been carefully located, remeasure the distances and angles very carefully. Before beginning the chaining, a standard should be established as described in Problem A23. (3) Fill in the topographic details with the transit and stadia, unless directed otherwise, using consistent accuracy. (4) Make a complete topographic map of the tract. (5) Design the townsite and sketch it in on the map. The questions of surface drainage, sewerage, possible overflow, street gradients, principal thoroughfares, diagonal streets, alleys, etc., should be carefully considered. The streets should be of ample width, and be laid out with reference to ease of grading both the street and adjacent property. Residences should face desirable streets and the cross streets in the residence district should not be too numerous. The principal thoroughfare should pass through the business portion and have minimum gradients. The system of sewerage and drainage should be worked out roughly before the design is completed. Much expensive construction can be avoided by using care in designing the town site. (6) Make preliminary profiles of all the streets on Plate A profile paper to the prescribed scale. (7) Carefully locate the block and other important corners and mark them by permanent monuments of stone, gas pipe, tiling, etc. (8) Subdivide the blocks into lots and mark the lot corners by means of gas pipes or hubs. (9) After the streets have been located carefully, take levels on the same, make profiles, and lay grade lines for all streets, sidewalks, and improvements.

Use accuracy consistent with the value of the property throughout the problem. Make a careful record of the notes. Complete the maps and profiles.

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CHAPTER VIII.

RAILROAD SURVEYING.

Classification.—For the purpose of class instruction, railroad surveying will be discussed under the following heads: (1) curve practice, (2) reconnaissance, (3) preliminary survey, (4) location survey, (5) construction, (6) maintenance.

Curve practice is designed to give the student familiarity with the methods of running curves so that the location survey may be made without needless delay. It consists of a series of typical problems covering the usual range of conditions found in such surveys.

The *reconnaissance* is a rapid preliminary examination of a district or area for the purpose of selecting ruling points to control the general routes of the preliminary survey lines. The distances are paced or scaled from a map; elevations are determined by means of the barometer or hand level.

The *preliminary survey* is designed to obtain information and to obtain it rapidly, as a guide in making the location survey. A rapid deflection angle traverse is run, following the general route of the proposed line, but keeping in clear ground as far as may be to gain time; levels are run, topography including contours taken, the map made, and one or more location lines projected on the map.

The *location survey* fixes the exact lines, including the curves, preparatory to building the proposed railroad. Some engineers prefer to run one or more trial location lines, but it is best practice to locate the line as projected on a reliable contour map.

Construction surveys are made for the purpose of fixing the roadbed limits and other constructive details, and estimating earthwork and other quantities.

Maintenance surveys and resurveys are made after the line is built, for ballasting, yard construction or other purpose.

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Field Organization of Class.—In order to carry out the foregoing steps, the following field parties are required: (a) transit party, (b) leveling party, (c) topography party, (d) land-line party, (e) cross-sectioning party, (f) bridge and masonry party, (g) resurvey party.

General Requirements.—Each party should work with snap and vigor and accomplish the best results practicable, both as to quality and quantity. To this end each member of the party should not only be careful, exact, and rapid in the discharge of his own duties, but avoid interfering with the work of others, such as obstructing the view of the transitman. In order to give each student practice in all the positions, the posts will be shifted daily, progressing to the higher positions in the party. The student should not underrate his practice in the subordinate positions, nor fail to make proper use of his more responsible duties. The usual decorum of field parties will be observed.

TRANSIT PARTY.—It is the duty of the transit party to establish the traverse line upon which to base the levels and topography. The student transit party will consist of the following members: (1) chief of party, (2) transitman, (3) head chainman, (4) rear chainman, (5) stakeman, (6) axeman, (7) front flagman, (8) rear flagman. The duties and equipment of the respective members are stated below.

Chief of Party.—(Party list, map of line, 50-foot metallic tape, railroad curve text book.) The chief of party is responsible for the general progress and quality of the work. It is his duty to direct the survey; see that each man does his work properly and with sufficient accuracy and despatch; check the transitman's work when necessary; keep the transit notes if the transitman is pushed; and make himself generally useful. He should be thoroughly acquainted, before going to the field, with the situation and with the data applicable to the work of the day. In requiring subordinate members of the party to perform their work properly, he should carefully preserve the dignity of his own position. Should there be no chief, these duties will be shared by the transitman and head chainman under the former's directions.

Transitman.—(Transit, reading glass, adjusting pin, transit note book, railroad curve text book, figuring pad.) The transitman runs the transit, keeps the notes, and in the absence of the chief, directs the work of the party. He should do careful and exact as well as rapid work, since the

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progress and character of the survey are usually controlled chiefly by the skill of the transitman.

In leveling up, keep the lower parallel plate about level. Avoid undue tightness of foot screws. In setting the vernier to zero, use a quick converging motion with the tangent movement and note the adjacent graduations. If the transit has lost motion, learn which way to get the slack on the tangent screws. As a rule, use the lower motion by preference. Habitually back sight to the rear with telescope reversed, then plunge the telescope on prolongation and read the deflection right or left. If practicable, base the calculated bearings on a true meridian; otherwise, allow for the magnetic declination at a station which seems to be free from local attraction and thus obtain a reference meridian. Check all deflection angles by needle reading, both as to amount and direction. Lack of proper adjustment is no excuse for error. Always prolong a tangent line by double sightings. Also check deflection angles from time to time, by double sightings. Check on back sight before finally approving any precise point; likewise never fail to conclude the observations at each transit station by checking on the back sight. In such check it is usually best to sight back precisely on the point and then note whether the vernier has the proper reading. Assist the flagman in plumbing the pole, and always sight as near the bottom of the pole as possible. The transitman should admonish the chainmen, etc., to keep clear of the line.

On preliminary surveys, usually let the rear chainman line in the head chainman by eye, at least for short stretches. Do not hesitate to offset or zig-zag more or less along open ground to gain time. A rapid method for passing through heavy timber is to zig-zag on slight deflection angles right and left, tabulate the lengths in stations and deflections in minutes, and the products of the two in separate columns on the right hand page. The original line is regained by making the algebraic sum of the products zero, and the original direction is resumed by turning off a deflection which balances the deflection angle columns.

On location, each stake should be lined in carefully by transit. Small obstructions, such as trees, may be passed by parallel lines, using offsets of one foot or so at two hubs a few stations apart; the line is resumed in like manner. Where plate readings are used in rectangular or other offset methods, no sights shorter than 50 feet should be used. The equilateral triangle one station or more on a side is

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often used. Obstructions on curves may usually be passed readily with the aid of tables of long chords and mid-ordinates.

Curve index-readings should be calculated as though the entire curve were to be run in from the P. C.; starting with the index-reading of P. C. always equal to zero, check the calculations by noting that the index of M. C. is ¼ I, and of P. T. is ½ I. In using the notes, remember that with the transit at any point whatever on the curve, the following rules apply: (1) When pointing to any station, the vernier must always be set to read the index-reading for that station; and (2) when pointing on tangent at any station, the vernier must be set to read the index-reading for that station. As a rule, the best program in curve location is: Having P. I. located, (1) measure I and assume D; (2) calculate T and E; (3) establish P. T. by chaining off T on front tangent; (4) establish M. C. by laying off E on bisecting line; (5) locate P. C. by interpolating hub at calculated station number on back tangent; (6) move transit to P. C. and foresight on P. I.; (7) calculate curve notes (if not already done); (8) check sight on P. T. and M. C. and if satisfactory; (9) run in curve, checking for distance and angle on M. C. and P. T., moving transit ahead if desirable or necessary; (10) set up at P. T. and resume front tangent. One minute is the limit of allowable error in any curve. Mistakes in calculations or in measurements of angles will be counted serious errors. On final location the curves will be spiraled. After the line is located, reference out P. C., P. T., and other important hub points by two intersecting lines and take careful notes of the same (see method (g), Fig. 5, Chapter II).

The transit notes should be reliable, complete, neat and distinct. Each entry should have but one reasonable meaning and that the correct one. Record station numbers from the bottom upwards, usually with ten stations per page. Repeat the last station at the bottom of the next page. Allow two lines per station so as to provide for sketching at 200 feet to the inch. On the middle line of the right hand page mark each station with a dot and number every fifth station which should also be enclosed in a circle. The transit notes should include sketches of prominent land and street lines, stream crossings and other prominent topographic details, with pluses shown in the sketch. The notes should include date, weather, organization of party, etc. An appropriate title page giving name of survey, date

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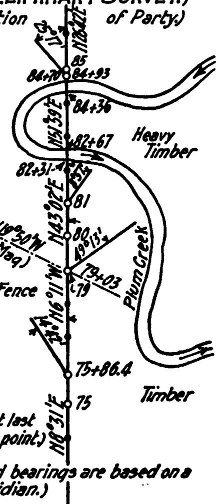
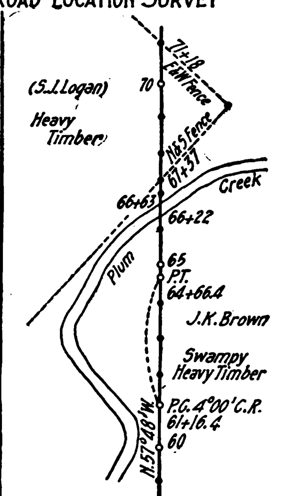
(TRANSIT NOTES FOR RAILROAD PRELIMINARY SURVEY.)

Sta.	Defl.	Mag.Ang.	Mag.B.S.	Mag.F.S.	Cal.Brg.	(Organization of Party.)
86						
85						
+93	25°17'L	25°20'L	N48°30'E	N23°10'E	N26°42'E	(Var.=3°32')
84						
83						
82						
81	8°57'R	9°00'R	N39°30'E	N48°30'E	N51°59'E	(Var.=3°29')
80						
79	49°13'R	49°10'R	N9°20'W	N39°30'E	N43°02'E	(Var.=3°12')
78						
77						
76						
+86.4	14°42'L	14°40'L	N4°35'E	N9°45'W	N6°11'W	(Var.=3°34')
75						
					(N8°31'E)	→(Calc.Brg. at last deflection point.)
74						
Note.	Magnetic Back Sight is needle reading on back tangent prolonged.					(Calculated bearings are based on a true meridian.)



TRANSIT NOTES FOR RAILROAD LOCATION SURVEY

Sta.	Curves	Index.	Defl.	Calc.Brg.	Needle.
71	(Repeat this sta. bottom next pages)				
70					
69					
68					
67					
66					
65					
+66.4	P.T.	7°00'	14°00'R	N43°48'W	N47°25'W
+50		6°40'	0°20'		
64	D = 4°00'	5°40'	1°00'		
+50	I = 14°00' R.	4°40'	1°00'		
63	T = 175.9	3°40'	1°00'		
+50	L = 3.50	2°40'	1°00'		
62		1°40'	1°00'		
+50		0°40'	1°00'		
+16.4	P.C.	0°00'	0°40'	N57°48'W	N61°20'W
61					
60					
59	(Repeated from top preceeding pages)				



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of commencement and completion, etc., should be prepared. The notes will be kept in the prescribed form. The field notes are to be returned at the close of the day's work. All estimated data should be noted as such.

Completeness and neatness of notes and records, facility and accuracy in handling the instrument, and promptness in advancing the progress of the survey will count in the estimate of the work of the transitman.

Head Chainman.—(Flag pole.) The progress of the chaining depends chiefly on the activity of the head chainman. After setting a stake he should move off briskly (preferably at a trot) and be prepared for the "halt" signal as he approaches the next station. When the full chain length is pulled out, the head chainman turns, holding the flag pole in one hand and the chain handle in the other, and sets the pole in line by signal from the rear chainman or transitman. Much time can be saved in this process if the head chainman habitually walks about on line and if he sights back over the two stakes last set. If on curve location, he should line himself in on the prolongation of the preceding station chord, and then offset by pacing or with flag pole a distance in feet equal to 1¾ times the degree of the curve; the calculation is made mentally and the pole can usually be set within a few inches of the correct position by the time a speedy transitman has the deflection angle set off. Having the line established, the pole is shifted to the correct distance, and the stake is driven plumb in the hole made by the flag pole spike. If the survey is a rapid preliminary line, the head chainman hastens ahead the instant the stake is started at the proper point, although in a more careful preliminary the chainmen check the distance to the driven stake. On location surveys it is customary for the chainmen to wait until the stake is driven and mark the exact distance on the top of the stake with the axe blade, and the exact line of signal from the transitman. In this process the head chainman should keep in mind the convenience of the transitman, and in case the line is being run to a front flag, the chainman should be careful to clear the line frequently to allow check sights ahead. In breaking chain on steep slopes the full length of chain should usually be pulled out ahead and the chain thumbed at the breaking points so as to avoid blunders; a plumb bob or flag pole should be used in the process. In passing over fences it often saves time to drive a 10-d nail, with "butterfly" attached, in the top plank to serve as a

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check back sight from the next transit point. The chainmen should carefully avoid obstructing the transitman's view, to which end they should walk on the outside when locating curves.

Rear Chainman.—(100-foot chain or tape, chaining pins (if allowed), figuring pad or note book.) As the rear chainman approaches the stake just set, he calls out "halt" and holds the end of the chain approximately over the stake, quickly lines in the flag pole in the hand of the head chainman (or the pole is lined in by the transitman), the precise distance is given, and the chainmen move on briskly. As a rule, pluses should be read by the rear chainman, the front end being held at the point to be determined. Fractions will usually be taken to the nearest 0.1 foot, although 0.01 foot may at times be properly noted. It is the duty of the rear chainman to keep a record of pluses and topographic details when the transitman is not at hand. This record may be kept on a figuring pad and the memoranda handed at the first opportunity to the transitman, who transfers the data to his book and carefully preserves the slips for future reference. It is usually better, however, to keep the auxiliary notes in a memorandum book instead of on the loose slips. The chainmen should carefully avoid disturbing the transit legs.

The responsibility for correct numbering of the station stakes rests chiefly on the rear chainman. It is his duty to remember the number of the previous station so as to catch blunders on the part of the stakeman. As he reaches the stake just driven, he mentally verifies its number and repeats it distinctly for the guidance of the stakeman in marking the stake to be driven; the stakeman responds by calling the new number, and each repeats his number as a check before final approval. The rear chainman then charges his mind with the numbers and checks the newly set stake on reaching it. In case of doubt he returns to the preceding stake and notes its number.

Stakeman.—(Sack of flat and hub stakes, marking crayon, handaxe.) The stakeman with his supply of flat and hub stakes in a sack, should keep up with the head chainman and be standing, with stake and marking keel in hand, ready to number the new station stake on hearing the rear chainman call out the preceding station number; the numbering is repeated, as already explained, before the stake is driven. Chaining pins are not used, but their equivalent in checking tallies may be had by numbering the

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stakes ahead and tieing them up in sets of ten. By numbering stakes at slack moments the stakeman gains time to assist the axeman in clearing the line, etc. However, special care should be taken to avoid omissions and duplicates. The stakeman should finish numbering the stake and hand it to the axeman by the time the head chainman has fixed the exact station point. The stakes should be numbered in a bold and legible manner, the keel being pressed into the wood for permanency. The number should read from the top of the stake downward. Stakes on an offsetted line should be so marked as 4′L or 2′R, beneath the station number. When survey lines are lettered, the serial letter should precede the station number. Guard stakes for P. I., P. C., P. T., reference points (R. P.), etc., should be clearly marked. The stakeman should assist the axeman in clearing the line and should drive stakes when the axeman is delayed. He should carefully avoid obstructing the transitman's view. The stakeman is under the direction of the head chainman.

Axeman.—(Axe, tacks, (and if so instructed) an extra sack of stakes with marking keel.) It is the duty of the axeman to drive stakes, remove underbrush from the line, clear an ample space about the transit station, etc. He is expressly warned, however, in student field practice, not to hack or cut trees or damage other property in any way, and in general, not to trespass on the rights of owners of premises entered in the progress of the survey.

The flat station stakes are driven firmly crosswise to the line with the numbered face to the rear. Hubs are driven about flush and usually receive a tack; they are properly witnessed by a flat guard stake driven 10 inches or so to the left, the marked face slanting towards the hub, as shown in Fig. 9, Chapter II. The axeman receives the marked stake from the stakeman and drives it plumb at the point marked by the spike of the flag pole. On location or careful preliminary surveys when the stakes are being lined in by transit, the axeman should stand on one side when driving and keep a lookout for signals from the transitman. In shifting the stake as signaled he should use combined driving and drawing blows with the axe. When the precise point comes much to one side of the top of the hub, another hub should be driven alongside and the first one driven out of sight before the tack is set. The axeman should move ahead briskly and avoid delay to the chaining. The stakeman should, when necessary, drive the stake with

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the spare handaxe. When the field force is scant, one man may serve in both capacities. The axeman is under the direct charge of the head chainman.

Front Flagman.—(Flag pole, small supply of hubs and guard stakes in stake sack, handaxe, a few 10-d nails.) It is the duty of the front flagman to establish hub points ahead of the chaining party under the direction of the chief and transitman. In selecting transit stations he should keep in mind visibility and length of both fore sight and back sight, and to this end, points should be taken on ridge lines and where underbrush, etc., is least in the way. The practice of planting the flag pole behind the hub may be warranted occasionally, as for example, when the field party is shorthanded, but never when the regular flagman is not specially detailed for other duties. The front flagman should keep close watch on the transitman and should habitually stand with the spike of the flag pole on the tack head and plumb the pole by standing squarely behind it and supporting it between the tips of the fingers of the two hands. Should the front flagman be flagging for an interpolated point depending on a foresight which his pole would conceal, he should clear the line for a check sight by leaning the pole to one side. When crossing fences he should, when convenient, establish check sights on the top plank by driving a spike and attaching a "butterfly" .

Rear Flagman.—(Flag pole, hatchet, slips of paper.) The rear flagman gives back sight on the preceding transit station. The details of his duties are much the same as those of the front flagman. It is an excellent plan for him to cut a straight sapling or limb and plant it exactly behind the hub when signaled ahead. This picket pole is made more visible by splitting the top and inserting a slip of paper, to make a "butterfly." A series of such pickets on a long tangent line often affords a fine check on the work when an elevated transit point is reached.

LEVEL PARTY.—It is the purpose of the level party to secure data concerning the elevations of the points along the line so that an accurate profile may be made and the grade line established. The leveling party should be on the alert to detect errors in the work of the transit party, such as omitted or duplicated stations, etc. The party consists of two members: (1) leveler, (2) rodman. In very brushy country an axeman may be added, but this is usually unnecessary if the line cleared by the transit party is followed.

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(LEVEL NOTES FOR RAILROAD SURVEY.) Keen, Leveler.

S	+	⊼	−	E	⊙	Oct. 13, 1893. Cool. Swift, Rodman.
B.M					712.39	Spike in notch at root of Elm tree, 68' R. of Sta. 15+48, 2' S. of rail fence.
⊼	+7.21	719.60				
16			8.4	711.2		Ground
17			7.2	712.4		"
18			5.4	714.2		"
19			6.4	713.2		"
20			4.5	715.1		"
+50			2.1	717.5		"
21			0.2	719.4		"
⊙		.3	−0.15		719.45	On hub at Sta. 21.
⊼	+8.83	728.28				
22			8.4	719.9		Ground
+28			6.6	721.7		" P.C. 1°00' C. R.
23			4.8	723.5		"
24			3.8	724.5		"
25			3.7	724.6		"
26			1.6	726.7		"
• B.M.			−1.57		726.71	Top of granite boulder, 74' R., Sta. 26+17.
⊼	+8.92	735.63				
27			5.6	730.0		Ground
+32			5.7	729.9		" P.T.
28			3.8	731.8		"
29			3.7	731.9		"
30			4.3	731.3		"
31			5.2	730.4		" (Checked ⊙'s, B.M.'s and H.I.'s with Rodman's Peg. Book.).
	+24.96	735.63	−1.72	Profile		
	−1.72	712.39		data		
	+23.24	+23.24	Check	above.		

Leveler.—(Level, adjusting pin, level note book.) The leveler should follow the most approved methods described under the head of differential and profile leveling in Chapter IV. The nearest 0.01 foot should be observed on turning points and bench mark rod readings and elevations and on occasional important profile points. The fore sight rod readings on ground profile points are to be taken only to the nearest 0.1 foot and the nearest 0.1 foot in the height of instrument is to be used in calculating the elevation. (Beginners sometimes calculate elevations to 0.01 foot when the rod readings are taken only to the nearest 0.1 foot.) The leveler should be rapid with his level as well as with figures. He should calculate elevations as fast as the rod readings are taken and should systematically check up the turning point and instrument heights as the work proceeds. As results are verified the same should be indicated by check marks. Each page of notes should be checked by summing up turning point back and fore sight rod readings, and comparing their difference with the difference between the first and last elevations or instrument heights, as the case may be, on the page. Follow the prescribed form. As far as

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possible, bench marks should be checked by including them in the circuit as turning points. Balance back and fore sight distances on turning points. Permanent bench marks should be established at least every 1500 feet, and located in places at once convenient and free from disturbance during construction. Later levels should check within 0.05 foot into the square root of the length of circuit in miles. When a discrepancy is found, a line of check levels must be run to fix responsibility for the error. In crossing streams, secure high water elevations, with dates, especially of extraordinary floods, also low water level. In crossing highways obtain elevations each side for some distance with a view to avoid grade crossings. In going up or down steep slopes, gain all the vertical distance possible each setting, and follow a zig-zag course. The bottom of deep gullies may be determined by hand level. Assist the rodman in plumbing the rod, and on turning points and benches have the rod gently swung in a vertical plane to and from the instrument and take the minimum reading. The self-reading rod is to be preferred. Many levelers use the Philadelphia rod without target. If the target is used on turning points, the leveler should check the rod reading when practicable.

Completeness, correctness and neatness of notes and records, and facility and accuracy in handling the level will be given chief weight in fixing the merit of the leveler's work. The level notes are to be returned at the end of the day's work.

Rodman.—(Leveling rod, peg book, hatchet, turning point pegs, spikes, keel.) The rodman holds the rod at station stakes and at such plus points as may be required to make a representative profile. It is his duty to identify each station point and be on the lookout for duplicated or omitted stations. To this end he should habitually pace in each station, especially in grass or underbrush, and call out or signal the station number to the leveler. Should a blunder in station numbering appear, he should positively confirm the fact by retracing several stations, and then carry the corrected stationing ahead. The rod should be held truly plumb, which is best done by standing squarely behind the rod and supporting it with the tips of the fingers of both hands. On turning points, the rod should be waved gently in a vertical plane to and from the instrument. The rodman should pay special attention to placing the target right for long rods and examine it to note if it has slipped

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before reading the rod. Errors of 1 foot, 0.1 foot, etc.. should be carefully guarded against. Turning points should be selected with special reference to their solidity, and care should be taken not to disturb them. Station pegs and hubs are often used for turning points; when so used, the precise fore sight to 0.01 foot should follow the usual ground rod reading to the nearest 0.1 foot. The rodman should use good judgment in selecting bench marks, locating them out of reach of probable disturbance during construction and describing them so as to be easily found. He should be active and do his best to keep close up with the transit party. The rodman should keep a peg book for recording turning points and instrument heights, and check his computations independently and compare results with the leveler.

TOPOGRAPHY PARTY.—It is the purpose of the topography party to secure full data for mapping contours, property lines, buildings, roads, streams, and other important topographic details. The width of territory to be embraced in the survey depends on local conditions; in places it may be as much as one-fourth or one-half mile from the line, although it is usually better to run alternate lines when the distance to be included becomes so great. The topography party often consists of only two men, but a party of four is much more efficient. Sometimes no regular topography party is provided, but after running a few miles of line ahead, the transit and level parties are formed into several parties to bring the topography up to the end of the preliminary line. For student practice the topography party will consist of four members: (1) topographer, (2) assistant topographer, (3) topography rodman, (4) tapeman.

Topographer.—(Topography board, topography sheet (or several sheets), hard pencil, compasses, eraser, etc.) The topography sheet should be prepared before going to the field, showing the alinement and other data needed from the transit notes, and elevations of all stations and pluses from the level notes. Cross-section paper is to be preferred. The center line may be plotted to one side of the center line of the sheet, when the topography is to be taken farther in one direction than the other. In order to secure full details, the scale of the field plat may well be double (or even more) that of the finished map. The topography sheet should show local conditions, such as gravel banks, rock ledges, etc., suitable for ballast or other constructive use; out-croppings of rock or other material which may

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affect the classification of the graduation; character of substrata at sites of bridge or other masonry work; springs, wells, streams, etc., suitable for water supply; approximate flood levels and other data relating to waterways or surface drainage; location of streams, especially with reference to desirable crossings, freedom from probable change of channel, etc.; location of highways including elevations some distance either way with special reference to avoiding grade crossings; other railroad lines, with the same point in view; character and condition of crops and other farm improvements, names of owners, etc.,—in short, any and all information that is at all likely to be of service in mapping the route, projecting the location, during construction, etc. In locating a group of buildings some distance from the line, fix the principal one by tie lines, by intersection or polar coordinates, and the others by measurement and sketch from it. Locate buildings near the line by rectangular offsets, or by intersections of the principal outlines with the survey line. Contours are located by means of the hand level used by the assistant topographer. The contour interval should be five feet ordinarily, but may be increased to ten or more feet on very steep slopes. The contour data should be selected with special reference to ridge and gully lines (see problem and plat on contour leveling, Chapter IV). Ordinarily hand level lines may be run out at right angles; angling lines along gulches and ridges may be located by estimation, pocket compass or tie lines. The plat is made by the topographer from data collected by the other members of the party. A common fault with the beginner in such work is the omission from the plat of important numerical data, such as station numbers of landline crossings, etc., owing to an undue attention to the minute details of the drafting work. A good topography record with contour notes on the left hand page and field sketch showing all numerical data on the right, is shown in the accompanying form.

Assistant Topographer.—(Hand level, pocket compass, topography note book.) It is the duty of the assistant topographer to collect data for the use of the topographer in making the plat. He uses the hand level, notes station numbers, distances, bearings, etc., and makes such record of the same as may be required to fit local conditions. In contouring, a special rod with adjustable base (see Fig. 19, Chapter IV.), if available, may be used; otherwise, an ordinary flag pole with alternate feet red and white is em-

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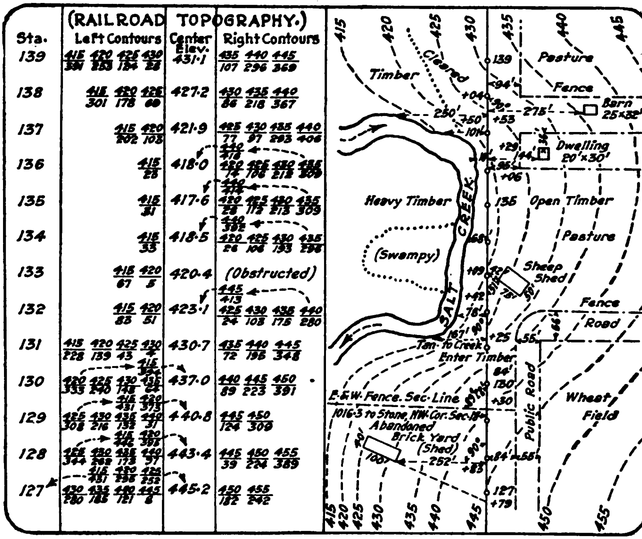

ployed. Beginning with the known profile elevation, as extracted from the leveler's record, even five-foot contours are located, as a rule, nominally every 200 to 500 feet at right angles to the line, except as ruling ridges or gullies may suggest other directions. His record should be ample and legible, and include data and information which may not properly be placed on the plat. All estimated elevations, distances or dimensions should be noted as such. The assistant topographer works under the direction of the topographer, but is expected to take the initiative in the collection of data so as to permit his superior to devote proper attention to the field plat.

Topography Rodman.—(Topography rod with adjustable base (see (f), Fig. 19, Chapter IV.) or flag pole, hatchet.) It is the duty of the rodman to hold the topography rod as directed by the assistant topographer. He should be active and continually on the alert for information or data which the record book or sheet should contain. The rodman holds the zero end of the tape in measuring the distances. He should acquire skill in pacing on rough as well as smooth ground, and when sufficiently exact es-

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pecially on ground remote from the surveyed line, he should gain time by pacing in the distances to contour lines.

Tapeman.—(Metallic (or band) tape, set of chaining pins, flag pole.) It is the duty of the tapeman to determine distances with the help of the rodman. He should be vigilant in checking up tallies, reading fractions, leveling the tape, breaking chain, plumbing down ends, etc., and should never be the cause of needless delay in the work. When required, he should measure angles, take tie lines, etc., with the tape.

OFFICE WORK.—The office work of each student includes: (1) reconnaissance map, profile and report; (2) map showing preliminary lines with topography and projected location lines; (3) preliminary profile with grade lines, approximate estimate of quantities, etc.; (4) final location map (traced from preliminary map); (5) location profile; (6) copies of field notes; (7) cross-section notes and estimate of graduation quantities; (8) estimate of cost of constrution; (9) monthly estimates, progress profile, haul, prismoidal and curvature corrections, vouchers, etc., final estimate.

Reconnaissance Report.—The reconnaissance map showing the area examined will be based upon such maps of the route as may be available. It should show the several ruling points and general routes selected for actual survey. The profile should be based upon barometric or hand level observations and distances scaled from the map or determined roughly by pacing or otherwise on the ground. The report should refer to the map and profile and state the general scheme, the several ruling considerations or conditions, the details of the examination, a rough comparison of the several alternative routes, and a final summary and conclusion with definite recommendations. The report should be made in accordance with best usage as to form, composition, etc.

(Considering the limited point of view of the beginner, the reconnaissance reports may not be required until the actual surveys are well along. In such case, however, the student is not to draw data from sources other than those above outlined.)

Preliminary Map.—The mapping should be the best product of the student's skill as a draftsman, and should conform closely to the department standards, which are based upon best current usage of leading American railroads. Unless otherwise instructed, the preliminary map

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PLOTTING SHEET, PRELIMINARY LINE "A"

	Sta.	Defl.	Cal. Brg.	Dist. Ft.	Latitudes N.	Latitudes S.	Departures E	Departures W	Total Lat. N.	Total Dep. W.	
a	0		N.32°34'W.	2489.1	2097.7			1339.8	0.0	0.0	a
b	$24+89^{1}$	32°07'L.	N.64°41'W.	78.5	33.6			71.0	2097.7	1339.8	b
c	$25+67^{5}$	28°15'R.	N.36°26'W.	464.1	373.4			275.6	2131.3	1410.8	c
d	$30+31^{2}$	10°38'R.	N.25°48'W.	225.5	203.0			98.1	2504.7	1686.4	d
e	$32+57^{7}$	2°27'R.	N.23°21'W.	436.7	400.9			173.1	2707.7	1784.5	e
f	$36+93^{9}$	18°40'R.	N.4°41'W.	164.8	164.2			13.5	3108.6	1957.6	f
g	$38+58^{7}$	46°59'L.	N.51°40'W.	152.9	94.8			119.9	3272.8	1971.1	g
h	$40+11^{6}$	37°05'R.	N.14°35'W.	176.0	170.3			44.3	3367.6	2091.0	h
i	$41+87^{6}$	16°07'R.	N.1°32'E.	310.1	309.9		8.3		3537.9	2135.3	i
j	$44+97^{7}$	21°27'L.	N.19°35'W.	105.9	99.6			36.1	3847.8	2127.0	j
k	$46+03^{6}$	6°17'L.	N.26°12'W.	307.8	276.2			135.9	3947.4	2163.0	k
l	$49+11^{4}$	75°06'L.	S.78°42'W.	331.9		65.0		325.5	4223.6	2299.0	l
m	$52+43^{3}$	73°14'R.	N.28°04'W.	202.7	178.9			95.4	4158.6	2624.5	m
n	$54+46^{2}$	21°29'L.	N.49°33'W.	156.4	101.5			119.0	4337.5	2719.9	n
o	$56+02^{6}$	32°05'L.	N.81°38'W.	332.6	48.4			329.1	4439.0	2838.9	o
p	$59+35^{2}$	36°52'R.	N.44°46'W.	308.7	219.2			217.4	4487.4	3168.0	p
q	$62+43^{7}$	11°54'L.	N.56°40'W.	128.1	70.4			107.0	4706.6	3385.4	q
r	$63+71^{8}$	42°20'L.	S.81°00'W.	251.8		39.4		248.7	4777.0	3492.4	r
s	$66+23^{6}$	50°03'R.	N.48°57'W.	334.2	219.5			252.0	4737.6	3741.1	s
t	$69+57^{8}$	35°11'L.	N.84°08'W.	266.9	27.3			265.6	4957.1	3993.1	t
u	$72+24^{7}$	2°05'R.	N.82°03'W.	317.0	43.8			313.8	4984.4	4258.6	u
v	$75+41^{7}$	7°22'L.	N.89°25'W.	557.8	5.7			557.8	5028.2	4572.4	v
w	$80+99^{6}$	43°15'R.	N.46°10'W.	1067.6	739.4			770.1	5033.9	5130.2	w
x	$91+67^{1}$			9167.1	5877.7	104.4	8.3	5908.6	5773.3	5900.3	x
					104.4			8.3			
					5773.3	← Check		5900.3	Check		

PLOTTING SHEET, LOCATION SURVEY, BELT R.R. EXTENSION.

From Sta.	To Sta.	Length Tang'ts Ft.	Length Curve Ft.	Angle I	Degree D.	Radius R. Ft.	Tan. Dist. T Ft.	Dist. P.I. to P.I. Ft.	Calc. Bearing	Lat. / Dep. Ft.	Tot. Lat. / Tot. Dep. Ft.
Tangent	Main	Track,	A.B.&C.	R.R.					S.35°15'E.	P.I.=	0.0 / 0.0
P.C. 0	P.T. 1		100.0	7°30'R.	7°30'	764.1	50.1	1320.4	S.27°45'E.	S.E.	
P.C. 1	P.T. 26		2500.0	25°00'R.	1°00'	5730.0	1270.3			P.I.=	S.E.
P.T. 26	P.C. $43+55^{2}$	1755.6						3963.7	S.2°45'E.	S.E.	
P.C. $43+55^{2}$	P.T. $61+68^{1}$		1812.5	36°15'R.	2°00'	2865.0	937.8			P.I.=	
P.T. $61+68^{1}$	P.C. 93+65	3196.9						4580.1	S.33°30'W.	S.W.	
P.C. 93+65	P.T. $102+51^{2}$		886.7	13°18'L.	1°30'	3820.0	445.4			P.I.=	
P.T. $102+51^{2}$	P.C. 143+90	4138.3						5185.7	S.20°12'W.	S.W.	
P.C. 143+90	P.T. $155+76^{2}$		1186.7	23°44'L.	2°00'	2865.0	602.0			P.I.=	
P.T. $155+76^{2}$	P.C. 170+43	1466.3						2618.3	S.3°32'E.	S.E.	
P.C. 170+43	P.T. $181+29^{7}$		1086.7	21°44'R.	2°00'	2865.0	550.0			P.I.=	
P.T. $181+29^{7}$	P.C. $184+50^{6}$	320.9						1307.3	S.18°12'W.	S.W.	
P.C. $184+50^{6}$	P.T. $193+19^{5}$		868.9	13°02'R.	1°30'	3820.0	436.4			P.I.=	
P.T. $193+19^{5}$	P.C. $213+06^{4}$	1986.9						2816.9	S.31°14'W.	S.W.	
P.C. $213+06^{4}$	P.T. $220+90^{8}$		784.4	11°46'L.	1°30'	3820.0	393.6			P.I.=	
P.T. $220+90^{8}$	P.C. $230+54^{6}$	963.8						1963.8	S.19°28'W.	S.W.	
P.C. $230+54^{6}$	P.T. $242+49^{6}$		1195.0	23°54'R.	2°00'	2865.0	606.4			P.I.=	
P.T. $242+49^{6}$	P.C. $252+35^{6}$	986.0						1861.3	S.43°22'W.	S.W.	
P.C. $252+35^{6}$	P.T. $257+52^{7}$		517.1	38°47'L.	7°30'	764.1	268.9			P.I.=	
		14814.7	10938.0	127°25'R.	Tang't Main Trk.		5560.9	25617.5	S.4°35'W.		(Check
		10938.0		87°35'L.	X.Y.&Z.R.R.		× 2		(See		last
	Check	25752.7		39°50'R.	(1st T.+last T.)=		11121.8		check		totals
							-319.0		in 5th		by Σ)
				S.35°15'E.			10802.8		column)		
			Check.	S.4°35'W.	Σ Tangents =		14814.7				
					Check		25617.5				

(Last two columns are to be filled in and checked by each student)

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will be made on eggshell or paragon paper. There are three ways to plot the skeleton of the preliminary survey: (1) by laying off each successive deflection angle and distance from the preceding line; (2) by laying off the successive calculated courses and distances from a precisely drawn meridian or other reference line; and (3) by rectangular coordinates. The first method should not be used, since cumulative errors are probable. The second is rapid and free from serious objection; if preferred, a modified base line may be assumed and the calculated bearings transferred to the same; the angles may be laid off by means of scale and table of natural trigonometric functions from a precisely drawn base line and then transferred, as required, by parallel ruler or triangle; this method is used most in practice. The third method is the most exact, and will be used by the student unless the second is specified. It involves the calculation of a plotting sheet, as shown in the accompanying form. The axis is usually a meridian line, but any line may be taken and the courses changed to suit. In making the plotting table, the data, calculated bearings, distances, etc., should be carefully checked through to the last point in the skeleton before the plotting is begun. Only one axis should be plotted, preferably the one having greater totals, so as to give short perpendiculars. Starting from the origin, 1000-foot points are pricked in along the axis to the specified scale, and marked 0, 10, 20, etc.; the totals are interpolated on the axis and lettered; exact perpendiculars about the right length are erected; the second point is established by scaling the perpendicular and the line is checked back on the preceding point; if correct, the stations are pricked in and every fifth station and deflection points are enclosed in a small circle and neatly numbered; the next course is so located and checked back by length of hypothenuse, the stations fixed and numbered, and so on to the end of the line; the courses should be taken in their order and none passed without checking satisfactorily. After the skeleton is completed, the topographic details are penciled in, and the map finished and inked. The title, border, meridian (both true and magnetic), etc., should be first-class in quality and in keeping with the rest of the map. Crude or careless lettering or other details of the map will cause its rejection. The title of the map, profile, etc., should be given in brief on the outside of the sheet or roll at each end.

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Preliminary Profile.—Use Plate A profile paper in making the profiles. The level notes should first be carefully verified and then one person should read off while another plots the data. A hard pencil, 6H or 7H, sharpened to a long needle point should be used. The stations are first numbered along the bottom from left to right (or the reverse, as prescribed) ; leaving six inches or so at the left for a title, and beginning at a prominent line with station 0, every tenth station is so numbered. The notes are examined for lowest and highest elevation and a prominent line is assumed as an even 50 or 100-foot value relative to the datum. The horizontal scale is 400 feet and the vertical scale 20 feet to the inch. Points should be plotted no heavier than necessary, since the surface of profile paper will not permit much erasing. The surface line should be traced in close up to the plotted points, owing to the danger of overlooking abrupt breaks such as streams, ditches, etc. Pluses should be fixed by estimation. The surface line when completed should be inked with a ruling pen used freehand; the weight of the line should be about the average of the ruled lines on the profile paper. (A special profiling or contouring pen is much used for this purpose.) The profile should show the grade line, grade intersection, elevations and rates of grade in red; water levels, and data relative to same in blue; surface line, station numerals, etc., in black; the alinement, important land-lines, streams, etc., should be shown at the bottom of the profile in black. The grade line should be laid nominally with a view to balance the cut and fill quantities, but this should be varied to suit local conditions, such as drainage, the elimination of grade crossings, classification of materials, etc. The maximum gradients, the rate of compensation for curvature, etc., will be made to suit the specified conditions. The compensation for curvature will be allowed for on the preliminary profile by dropping the grade line on maximum gradients at each deflection point. Grade intersection elevations and rates of grade will be given to the nearest 0.01 foot.

Approximate Estimates.—Rapid estimates of earthwork quantities may be made direct from the profile either by reference to a table of level sections, or preferably by means of an earthwork scale. Estimates made in this way from the profile of a careful preliminary survey, often do not vary more than five per cent from the final construction quantities.

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Location Map.—The location map may be traced from the preliminary map and should include the topography and such details as usually appear in the final record map of the located line. Contour lines may be traced in cadmium yellow to insure satisfactory blue printing.

Location Profile.—The location profile should be executed according to the standard specimen, and should include estimates of earthwork as determined from the actual cross-section notes, and quantities of other construction materials. Curvature compensation will be shown on the location profile by reduced maximum gradients. Vertical curves will be calculated at a rate of change not to exceed 0.05 foot per station, except at summits where it may be 0.10 foot or more per station. It should be prepared as the final record profile. Approximate profiles of projected lines, determined from the contour map, with rough estimates of quantities will also be prepared, as specified.

Office Copies of Notes.—The complete level and transit notes, and topography notes as assigned, must be copied in the individual books by each student. These copies will be in pencil (or ink if so specified) and will be executed in a faithful and draftsmanlike manner according to the department standards of lettering, etc.

Estimates of Quantities.—The cross-section notes will be copied and the quantities of excavation and embankment calculated, as assigned. The cross-sectional areas will be calculated arithmetically and checked, especially on rough ground, by means of planimeter. The quantities will be calculated by average end areas, by tables, and by diagrams, so as to afford ample practice for the student in all the current methods. The estimate will also include all the other materials of construction.

Estimate of Cost.—Each student will make a detailed summary of the quantities, fix prices, and estimate the probable total cost of the work, or of the assigned section. The prescribed form will be followed. The prices should be based on local conditions as far as possible.

Construction Estimates.—Monthly estimates, estimates of haul, borrow pit estimates, classification, prismoidal and curvature corrections, progress profile, vouchers, force account, etc., and final estimate will be prepared by each student in accordance with prescribed forms and standards.

Right of Way Records.—Each student will be assigned a share of work in the preparation of right of way deeds and record maps. The following forms (from the "Engi-

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neering Rules and Instructions," Northern Pacific R. R.) will be used as models in preparing right of way descriptions.

(Through government subdivisions): "A strip, piece or parcel of land one hundred feet in width, situated in the northwest quarter of the northwest quarter of section ten, in township two north, range one west (S. 10, T. 2 N., R. 1 W.), Madison county, Montana, and having for its boundaries two lines that are parallel with and equidistant from the center line of the railroad of the ——— Railway Company, as the same is now located (and constructed). For a more particular description, reference may be had to the plat drawn upon and made a part of this deed."

(Lots in platted tracts): "Lot seven (7), block six (6), in Smith's addition to Helena, Lewis and Clark county, Montana, according to the recorded plat thereof."

CROSS-SECTIONING PARTY.—It is the duty of the cross-sectioning party to set slope stakes for the proposed roadbed and to secure data for the calculation of earthwork quantities. The data should first be transcribed from the location level notes and profile into the cross-section book, including station numbers, surface and grade elevations, rates of grade, bench mark record, etc. In order to avoid confusion in relation to directions right and left, the station numbers should run up the page, and plenty of space left for pluses in the notes, especially on rough ground. As shown in the form, the left hand page should be used for data and the other for the cross-section notes.

The organization and equipment of the cross-sectioning party when using the engineers' level is: (1) recorder (note book), (2) leveler (engineer's level), (3) rodman (self-reading leveling rod, 50-foot tape), (4) axemen (axe, sack of flat stakes, marking keel). The usual routine is: (1) Determine height of instrument by back sight on identified bench or turning point. (When a bench mark is remote and an original turning point can not be found, it may suffice in an emergency to check on the ground at several stations to the nearest 0.1 foot and use the mean height of instrument. Such places should be verified later.) (2) Having the height of instrument, check the original elevation of the station about to be cross-sectioned, reading the rod and checking off the elevation if it does not differ more than 0.1 foot or so; in case of a new plus, take a rod reading and record the elevation. (3) Determine the "grade rod" for the station by subtracting the height of instru-

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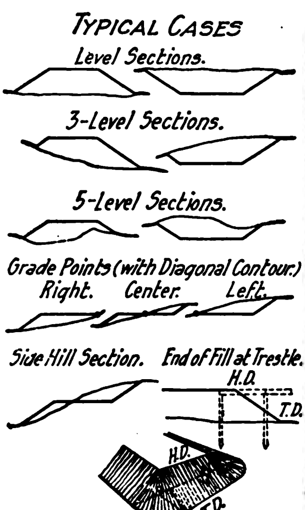
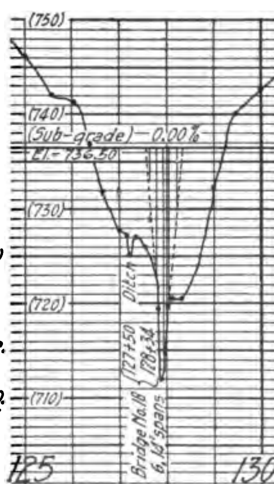
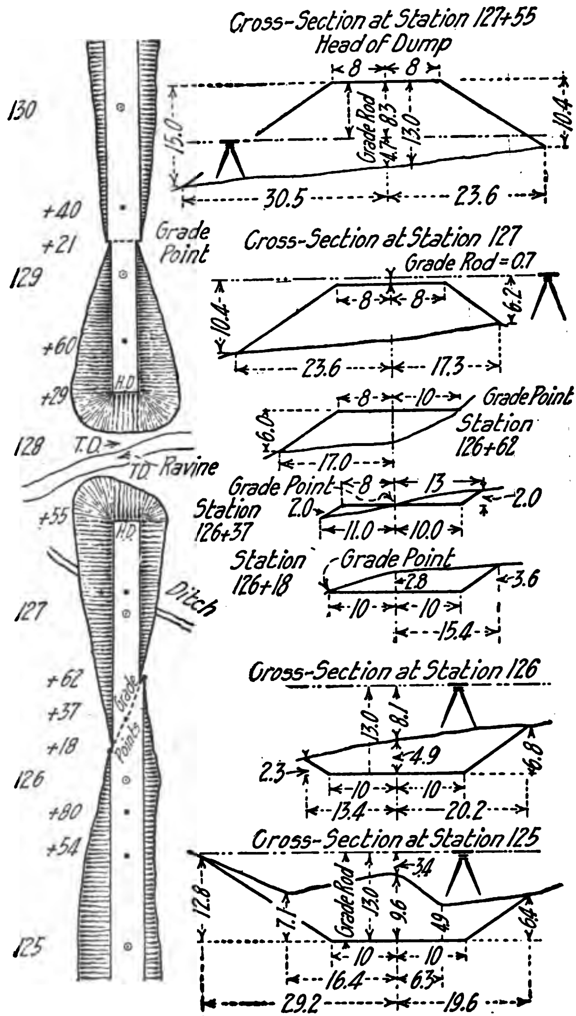
(FORM FOR CROSS-SECTION NOTES)

Sta.	Elev.	Grade	Rate	Surf. Rod	Grade Rod	⊼	L	C	R	Remarks
					⊼ 747.67					
130	742.5	736.50		5.2	11.2	⊼ 728.22	[illegible]/18.1	+6.0	[illegible]	(3-level section in cut)
+40	739.8	736.50		7.9	11.2	F.S. 0.57 ● 727.65	[illegible]	+3.3	[illegible]	(Level section in cut)
					⊼ 739.61	B.S. 11.96				
+21	736.5	736.50		3.1	3.1	⊼ 739.61 F.S. 2.03	0.0/10.0	0.0	0.0/10.0	(Grade point, L, C and L)
129	732.3	736.50			3.1	● 737.58	[illegible]	-4.2	[illegible]	(3 level section in Fill)
+60	723.9	736.50		4.3	⊼ 728.22 -8.3	B.S. 10.09 ⊼ 747.67	[illegible]	-12.6	[illegible]	(Level section in Fill)
+34						F.S. 5.48				N. end stringer, Br. No. 18.
H.D.+29	720.5	736.50		7.7	-8.3	742.19 Check on	-16.0/32.0	-16.0	-16.0/32.0	(Head of Dump)
T.D.+05	720.5	736.50		7.7	-8.3	B.M. No. 12	0.0/8.0	-16.0	0.0/8.0	(Toe of Dump)
128	720.1	736.50		8.1		(below)				Bridge No. 18 {128+34
+90	712.2		0.00% Gradient Sta. 118 to Sta. 135+50							6, 14' spans {127+50
T.D. {+81.4 L					-8.3	⊼ 737.23 F.S. 10.15	0.0/8.0 -17.6	←		(Toe of Dump {left.)
+73.8 R					-8.3	● 727.08		→	-12.4 0.0/8.0	right.)
H.D.+55	723.5	736.50		4.7	-8.3	B.S. 1.14 ⊼ 728.22	[illegible]	-13.0	[illegible]	(Head of Dump)
+50										S. end stringer, Br. No. 18
+13					⊼ 737.23					Ditch 2.4' × 4.7' × 53.'
127	727.8	736.50		9.4	0.7	⊼ 749.51	[illegible]	-8.7	[illegible]	(3 level section in Fill)
+62	731.9	736.50		5.3	0.7	F.S. 12.58	[illegible]	-4.6	0.0/10.0	(Grade point right)
+37	736.5	736.50		0.7	0.7	● 736.93 B.S. 0.30	[illegible]	0.0	[illegible]	(Grade point center)
+18	739.3	736.50		10.2	⊼ 749.51 13.0	⊼ 737.23	0.0/10.0	+2.8	[illegible]	(Grade point left)
126	741.4	736.50		8.1	13.0		[illegible]	+4.9	[illegible]	(3 level section in cut)
+80	741.7	736.50		7.8	13.0		[illegible]	+5.2	[illegible]	(Level section in cut)
+54	742.2	736.50		7.3	13.0	B.M. No. 12 742.17	[illegible] [illegible]	+5.7	[illegible]	(4 level section in cut)
125	746.1	736.50		3.4	13.0	B.S. 7.34 ⊼ 749.51	[illegible] [illegible]	+9.6	[illegible] [illegible]	(5 level section in cut) Cuts 20', 1½:1, Fills 16', 1½:1.







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ment from the grade elevation; then note that cut or fill at any point of the cross-section is equal to surface rod minus grade rod (counting rods as minus when downward from the plane of the level and those upward as plus, this rule gives results always plus for cut and minus for fill, which agrees with the conception that cross-section notes are rectangular coordinates of the sectional area referred to the center of the finished roadbed as an origin). (4) If the ground is level transversely, that is, does not vary more than 0.1 foot or so within the limits of the proposed grading, then the distance from the center out to each side slope stake is half width of roadbed plus center cut or fill times rate of side slope; (thus for 20-foot roadbed, side slopes 1 to 1, and a cut of 18.6 feet, the distance out to slope stake on a level section would be 28.6 feet, or with a slope of 1½ to 1, the distance out would be 10 plus 1½ times 18.6, or 37.9 feet. Calculations of this sort should be done mentally in an instant). (5) On three-level ground estimate the rise or fall of the surface from the center to about where the side slope stake should come, and add the same to, or subtract it from the center cut or fill, as the case may be; compute the distance out to the point where the side slope line would pierce the ground surface and test the same with tape, rod and level by the foregoing rule for cut or fill; continue to construct points on the side slope line until the common point is found. (6) The axeman marks "S. S." (slope stake) on one side of the stake with the cut or fill to the nearest 0.1 foot (as C 6.8 or F 10.2) and the station number on the other side; the stake is driven slanting towards or away from the center line according as it is cut or fill. (7) On five-level ground or, in general, on ground involving any number of points or angles in the section, the cut or fill is taken at each break. (8) Should there appear to be danger of land slips, the cross-sectioning should be carried well beyond the limits of the slope stake points. (9) The cross-section notes are recorded as in the accompanying form, expressing the coordinates of each point in the form of a fraction, and distinguishing the slope stake points by enclosure in a circle. (10) Having completed the cross-sectioning at the station, the same program is followed at the next point, first checking the elevation obtained in the original location levels; the grade rod should be determined as before by subtracting the height of instrument from the grade elevation, and then checked by applying to the preceding grade rod the

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rise or fall of grade from the preceding point. (11) Cross-sections should be taken as a general rule at every station and at such intermediate points as will insure a reliable measurement of the earthwork quantities. It is not necessarily the lowest and highest points that are required, but those points which, when joined by straight lines, will give the contents as nearly as possible equal to the true volume; if the "average end areas" method is to be used in calculating the quantities, sections should be taken every 50 feet when the difference of center height is as much as 5 feet; as a rule, slope stakes need not be set at cross-sections taken between stations. (12) "Grade point" stakes (marked 0.0), should be set where the center line and each edge of the roadbed pierce the ground; and also in side-hill sections in both cut and fill, where the roadbed plane cuts the ground line; if the width of roadbed is different in cut and fill, the greater half-width is commonly used in locating the side grade point; in the simplest case a contour line is perpendicular to the center line and the three grade points are at the same cross-section, forming two wedges; in the more usual case the contour line is diagonal, and the three grade points are not in the same section, so that two pyramids are formed; if the station numbers of the two side grade points differ by only a few feet, it is usual to simplify the record by taking the notes as for a wedge at the station number of the center grade point, although the side grade point stakes are set in their true positions; as a rule, a complete cross-section is taken at each grade point. (13) In cross-sectioning for the end of an embankment at a wooden trestle the end slope is made the same as the side slope, and the end and side planes are joined by conical quadrants; the distance between "heads of dump" (H. D.) is usually 10 feet (5 feet at each end) less than the total length of stringers; a complete cross-section is taken at the "head of dump," and the "toe of dump" (T. D.) on each edge of the end slope is located and recorded; on level ground the volume of the wedge-like solid so formed is found by dividing it into a triangular prism and two right conical quadrants; on ground sloping transversely the end of dump is made up of a middle prismoid and two conical quadrants, each of the latter being generated by a variable triangle revolved about a vertical axis through a corner of the top roadbed plane at "head of dump."

The calculations in the foregoing method of cross-section-

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ing may be simplified by preparing a table of distances out for the standard roadbed widths and slopes, or by using a special tape having the zero graduation at a distance from the end equal to the half-width of roadbed, and the remaining graduations modified to suit the side slope ratio. The calculations may be further simplified by using a special rod having an endless sliding tape graduation. The student will be given practice with these labor saving devices after he has first acquired familiarity with the principles of cross-sectioning without these aids.

Cross-sectioning with rods alone is done in much the same manner as that described above. Two rods are used. The usual length of the rods is ten feet, and each is graduated to tenths and has a bubble vial in one or both ends. The slope stake point is determined by leveling out from the ground at the center stake with reference to the center cut or fill, each rod being held alternately level and plumb. Other points in the cross-section, as well as grade points, etc., are determined in the same manner. The notes are kept as in the other method. On very rough ground, the rod method is usually the more rapid. Some engineers cross-section on rough ground by taking the elevation of each point and plotting the notes on cross-section paper, then using the planimeter to determine the areas. Borrow pits are often cross-sectioned by taking elevations at the intersections of two series of parallel lines forming squares.

Land-Line Party.—It is the duty of the right of way party to secure data for the preparation of right of way deeds. The party should consist of at least four: (1) recorder, (2) transitman, (3) head chainman, (4) rear chainman, (the chainmen also to serve as axemen and flagmen as required). Their equipment is the usual one of a transit party for such work. The party should secure ties with all section and other land lines whenever crossed. The notes should show station numbers and angles of intersection and distance along land line to the nearest identified land corner and also to important fences. As a rule, make the intersection by running through from one corner to the other. Where the line passes through a town, tie the center line to the plats, block lines, monuments, etc. Secure any records and make tracings of any plats, etc., at the recorder's office, that may be of service in preparing deeds.

Bridge and Masonry Party.—The bridge and masonry survey party will determine drainage areas for culverts and other waterways, prospect for foundations, and stake out

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trestles, masonry work, etc. The usual organization will be four men: (1) recorder (in charge), (2) transitman or leveler, (3) chainman, rodman, flagman, etc., (4) chainman, axeman, flagman, etc., as the work assigned may demand.

Resurvey Party.—The resurvey party will be assigned to such duties as the resurvey of yards, the collection of data for crossings frogs, running centers on old track, including spiraling, etc. It will usually be a party of four.

PROBLEMS IN RAILROAD SURVEYING.

PROBLEM G1. ADJUSTMENTS OF LEVEL AND TRANSIT.

(a) *Equipment.*—Engineers' level and transit, adjusting pin.

(b) *Problem.*—Test the essential adjustments of the assigned instruments and correct any discrepancies found.

(c) *Methods.*—This problem is designed to freshen the student's knowledge of the adjustments of the instruments, as well as to place the equipment in condition for accurate work. The adjustments will be made under the personal direction of the instructor. The student should attempt to be speedy as well as accurate in testing and making the adjustments.

PROBLEM G2. USE OF FIELD EQUIPMENT.

(a) *Equipment.*—Complete equipment for railroad transit and level party, as specified in foregoing pages.

(b) *Problem.*—Practice the detailed duties of each position in the transit and level party.

(c) *Methods.*—This problem is designed as a "breaking in" exercise preparatory to engaging in the regular field work of railroad location. With the manual in hand the duties of each position will be studied and practiced in turn.

For example, each student will go through the following exercise with the transit as briskly as possible: (1) set transit over tack in hub, (2) level up, (3) set plate to zero, (4) reverse telescope and sight on back flag, (5) release needle, (6) plunge telescope, (7) read and record needle on back line prolonged, (8) sight at front flag pole, (9) read and record deflection angle right or left, (10) read and record needle on front line, (11) lift needle, (12) plunge telescope and check on back flag, (13) calculate needle

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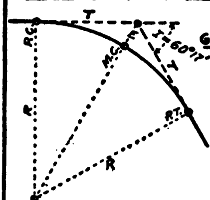
angle and compare with plate reading, and if checked, shoulder transit; now repeat entire process at the same hub, more briskly than at first, if practicable, avoiding reference to preceding record until the full series of steps is completed.

Problem 2. Calculation of Curve Elements.



Given: $I = 60°17'R.$, $D = 4°17'$, $R(4°17') = 1337.65$

Required: L, T and E. (a) By trigo. (b) By Table I°C. (Results to 0.01 ft.)

$\frac{1}{2}I = \frac{60°17'}{2} = 30°08'.5$

tan $30°08'.5 = 0.58066$

exsec $30°08'.5 = 0.15636$

$60°17' = 60°.2833+$

$4°17' = 4°.2833+$

Part	Method (a)	(b)	Diff.
L	14.07[39]	14.07[39]	
T	776.71	776.77	0.06
E	209.15	209.17	0.02

Indicated Work. / **Calculations.**

Length of Curve, L.

$L = \frac{60°17'}{4°17'}$

(a) $= \frac{3617'}{257'} = 14.07^{39}$

(b) $= \frac{60°.2833}{4°.2833} = 14.07^{39}$

Calculations: 257)3617)14.0739 o.k. — 257, 1047, 1028, 1900, 1799, 1010, 771, 2390, 2313, 77

60.2833)4.28333 14.0739 o.K. — 428333, 174500, 171333, 3167, 2998, 169, 128, 41, 38

Tangent Distance, T.

(a) $T = R \tan \frac{1}{2}I$

$= 1337.65 \times 0.58066$

$= 776.71$

(b) $T = \frac{T_{1°c.}}{D}$

$= \frac{3327.15}{4.2833}$

$= 776.77$

Calculations: 1337.65 × 660850 — 66882, 10701, 80, 8 — 776.71 o.k.

$T_1(60°16') = 3326.0$

$T_1(60°18') = 3328.3$

$T_1(60°17') = 3327.15$)4.2833 (3) 776.77 o.k. — 2998.33, 32882, 29983, 2899, 2570, 329, 300, 29, 30

776.77 − 776.71 = 0.06

Diff. due to approx. basis of method (b).

External Distance, E.

(a) $E = R$ exsec $\frac{1}{2}I$

$= 1337.65 \times 0.15636$

$= 209.15$

(b) $E = \frac{E_{1°c.}}{D}$

$= \frac{895.95}{4.2833}$

$= 209.17$

Calculations: 1337.65 × 63651.0 — 13376, 6688, 803, 40, 8 — 209.15 o.k.

$E_1(60°16') = 895.4$

$E_1(60°18') = 896.5$

$E_1(60°17') = 895.95$)4.2833 (3) 209.17 o.k. — 85667, 3928, 3855, 73, 43, 30, 30

209.17 − 209.15 = 0.02

Diff. due to approx. basis of method (b).

Let the student prepare a similar numbered program for each of the other positions and practice the same systematically. This series of exercises may profitably occupy two or more assignments, since the speed and quality of the actual surveys to follow are certain to be much enhanced.

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PROBLEM G3. PRELIMINARY FIELD CURVE PRACTICE.

(a) *Equipment.*—Transit party equipment, as prescribed in instructions.

(b) *Problem.*—Run out the assigned practice curves in the field, with the prescribed organization and conditions.

(c) *Methods.*—The preliminary curve practice is designed to give the student a practical knowledge of the principles of railroad curves and the routine methods used in location surveys. The several positions in the field party will be filled in succession, and each student is expected to respond heartily to the spirit of the practice, whatever his assigned duties. Each member of the party should engage in the calculations as far as practicable. The report of the field work should state the precision of linear and angular checks. The field practice will be based in part on the indoor curve problems.

PROBLEM G4. CURVE PROBLEMS.

(a) *Equipment.*—Drafting instruments, paper, etc.

(b) *Problem.*—Solve the assigned problems in railroad curves and submit results in a neat and draftsmanlike form.

(c) *Methods.*—(1) Draw a plain figure to the largest convenient scale. (2) State problem and present data in a concise and systematic manner. (3) Show the separate steps clearly; first state formulas in general terms, then substitute values and give results; as a rule, show actual calculations adjacent to the indicated work; habitually verify results by an independent process; use common sense checks and contracted methods of calculation; in general, make full use of the opportunity to gain skill as a computer. (As a rule, the nearest 0.1 foot only is required in field measurements on curve location, but it is excellent practice, especially for the beginner, to preserve the nearest 0.01 foot in the calculations.)

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CHAPTER IX.

ERRORS OF SURVEYING.

Errors.—Errors of observations are of three kinds, viz., (1) mistakes; (2) systematic errors; (3) accidental errors. Systematic errors include all errors for which corrections can be made, as erroneous length of standard, errors of adjustment, refraction, etc. Accidental errors are those which still remain after mistakes and systematic errors have been eliminated from the results.

It has been found from experience that accidental errors are not distributed at random but follow mathematical laws. These laws are fundamental in the Theory of Least Squares and are: (1) small errors are more frequent than large ones; (2) positive and negative errors are equally numerous; (3) very large errors do not occur.

Arithmetical Mean.—The most probable value of a quantity obtained by direct measurements is the arithmetical mean of all the determinations where the observations are of equal weight, or is the weighted mean where the observations are of unequal weight.

Precision of Observations.—In the adjustment of observations it is often necessary to combine results of different degrees of precision or weight. It is also desirable to have some means of comparing observations so that the computer may know what degree of confidence to place in the results. The quantity commonly used for comparing the precision of observations is the probable error.

Probable Error.—The probable error is such a quantity that it is an even wager that the number of errors greater is the same as the number of errors less than the probable error. It is also the limit within which the probability is one-half that the truth will fall. For example, if 4.63 ± 0.12 is the mean of a number of observations, the true value is as likely to be between 4.51 and 4.75 as it is to be some value greater or less.

Probable error is also useful in finding the relative weights that should be given different sets of observations, as it has been found that the weights of observations vary inversely as the squares of their probable errors.

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Formulas:

Let E_1 = probable error of a single observation.

E_m = probable error of the mean of all the observations.

n = the number of observations.

d = the difference between any observation and the mean of all the observations.

Σ = symbol signifying sum of.

Then from the Theory of Least Squares

$$E_1 = 0.6745\sqrt{\frac{\Sigma d^2}{n-1}} \tag{1}$$

$$E_m = 0.6745\sqrt{\frac{\Sigma d^2}{n(n-1)}} \tag{2}$$

$$= \frac{E_1}{\sqrt{n}} \tag{3}$$

The probable error of the weighted or general mean is

$$E_0 = 0.6745\sqrt{\frac{\Sigma p \cdot d^2}{(n-1)\Sigma p}} \tag{4}$$

where $\Sigma\ p$ = summation of the weights.

The probable error of a quantity with a weight p is equal to E_0 divided by the square root of p.

The probable error of Z, where $Z = z_1 \pm z_2$, and R_1, r_1, and r_2 are the probable errors of Z, z_1 and z_2, respectively, is

$$R_1^2 = r_1^2 + r_2^2 \tag{5}$$

The probable error of Z, where $Z = a.\ z$ is

$$R_1^2 = a^2 \cdot r^2 \tag{6}$$

The probable error of Z, where $Z = z_1.\ z_2$ is

$$R_1^2 = z_1^2 \cdot r_2^2 + z_2^2 \cdot r_1^2 \tag{7}$$

This would be the probable error of the area of a rectangle where r_1 and r_2 are the probable errors of the sides z_1 and z_2, respectively.

Example.—As an example of the application of these formulas consider the two following series of measurements of an angle given in Table I. The first set was taken with a transit reading to 10 seconds, the second with a transit reading to 30 seconds.

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TABLE I

FIRST TRANSIT.				SECOND TRANSIT.			
No.	Angle.	d	d^2	No.	Angle.	d	d^2
1	34° 55′ 35″	2	4	1	34° 56′ 15″	39	1521
2	35	2	4	2	55 30	6	36
3	20	13	169	3	54 30	66	4356
4	05	28	784	4	55 15	21	441
5	56 15	42	1764	5	56 00	24	576
6	55 40	7	49	6	55 45	9	81
7	10	23	529	7	55 30	6	36
8	30	3	9	8	55 30	6	36
9	50	17	289	9	56 00	24	576
10	30	3	9	10	55 45	9	81
Mean	34° 55′ 33″	$\Sigma d^2 = 3610$		Mean	34° 55′ 36″	$\Sigma d^2 = 7740$	
$E_m = 0.6745\sqrt{\frac{3610}{9 \times 10}} = \pm 4''.3$				$E_m = 0.6745\sqrt{\frac{7740}{9 \times 10}} = \pm 6''.3$			

The weights of these mean values vary inversely as the squares of the probable errors, or in this case the weights are as $\frac{1}{4.3^2}$ to $\frac{1}{6.3^2}$ or as 12 to 5. The most probable value of the angle measured with the two transits will be the weighted mean.

$$Z = 34^\circ\, 55' + \frac{33 \times 12'' + 36 \times 5''}{17}$$
$$= 34^\circ\, 55'\, 33''.9$$

The probable error of this result from (5) since

$$Z = \tfrac{12}{17} z_1 + \tfrac{5}{17} z_2, \quad \text{is}$$

$$R_1^2 = (\tfrac{12}{17})^2 r_1^2 + (\tfrac{5}{17})^2 r_2^2$$

Substituting $r_2^2 = \frac{12}{5} r_1^2$ we have

$$R_1^2 = (\tfrac{12}{17})^2 r_1^2 + (\tfrac{5}{17})^2 (\tfrac{12}{5}) r_1^2$$
$$= \tfrac{12}{17} r_1^2$$

$$R_1 = \pm 4.''3 \sqrt{\tfrac{12}{17}} = \pm 3''.6.$$

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For other examples in the use of probable error see probable error of measuring a base line, probable error of setting a level target, probable error of setting a flag pole.

Angle Measurement.—The measurement of an angle requires two pointings and two readings. If r_r and r_s are the probable errors of reading and pointing, respectively; the probable error of the measurement of an angle will from (5) be

$$R_1 = \sqrt{r_r^2 + r_s^2}$$

If r_1 is the probable error of a single reading

$$r_r = r_1 \sqrt{2}$$

If the value of an angle is determined by n separate measurements the probable error due to reading will be

$$r_r = \frac{r_1 \sqrt{2}}{\sqrt{n}} .$$

If the value of an angle is determined by measuring the angle n times by repetition the probable error due to reading will be

$$r_r = \frac{r_1 \sqrt{2}}{n}$$

It will thus be seen that the probable error due to reading is very much reduced by measuring an angle by the method of repetition. The errors of pointing, etc., however, make it doubtful whether it is ever advantageous to make n exceed 5 or 6 with an engineers' transit.

Angle Adjustment.—When the three angles of a triangle have been measured with equal care they should be adjusted by applying one-third of the error as a correction to each angle.

When the interior angles of a polygon having n sides have been measured with equal care they should be adjusted by applying *one-nth* of the error as a correction to each angle.

When $n - 1$ angles and their sum angle at a point have been measured with equal care they should be adjusted by applying *one-nth* part of the error as a correction to each angle.

In a quadrilateral the true values of the angles fulfil the following geometrical conditions: (1) the sum of the angles of each triangle is equal to 180° plus the spherical excess

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(the spherical excess in seconds of arc is equal approximately to the area in square miles divided by 78); (2) the computed length of any side when obtained from any other side through two independent sets of triangles is the same in both cases.

When the angles of a quadrilateral have been measured, errors are certain to be present and the corrections that satisfy one of these conditions will not satisfy the other. The most probable values of the corrections to the angles are then determined by the Theory of Least Squares.

TESTS OF PRECISION.

Practical Tests.—In careful surveying where blunders are eliminated and the systematic and accidental errors are small and under control, it is found that the magnitude of the errors increases in close accord with the foregoing rational basis, that is, as the square root of the number of observations. The following practical tests of precision are based on this truth.

Linear Errors.—Cumulative or systematic errors usually increase directly as the length of the line chained, while compensating or accidental errors vary about as the square root of the length. While both kinds of errors affect all linear measurements, the former chiefly control the results of crude and the latter of accurate chaining. It is thus fairly consistent to express the precision of chaining in crude work in terms of the simple ratio of the length; but as the chaining becomes more and more exact, the variation of the differences between duplicate measurements approximates more and more closely to the law of square roots.

Coefficients of precision derived from the latter relation may be based on either 100-foot units or foot units in the distance chained, as preferred. The former basis is used in the chaining diagram while the latter is found in the last paragraph of the explanatory matter on the second page referring to the precision of traverse surveys.

The diagram of chaining errors shows chaining ratios by right lines radiating from the origin, and the law of square roots by means of parabolas. The coefficient of precision for a given observed difference between duplicate chainings is determined by inspection from the diagram, interpolating between curves if an additional decimal place is desired in the result. In actual practice a pair of careful chain-

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men may determine the coefficient corresponding to a given degree of care, and then use this value either in testing their duplicate results, or in estimating the probable uncertainty of the lengths chained.

For accurate chaining with the steel tape, duplicate measurements reduced for temperature, etc., or made under sensibly identical conditions, should not differ more than 0.05 foot into the square root of the distance in 100-foot units. Careful work with the common chain (estimating fractions to 0.1 foot) should not differ more than 0.1 foot into the square root of the distance in 100-foot units.

Angular Errors.—In measuring deflection angles by altitude reversals, as in railroad traversing, there is, of course a cumulative discrepancy due to the collimation error, but generally speaking, careful angular measurements with good instruments are subject only to compensating or accidental errors. Under the latter conditions the magnitude of the error of closure in a series of angles, either in a closed polygon or about a point, varies about as the square root of the number of angles. This relation is indicated graphically in the diagram of angular errors.

In measuring angles with a transit reading to the nearest minute, the compensating uncertainty of a single reading is probably somewhat under 0.5 minute per angle, or about one minute for the closure of a triangle. If a reading glass be used and the vernier reads to the nearest half minute, the uncertainty is still further reduced.

Again, in estimating the needle reading of a compass to the nearest 5 minutes (one-sixth part of a half-degree), the uncertainty of reading alone is perhaps 3 minutes, although this is increased by other conditions such as sluggishness of needle, etc., probably causing an uncertainty of as much as 5 minutes per angle, which later limit would produce an error of closure of a triangle of say 10 minutes, and of a five-sided polygon of perhaps the same amount. (See diagram.)

Traversing Errors.—The errors of traversing are made up of the combined errors of linear and angular measurements. If the error of closure as determined from the latitudes and departures is large, the work should be scanned closely to detect blunders such as the substitution of sine for cosine, errors of 100 feet in chaining, misplacing decimal point, etc. After establishing the consistency of the residual errors, they should be distributed either in proportion to the lengths of the several courses, as in the more

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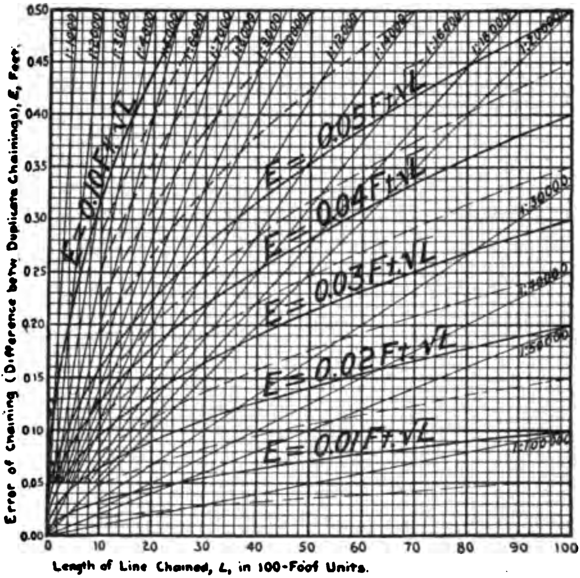
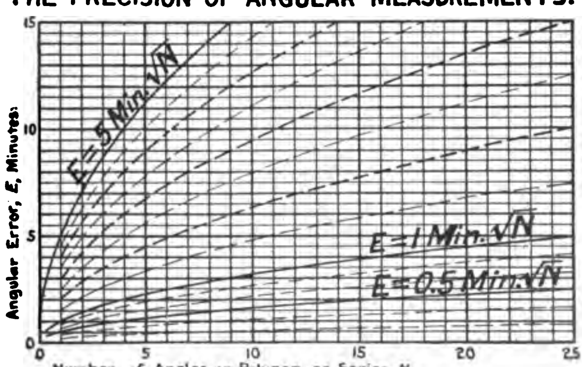
THE PRECISION OF CHAINING.



THE PRECISION OF ANGULAR MEASUREMENTS.



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THE PRECISION OF TRAVERSE SURVEYS.

The error of closure of a traverse is usually expressed as the ratio of the calculated linear error to the length of the perimeter of the field or polygon. The following table shows the limits prescribed by various authorities

Prescribed Limits For Closure Of Traverses.

Authority.	Conditions.	Limits.
Gillespie. (1855). "Surveying," p. 149.	Compass Surveys.	1:300 to 1:1000
Alsop. (1857). "Surveying," p. 199.	Compass Surveys. Transit Surveys.	1:500 1:1000 to 1:1500
Davies. (1870). "Surveying," p. 127.	Farm Surveys.	1:500 to 1:1000
Jordan. (1877). "Handbuch der Vermessungs-kunde," Vol. I, p. 296.	German Gov't Surveys. Baden Instructions. Prussian Instructions. Swiss Gov't Surveys. Ordinary Country. Mountainous Country.	 1:400 1:333 to 1:1000 1:400 to 1:800 1:267 to 1:533
Hodgman. (1885). "Surveying," p. 119.	Compass Surveys.	1:300 to 1:1000
Johnson. (1886). "Surveying," p. 201.	Farm Surveys. City Surveys.	1:300 1:1000 to 1:5000
Baker.* (1888). "Engineers' Surveying Instruments," p. 53.	(See Footnote).	(See Footnote).
Carhart. (1888). "Surveying," p. 161.	Ordinary Farm Surveys. Level Ground. Rough Ground. Average Transit Surveys.	1:500 1:1000 1:200 to 1:300 1:1200
Wood. (Roanoke, Va., 1892). (Baltimore, Md., 1894).	(See Footnote). Precise Traverses with Repeated Angles.	(See Footnote). 1:10 000 1:15 000 + .04 Ft.
Raymond. (1896). "Surveying," p. 144.	Ordinary Farm Surveys. Good Farm Surveys.	1:500 1:2000

*Baker derives the formula $E = P\sqrt{\frac{1}{d^2} + \frac{a^2}{12\,000\,000}}$ where E is the permissible linear error of closure, P the length of the perimeter, $1:d$ the ratio of the chaining error, and a the angular error of closure in minutes. A thorough test of this formula under a wide range of conditions proves it to be trustworthy.

However, the use of a chaining ratio, $1:d$, presumably of fixed value for the same chainmen, does not accord with the results of experience in careful work; for it is found that the differences between duplicate chainings vary about as the square root of the length of line.

On the following page a simplified formula is obtained by assuming the more consistent relation just stated for the chaining errors. The results are about the same as those obtained with Baker's formula, and the form of the expression is identical with that used by Wood in the Baltimore Survey.

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THE PRECISION OF TRAVERSE SURVEYS.

The reasonable or permissible error of closure of a traverse survey may be determined by the formula derived below, provided the errors of field work are under control and their magnitude is known, at least approximately.

Let P = length of perimeter.
L = calculated error of latitudes.
D = calculated error of departures.
E_a = actual or calculated linear error of closure of traverse.
c = coefficient of precision of chaining.
C = linear error of closure due to chaining errors.
a = angular error of closure in minutes.
A = linear error of closure due to angular errors.
E_p = permissible or reasonable linear error of closure due to errors of chaining and angle.

In the triangle of error the hypothenuse is $E_a = \sqrt{L^2+D^2}$. In Diagram A below values of E_a may be read close enough for most cases. Diagram A may also serve as a crude graphical traverse table, and blunders in the field work may be located by it.

In careful chaining by men of some training, the error varies about as the square root of the distance. If c be the compensating error for the unit distance, then $C = c\sqrt{P}$.

The angular error of closure in careful surveys probably occurs among the sides in proportion to their lengths. Assuming this to be the case, the resulting linear error is $A = aP.\text{arc}\,1' = .0003aP$.

In good work the errors are small in amount and equally liable to be plus and minus. Hence, the probable error of closure due to the two causes, i.e. the reasonable or permissible linear error of closure is $E_p = \sqrt{A^2+C^2} = \sqrt{(.0003aP)^2+c^2P}$

This formula may be much simplified by completing the square and dropping the negative term under the radical, whence with sufficient exactness, there results the general formula

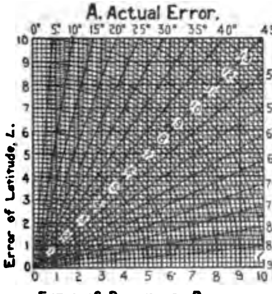
$$E_p = .0003aP + 1700c^2 \; . \; . \; . \; . \; . \; (1)$$

The very exact standard, $P \div 15000 + .04$ ft., used at Baltimore, (see table, preceding page), may be obtained from (1) by making a somewhat less than $\frac{1}{4}$ minute, and $c = .005$ ft., these values being consistent with the field work of that survey.

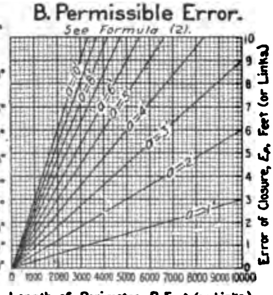
The value of c may be determined for the given chainmen, or the chaining term of (1) may be taken as follows:- For best work ($c < .005$ ft.), .05 ft.; for average work ($c < .010$ ft.), .2 ft.; for fair work ($c < .015$), .4 ft.; and for poor work ($c < .020$), .8 ft. In careful traverse surveys the angle term alone affords a rigid test, so that formula (2) may be used except when $a = 0$. Diagram B gives (2) for the general run of traverse problems.

$$E_p = .0003aP = \frac{3aP}{10000} \; . \; . \; . \; . \; . \; (2)$$

A. Actual Error.



B. Permissible Error.
See Formula (2).



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THE PRECISION OF LEVEL CIRCUITS.
(For Good Average Practice.)

When the length of the level circuit is known in 100-ft. stations, or when merely the number of settings of the instrument and the approximate average distance covered per setting are known, the following modifications of the preceding test are valuable.

Let E = *maximum permissible error of closure of level circuit.*
M = *length of level circuit in miles.*
L = " " " " " " *100-ft. stations.*
L' = *approximate average distance covered per setting of the instrument in 100-ft. stations.*
S = *number of instrumental settings in the circuit.*

For good average work with the engineers' level

$$E = 0.05\text{ ft.}\sqrt{M}$$

from which $E = 0.007\text{ ft.}\sqrt{L}$

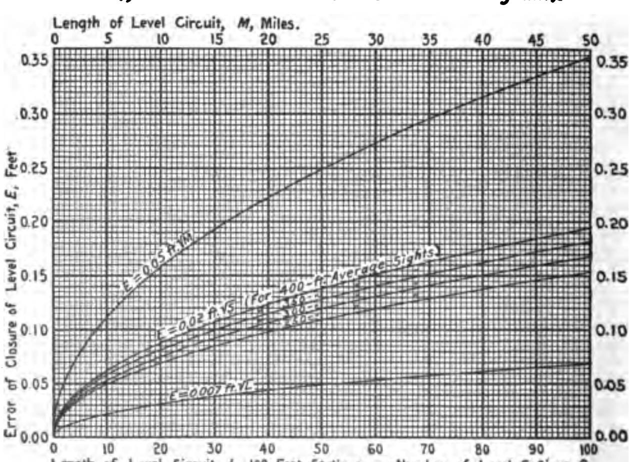
and $E = 0.007\text{ ft.}\sqrt{L'S}$

Substituting for	*400-ft. average sights,*	$L' = 8$,	$E = 0.0195\text{ ft.}\sqrt{S}$
"	*350-" " "*	$L' = 7$,	$E = 0.0182\text{ ft.}\sqrt{S}$
"	*300-" " "*	$L' = 6$,	$E = 0.0169\text{ ft.}\sqrt{S}$
"	*250-" " "*	$L' = 5$,	$E = 0.0154\text{ ft.}\sqrt{S}$

For a very rapid approximate check under ordinary conditions, it may be assumed that $E = 0.02\text{ ft.}\sqrt{S}$. *A graphical representation of these formulas is given below.*

Permissible Error of Closure of Level Circuits For Careful Work with a Good Engineers' Level.

Length of Circuit Given in Miles (Upper Curve); or in the Number of Instrumental Settings (Middle Group of Curves); or in 100-Foot Units (Lower Curve in Diagram).



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THE PRECISION OF LEVEL CIRCUITS.

The precision of spirit leveling is expressed by the formula

$$\text{Error of Closure} = \text{Constant}\sqrt{\text{Length of Circuit}}$$

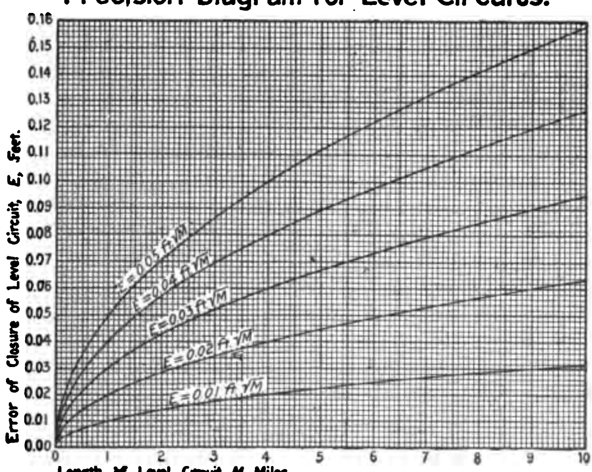
In the following summary of practice in representative surveys of the United States, E is the maximum limit of error of closure of a level circuit having a length of K kilometers or M miles.

Precision of Leveling in Representative Surveys.

NAME OF SURVEY.	MAXIMUM PERMISSIBLE ERROR OF CLOSURE. Metric Units. Coefficient to nearest mm.	British Units. Coefficient to nearest 0.001 ft.	0.01 ft.
Chicago Sanitary District.	$E = 3\text{mm}\sqrt{K}$	$= 0.012\text{ ft.}\sqrt{M}$	$= 0.01\text{ ft.}\sqrt{M}$
Missouri River Commission.	$E = 3\text{mm}\sqrt{2K}$	$= 0.018\text{ ft.}\sqrt{M}$	
Mississippi River Commission. (1891).	$E = 3\text{mm}\sqrt{2K}$	$= 0.018\text{ ft.}\sqrt{M}$	$= 0.02\text{ ft.}\sqrt{M}$
Mississippi River Com'n (Before 1891).	$E = 5\text{mm}\sqrt{K}$	$= 0.021\text{ ft.}\sqrt{M}$	
United States Coast Survey.	$E = 5\text{mm}\sqrt{2K}$	$= 0.029\text{ ft.}\sqrt{M}$	$= 0.03\text{ ft.}\sqrt{M}$
United States Lake Survey.	$E = 10\text{mm}\sqrt{K}$	$= 0.042\text{ ft.}\sqrt{M}$	$= 0.04\text{ ft.}\sqrt{M}$
United States Geological Survey.	$E =$	$0.050\text{ ft.}\sqrt{M}$	$= 0.05\text{ ft.}\sqrt{M}$

A simple practical test of the degree of precision attained in spirit leveling is found in the last column of the above table. This graduated scale of precision is given below graphically for distances to ten miles.

Precision Diagram for Level Circuits.



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common usage, or in the proportion of the respective latitudes and departures, as would seem to be more consistent. If the several courses have not been surveyed with like precision, weights should be assigned in distributing the errors. Absurd refinement should be avoided in making the distribution of errors.

Leveling Errors.—Perhaps in no phase of surveying measurements is it more clearly established that accidental errors follow the law of square roots than in careful leveling. The precision diagrams are based on best current usage.

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CHAPTER X.

METHODS OF COMPUTING.

Introduction.—To no one is the ability to make calculations accurately and rapidly of more value than to the engineer. Many fail to appreciate the value of rapid methods of calculation, and have no conception of the amount of time that can be saved by the skillful use of arithmetic, logarithms, reckoning tables and computing machines.

In the field the engineer has to depend upon the ordinary methods of arithmetic, or a table of logarithms for his results. The use of these aids should therefore receive special attention, for the engineer cannot afford to lose the time of his assistants while he makes unnecessary or extended computations.

In the office tables of squares, reckoning tables, slide rules and computing machines can be used in many cases with profit.

Consistent Accuracy.—It is safe to say that at least one-third of the time expended in making computations is wasted in trying to attain a higher degree of precision than the nature of the work requires.

In making arithmetical computations where decimals are involved it is a common practice to carry the result out to its farthest limit and then drop a few figures at random.

In using logarithms time and labor are lost by using tables that are more extensive than the data will warrant. The relative amount of work in using four, five, six and seven-place tables is about as 1, 2, 3 and 4. Besides the extra labor involved, the computer has a result that is liable to give him an erroneous idea of the accuracy of his work.

In making computations, in general, calculate the result to one more place than it is desired to retain.

If several numbers are multiplied or divided, a given percentage of error in any one of them will produce the same percentage of error in the result.

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In taking the mean of a series of quantities it is consistent to retain one more place than is retained in the quantities themselves.

In direct multiplication or division retain four places of significant figures in every factor for an accuracy of about one per cent; retain five places of significant figures in every factor for an accuracy of about one-tenth of one per cent.

LOGARITHMIC CALCULATIONS.

Logarithm Tables.—Logarithm tables contain the decimal part of the logarithm called the mantissa, the integral part called the characteristic is supplied by the computer.

Four-place tables give the mantissa to four decimal places of numbers from 1 to 999, and by interpolation give the mantissa of numbers from 1 to 9,999. Four-place logarithms should be used where four significant figures are sufficient, and should not be used where an accuracy greater than one-half of one per cent is required.

Five-place tables give the mantissa to five decimal places of numbers from 1 to 9,999, and by interpolation give the mantissa of numbers from 1 to 99,999. Five-place logarithms should be used where five significant figures are sufficient, and should not be used where an accuracy greater than one-twentieth of one per cent is required. Five-place tables are sufficiently accurate for most engineering work.

Six-place tables give the mantissa to six decimal places of numbers from 1 to 9,999, and by interpolation give the mantissa of numbers from 1 to 99,999, the same as the five-place tables. Six-place tables give practically no gain in precision over five-place tables since the same numbers of significant figures are given in both tables, and in addition the labor of using a six- instead of a five-place table is about as 3 to 2, due to interpolation with larger differences. For the above reasons five-place tables have been selected for use in this book as being the most suitable tables for use in surveying.

Seven-place tables give the mantissa to seven decimal places of numbers from 1 to 99,999, and by interpolation of numbers from 1 to 999,999. Seven place tables are rarely needed in engineering work, except in triangulation work where the angles are measured by repetition.

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ARITHMETICAL CALCULATIONS.

Requirements.—To become a rapid computer the following requirements are essential:

(1) A good memory for retaining certain standard numbers for reference.

(2) The power of performing the ordinary simple arithmetical operations of multiplication, division, etc., on numbers with facility, quickness and accuracy.

(3) The power of registration, *i. e.*, of keeping a string of numbers in the mind and working accurately upon them.

(4) The power of devising instantly the best method of performing a complicated problem as regards facility, quickness and certainty.

It is obvious that all do not have the ability to become rapid computers, but even these can become fairly skillful by constant practice and perseverance. The ordinary processes of arithmetic should be performed with numbers in all possible positions. No more figures should be put down than necessary, and all operations should be performed mentally whenever possible. In the mental part the results should alone be stated, much time being lost by repeating each separate figure.

Checks.—In order to check his work the computer should keep the following well known properties of numbers well fixed in his mind:

(1). The sum or difference of two even or of two odd numbers is even.

(2) The sum or difference of an even and odd number is odd.

(3) The product of two even numbers is even.

(4) The product of two odd numbers is odd.

(5) The product of an even number and an odd number is even.

(6) Checking results by the familiar operation of casting out the 9's depends upon the following properties of numbers:

(*a*) A number divided by 9 leaves the same remainder as the sum of the digits divided by 9. For example:

$$4384 \div 9 = 487 + 1$$

$$(4 + 3 + 8 + 4) \div 9 = 2 + 1$$

(*b*) The excess of 9's in the product equals the excess of 9's in the product of the excesses of the factors.

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473,295	Excess = 3			
4,235	Excess = 5			
		15	Excess = 6	Check
2,004,404,325	Excess =		6	

(*c*) The excess of 9's in the dividend equals the excess of 9's in the product of the excesses in the divisor and quotient, plus the excess in the remainder:

56)2443	Excess in divisor	= 2	
43 + 35	Excess in quotient	= 7	
	Excess in remainder	= 8	
	Excess in $(2 \times 7 + 8)$	= 4	Check
	Excess in dividend	= 4	

(7) Results should be checked by taking aliquot parts wherever possible, and by performing the operations in inverse order or performing inverse operations. Computations performed by means of logarithms should be checked by making the computations roughly by means of arithmetic. *The probability of error should be recognized and precaution taken to verify results.*

ADDITION.—Since the eye is accustomed to pass from left to right time can be saved, where the columns are not too long, by adding in the same way. The device of increasing or diminishing the numbers to make them multiples of ten and then subtracting or adding to the result is very convenient, especially where several columns are added at one time.

Ex. 1.—

	96	
	47	143
212	69	
	32	
	87	331
	49	
	380	

The mental work in detail is as follows:
$100 + 47 = 147$; $147 - 4 = 143$; $143 + 70 = 213$; $213 - 1 = 212$; $212 + 30 + 90 = 332$; $332 - 1 = 331$; $331 + 50 = 381$; $381 - 1 = 380$.

Expert accountants use the method of adding columns in groups of 10, 20, 30, etc., small figures, indicating the number of the group, being placed along the column at intervals depending upon the computer. This method is well

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adapted to the addition of long columns where one is liable to be called away from his work. The progress of the work being then shown by the number of the group, plus the excess.

MULTIPLICATION.—In order to make the best use of the methods given, the computer should have perfect command of the multiplication table as far as 20 at least.

(1) When the tens differ by unity and the sum of the units equals 10, numbers may be multiplied by the following rule: From the squares of the tens of the larger number subtract the square of the units of the larger number. For the numbers may be represented by $(a+b)$ and $(a-b)$, and the product will be $(a+b)(a-b)=a^2-b^2$.

Ex. 1.—$(93 \times 87) = 90^2 - 3^2 = 8{,}100 - 9 = 8{,}091$.

(2) The product of composite nnmbers is best obtained mentally by resolving them into their factors and taking the products of the factors.

Ex. 2.— $26 \times 36 = 9 \times 13 \times 8 = 936$.

Ex. 3.— $48 \times 24 = (24)^2 \times 2 = 1{,}152$.

Multiples of 10.—To multiply by some number which is a factor of 10 or some multiple of 10, for example: Multiply

A by B, where $B = \dfrac{C10^n}{d}$

Annex n ciphers to A, multiply by C and divide by d.

Ex. 1.—$4{,}324 \times 625 = 4{,}324 \ \dfrac{5 \times 10^3}{8} = (4{,}324{,}000 \times 5) \div 8 = 2{,}702{,}500$.

Ex. 2.—$7{,}924 \times 25 = 792{,}400 \div 4 = 198{,}100$.

Squaring Small Numbers.—Numbers may be squared mentally by the following rule: Add to or subtract from one factor enough to make its units figure zero. Subtract from or add to the other factor the same amount. Multiply together this sum and difference, and to the product add the square of the amount by which the factors were increased or diminished.

Proof.—
$$a^2 - b^2 = (a+b)(a-b)$$
$$a^2 = (a+b)(a-b) + b^2.$$

Ex. 1.— $(76)^2 = (72 \times 80) + 4^2 = 5{,}776$.

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Ex. 2.— $(127)^2 = (124 \times 130) + 3^2 = 16{,}129.$

Ex. 3.— $(6\tfrac{1}{4})^2 = (6 \times 6\tfrac{1}{2}) + (\tfrac{1}{4})^2 = 39\tfrac{1}{16}.$

Ex. 4.— $(6\tfrac{1}{2})^2 = (6 \times 7) + (\tfrac{1}{2})^2 = 42\tfrac{1}{4}.$

Ex. 5.— $(7.5)^2 = (7 \times 8) + (.5)^2 = 56.25.$

It will be seen that the process is very simple where the units place is 5.

(2) Having the square of any number the square of the number next higher is obtained by the following rule: To the known square add the number and the next higher and the result will be the square of the next higher number.

Ex. 6.— $(25)^2 = 625.$ $(26)^2 = 625 + 25 + 26 = 676.$

(3) A very close approximation to the square of a quantity which is very near unity is obtained by adding algebraically two times the difference between the quantity and unity to the quantity.

Proof.—$(1 \pm b)^2 = 1 \pm 2b + b^2 = 1 \pm 2b$, (approximate).

Ex. 7.—$(1.05)^2 = 1 + 2(1.05 - 1) = 1 + 10 = 1.10.$

Ex. 8.—$(.94)^2 = 1 - 2(1 - .94) = 1 - .12 = 88.$

Ex. 9.—$(2.034)^2 = 2^2(1 + 2 \times .017) = 4(1.034) = 4.136.$

Cross-Multiplication.—This consists in taking the product of each digit in the multiplicand by each digit in the multiplier and taking the sums, products of the same denomination being determined thus: units $\times$ units gives units; tens $\times$ units and units $\times$ tens gives tens; units $\times$ hundreds, tens $\times$ tens and hundreds $\times$ units give hundreds etc. All products are added mentally, only the final result being put down.

Ex. 1.—$(2{,}347)^2 = 5{,}508{,}409$ the final result being all that it is necessary to write down. The mental work is as follows, the figures in heavy type being figures in the product; $7 \times 7 = 4\mathbf{9}$; $4 + 2(7 \times 4) = 6\mathbf{0}$; $6 + 2(7 \times 3) + 4^2 = 6\mathbf{4}$; $6 + 2(2 \times 7) + 2(3 \times 4) = 5\mathbf{8}$; $5 + 2(2 \times 4) + 3^2 = 3\mathbf{0}$; $3 + 2(3 \times 2) = 1\mathbf{5}$; $1 + 2^2 = \mathbf{5}$.

Ex. 2.—The product of any two numbers may be found in the same manner.

$$\begin{array}{r} 9{,}432 \\ 2{,}583 \\ \hline 24{,}362{,}856 \end{array}$$

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The mental work is as follows: $3 \times 2 = 6$; $3 \times 3 + 8 \times 2 = 25$; $2 + 3 \times 4 + 8 \times 3 + 5 \times 2 = 48$; $4 + 3 \times 9 + 8 \times 4 + 5 \times 3 + 2 \times 2 = 82$; $8 + 8 \times 9 + 5 \times 4 + 2 \times 3 = 106$; $10 + 5 \times 9 + 2 \times 4 = 63$; $6 + 2 \times 9 = 24$.

Ex. 3.—The process of cross-multiplication may be simplified as follows: Required to multiply 4,328 by 736; write the multiplier on a slip of paper in inverse order and place it below the multiplicand with the left hand figure below the units place of the multiplicand thus:

$$\begin{array}{r} 4{,}328 \\ \boxed{637} \end{array}$$

Multiply together the figures in the same vertical column $6 \times 8 = 48$; set down the 8 and carry the 4; then move the slip one space to the left thus:

$$\begin{array}{r} 4{,}328 \\ \boxed{637} \\ 8 \end{array}$$

Multiplying together the figures in the same vertical columns and taking the sum, $4 + 6 \times 2 + 3 \times 8 = 40$; set down the 0 and carry the 4; then move the slip one space to the left, multiplying together the figures in the same vertical columns, adding, etc., we will finally have the work standing thus:

$$\begin{array}{r} 4{,}328 \\ \boxed{637} \\ 3{,}185{,}408 \end{array}$$

Removing the slip we have

$$\begin{array}{r} 4{,}328 \\ 736 \\ \hline 3{,}185{,}408 \end{array}$$

The multiplier may be written on the bottom of a sheet in inverse order and placed above the multiplicand instead as above described. The work, however, is very much simplified by simply writing the multiplier in inverse order without using the slip:

$$\begin{array}{r} 4{,}328 \\ 637 \\ \hline 3{,}185{,}408 \end{array}$$

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The mental work being as follows: $6 \times 8 = 48$; $4 + 6 \times 2 + 3 \times 8 = 40$; $4 + 6 \times 3 + 2 \times 3 + 7 \times 8 = 84$; $8 + 6 \times 4 + 3 \times 3 + 7 \times 2 = 55$; $5 + 3 \times 4 + 7 \times 3 = 38$; $3 + 7 \times 4 = 31$. It will be seen that this device removes most of the mental strain, there being no cross-products.

CONTRACTED MULTIPLICATION.—In multiplying decimals, when the product is required to a few places of decimals, the work may be shortened as follows: Required a product correct to the *nth* decimal place. Write the multiplier with its figures in reverse order, its units place under the *nth* decimal place of the multiplicand. Multiply the multiplicand by the figures in the multiplier, beginning with the right hand figure; rejecting those figures in the multiplicand which are to the right of the figure used as a multiplier, increasing each product by as many units as would have been carried from the rejected part of the multiplicand, taking the nearest unit in each case; place the right hand figure of each partial product in the same column, and add as in common multiplication.

In most cases it is best to compute one more place than required. The following examples illustrate the process:

Ex. 1.—The radius of a circle is 420.17 ft. What is its semicircumference to nearest 0.01 ft.? ($\pi = 3.14159265$.)

In the work below the partial products in the contracted multiplication are seen to correspond to the partials of the common method, taken in reverse order, the part to the right of the vertical line being rejected. The contracted multiplication is carried one more place than required. A dot is placed above each figure when it is rejected from the multiplicand.

```
   ......
   420.170                            420.17
 56295141.3                         3.141593
 ----------                       ---------------
   1260510                           1|26051
     42017                          37|8153
     16807                         210|085
       420                         420|17
       210                       16806|8
        38                       42017|
         1                      126051|
 ----------                     ---------------
  1320.008                      1320.008|13081
```

Ex. 2.—The observed length of a line is 2231.63 ft. with a tape having a length of 100.018 ft. Required the reduced length of the line to the nearest 0.01 ft.

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Noting that each foot of the tape = 1.00018 ft.

```
 ......                       
2231.63               •2231.63
81000.1                1.00018
-------              ---------------
223163               17|85304
    22               22|3163
    18           223163|000
-------          -----------------
2232.03          2232.03|16934
```

Ex. 3.—Same observed length with a tape 99.982 ft. long. Required the reduced length.

Each foot of the tape = 0.99982 = (1 — 0.00018) ft.

```
 ......
2231.63                2231.63
81000.0—               0.99982
--------               -------
     22                 446326
     18                1785304
  —0.40               2008467
-------              2008467
2231.23             2008467
                    ------------
                    2231.2283066
```

Ex. 4.—To compare contracted multiplication with logarithmic work, calculate 861.3 ft. × sin 17° 19′ to the nearest 0.1 ft.

```
   ....
  861.3        log. 861.3       = 2.93515
56792.0        log. sin 17° 19′ = 9.47371
-------                           -------
   1723        log. (256.4)     = 2.40886
    776
     60
      5
-------
  256.4
```

CONTRACTED DIVISION.—If the quotient is desired correct to the *nth* decimal place, the following method may be used: Find one-half of the desired figures in the quotient in the usual way and do not bring down a figure for the last remainder. Drop a figure from the right of the divisor and find another figure in the quotient. Then without bringing down any more figures continue to discard figures from the divisor until the required places are obtained.

Ex 1.—Divide 443.9425 by 24.311 to nearest hundredth. There will be four figures in the quotient, so we will find

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the first two in the ordinary way. A dot is placed over each figure in the divisor when it is rejected.

```
2432)4438.9425(1825
     2432
     20074
     19456
       618
       486
       132
       122
        10
```

Divisor Near Unity.—When the divisor is near unity a very close approximation is given by the method shown in the following problems:

Ex. 1.—$\frac{5}{1.003254} = 5(1 - .003254) = 5 \times .996746 = 4.98373$

correct to within one unit in the fifth place.

Ex. 2.—$\frac{7}{.9982} = 7(1 + (1 - .9982)) = 7 \times 1.0018 = 7.0126$

correct to the last place.

CONTRACTED SQUARE ROOT.—A result correct to a required number of decimal places may be found by a process similar to the method employed for contracted division.

Ex. 1.—Required the square root of 12,598.87325 correct to thousandths. We see by inspection that the root will contain six figures. Find in the ordinary way the first three figures. Form a new trial divisor in the usual way,

```
     12598.87325(112.245
     1
21)25
   21
222)498
    444
  224)548
      448
      100
       89
       11
       11
        0
```

and bring down only one figure for the dividend in place of two. Find the remaining figures by contracted division.

The last figure brought down is not increased whatever it may be followed by, since the contracted process tends to make the result a little too large. This method may be applied to the extraction of cube roots, where it saves much work in finding long trial divisors.

Square Root of Small Numbers.—The approximate square roots of small numbers may be found by means of the following rule: Divide the given number by the number whose square is nearest the given number. The arithmetical mean of the quotient and divisor will be the approximate square root of the number. The nearer the number is to a perfect square the less the error. For example,

Ex. 1.—$\sqrt{35} = (\frac{35}{6} + 6) \div 2 = 5.92.$

Ex. 2.—$\sqrt{8} = (\frac{8}{3} + 3) \div 2 = 2.83.$

Ex 3.—$\sqrt{79} = (\frac{79}{9} + 9) \div 2 = 8.89.$

Ex. 4.—$\sqrt{128} = (\frac{128}{11} + 11) \div 2 = 11.31.$

Square Root by Subtraction.—While it possesses no points of merit in this connection, it would not be proper to pass the subject of square root without presenting the novel method of extracting square roots used with the Thomas Computing machine. The method depends upon the relation existing between odd numbers and squares in the system of numbers having a radix ten. If we sum up the odd numbers, beginning at 1, we will observe the following relation:

$1 = 1^2$; $1 + 3 = 4 = 2^2$; $1 + 3 + 5 = 9 = 3^2$; $1 + 3 + 5 + 7 = 16 = 4^2$, etc. It will be seen that the square root of the sum in each case is the number of the group.

The method of extracting square roots is as follows: Point off in periods of two figures each. Subtract from the left hand period the odd numbers in order, beginning at unity, until a remainder is obtained less than the next odd number. Write for the first figure in the root the number which represents the number of subtractions made. Double the root already found and annex unity. Subtract as before, using for subtrahends the successive odd numbers, the root figure being the number of subtractions made.

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Ex. 1.—Extract the square root of 53,824.

```
         .  .  .
         53824(232
         1
         -
         4
         3.......2 subtractions.
        ----
   41)138
       41
      ----
       97
       43
      ----
       54
       45.....3 subtractions.
        ----
   461)924
        461
        ---
        463
        463...2 subtractions.
        ---
          0
```

RECKONING TABLES.—Tables for use in computing are so numerous and well known that it would be useless to try to refer to them by name. Two valuable tables for obtaining products of numbers—which are well known in Germany, but comparatively unknown in this country—are, "Crelle's Rechentafeln," which gives the products of numbers of three significant figures by three significant figures to 999 by 999; and "Zimmerman's Rechentafeln," which gives the products of numbers of two places of significant figures by numbers of three significant figures to 100 by 999.

COMPUTING MACHINES.—In Fig. 40, (a) is a Kuttner reckoning machine; (b) a Thomas computing machine; (c) a Fuller slide rule; (d) a Thacher slide rule; (e) an ordinary slide rule; (f) a Colby Stadia slide rule; (g) a Colby sewer slide rule; (h) a Grant calculating machine; (i) a full circle protractor; (j) a Crozet protractor; (k) a protractor tee square; (l) a polar planimeter; (m) a "jack knife" planimeter; (n) a pantagraph; (o) a section liner; (p) a spherical planimeter.

In using the "jack knife" planimeter, the point is placed at the center of gravity, and the knife edge is placed on a line passing through the center of gravity of the figure. The point is then made to traverse the perimeter of the figure to be measured; passing out to the perimeter and returning to the center of gravity of the figure on the same line. The distance from the final position of the knife edge to the line through the center of gravity, multiplied by the

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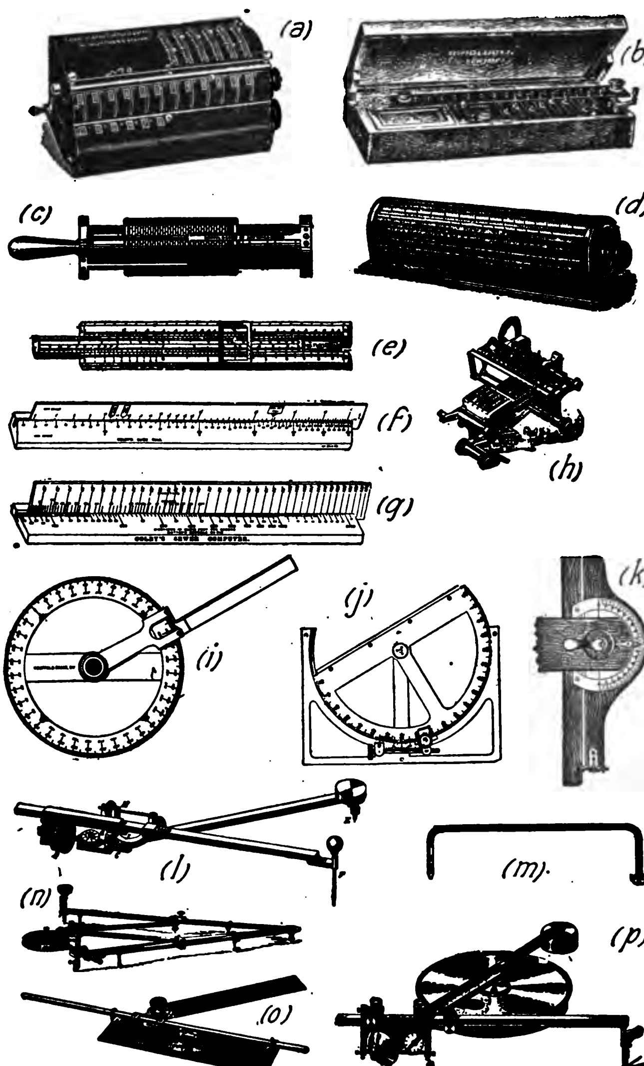
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
(o)
(p)

length of the arm of the planimeter will give the area of the figure. The arm of the planimeter is usually made ten inches long and the distance measured in inches.

The correct area may be obtained by means of the hatchet planimeter, without using the center of gravity of the figure, as follows: (1) Draw a tangent to the figure. (2) Trace the figure with the point starting with the hatchet on the tangent and the point at the point of tangency. (3) Trace the figure as before except that the point is to move around in the opposite direction. (4) The arithmetical mean of the two areas will be the true area. That this method is correct can be easily proved by the student.

The other machines are described in the instructions accompanying them when purchased.

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CHAPTER XI.

TOPOGRAPHIC DRAWING AND LETTERING.

LETTERING.—A magnified scale is used in the first six plates to give familiarity with form of letter and numeral, and also to produce freedom of hand motion. The six plates should first be made with a soft pencil sharpened to a needle point, and afterward with pen and india ink. In Plate 7 the height of letter is that prescribed in Chapter I. This standard size is not only well adapted to field notes and general drafting, but is economical of execution.

The student should train the eye and acquire a "swing" of the hand by industrious practice in such exercises as the following: (1) Pass a line freehand through two points; first sketch in the line roughly by a free swing of the forearm; then partially erase and retrace; finally test result with ruler. (2) Pass a circular arc through three points freehand; follow sketch method just described and, after perfecting the arc, sketch in the chords and locate the center freehand; test result mechanically. (3) Inscribe a circle in a square. (4) Inscribe an ellipse in a rectangle. (5) Inscribe an ellipse in an oblique parallelogram. (In the last three exercises give particular attention to points and lines of tangency and axes of symmetry.) After making the line or figure satisfactorily with pencil, it should be executed freehand in India ink.

Practice should include spacing of letters and words, and for this purpose it is suggested that the student use the "specifications for a good engineer" following the preface.

The student should not be content until he can make letters freehand so well that a close inspection is required to determine that they were not made mechanically.

Freehand Titles.—Good freehand titles suffice for most drawings. In a good title consistent emphasis is given to the several parts, and the title as a whole accords with the purpose and character of the drawing. Elaborate and ornamental titles have a limited application, and should not be attempted at all unless the draftsman has special skill

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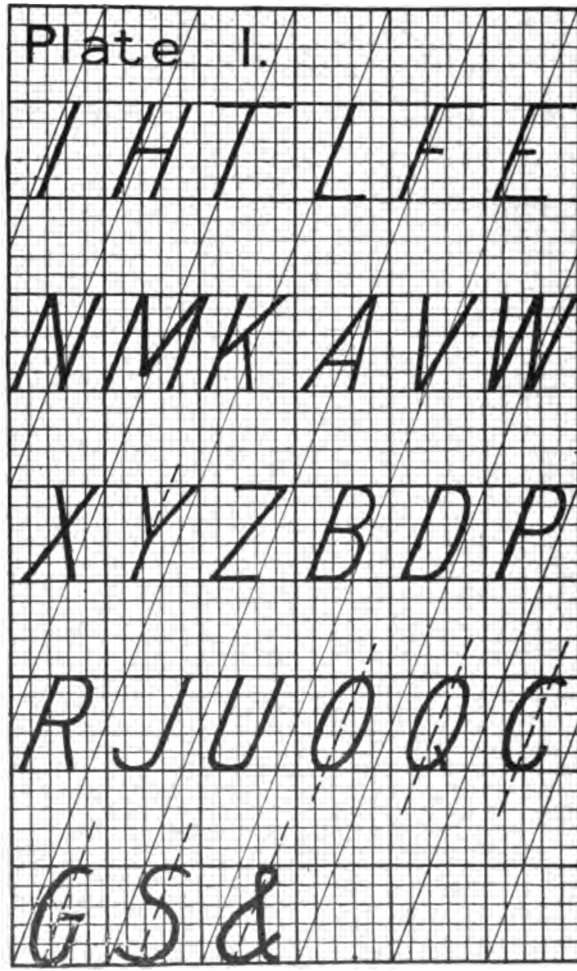

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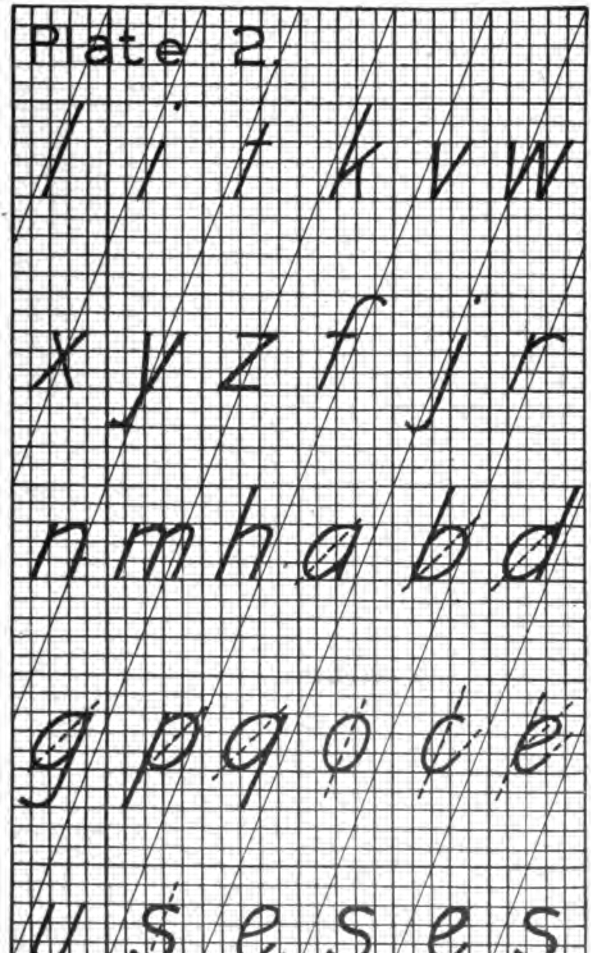

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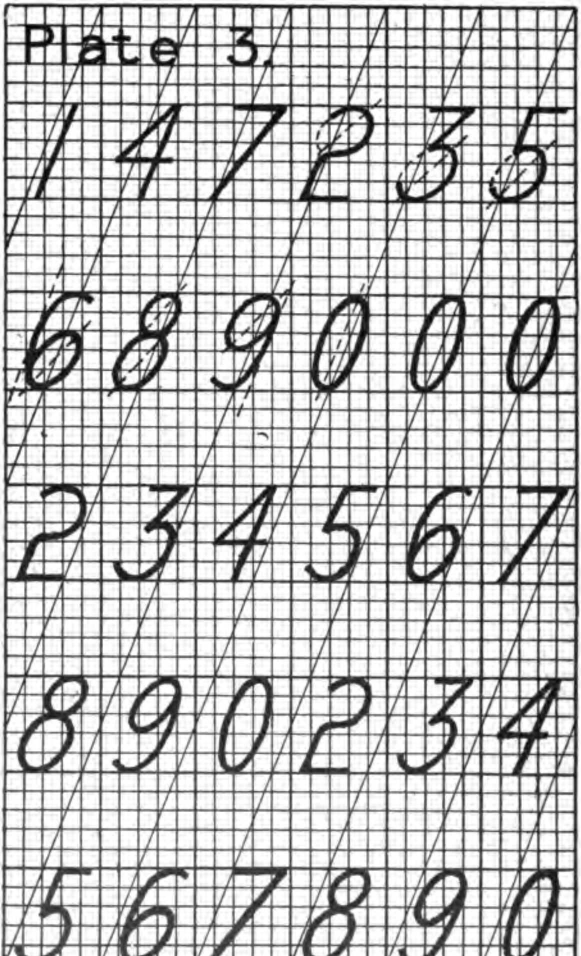

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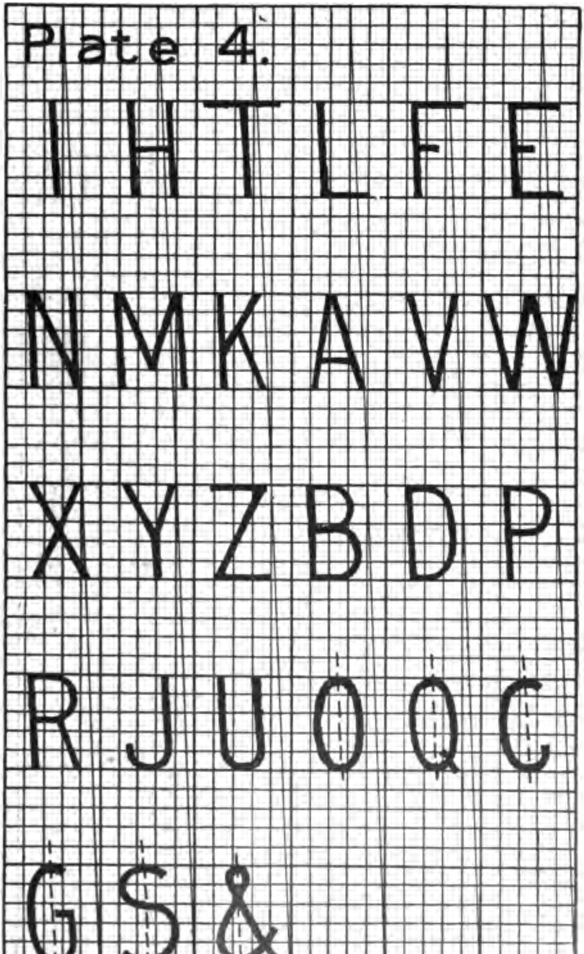

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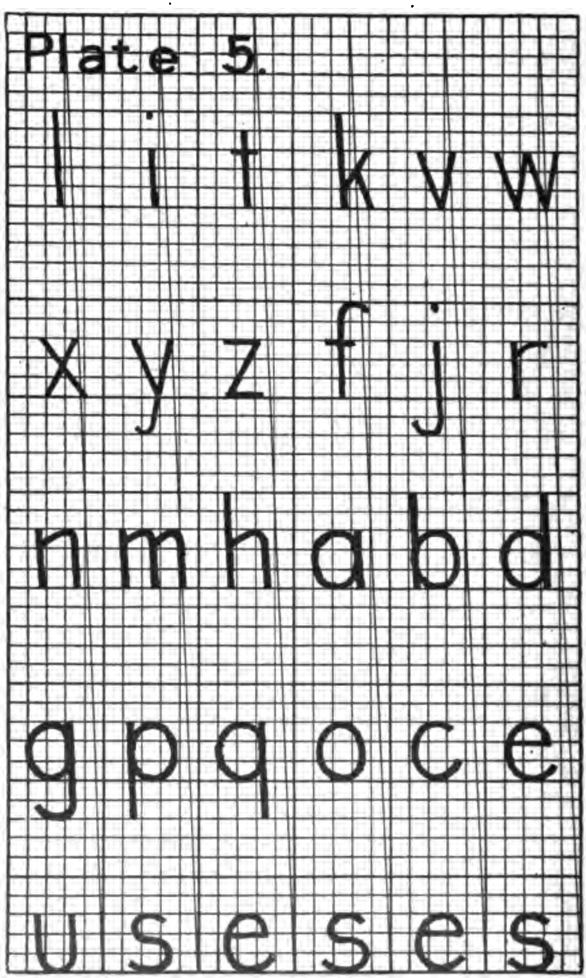

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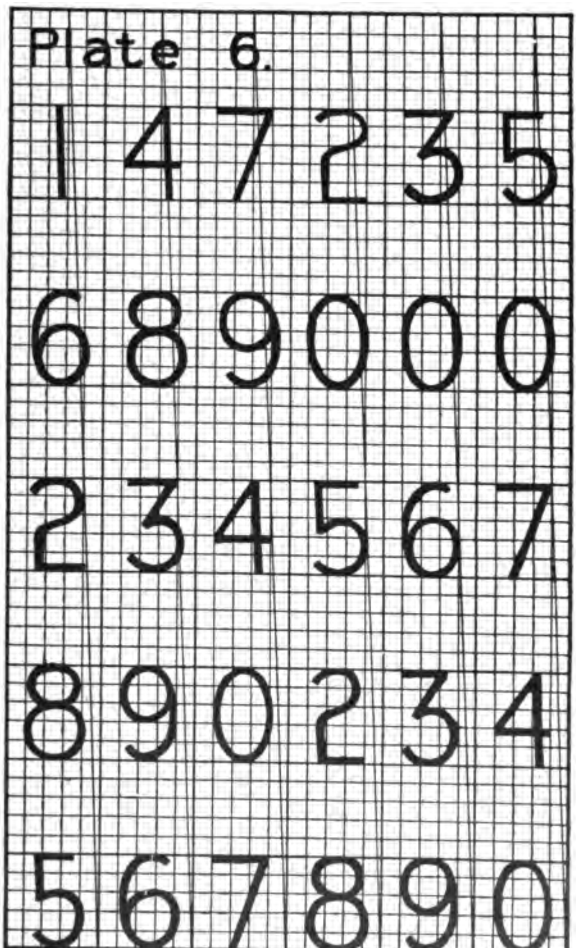

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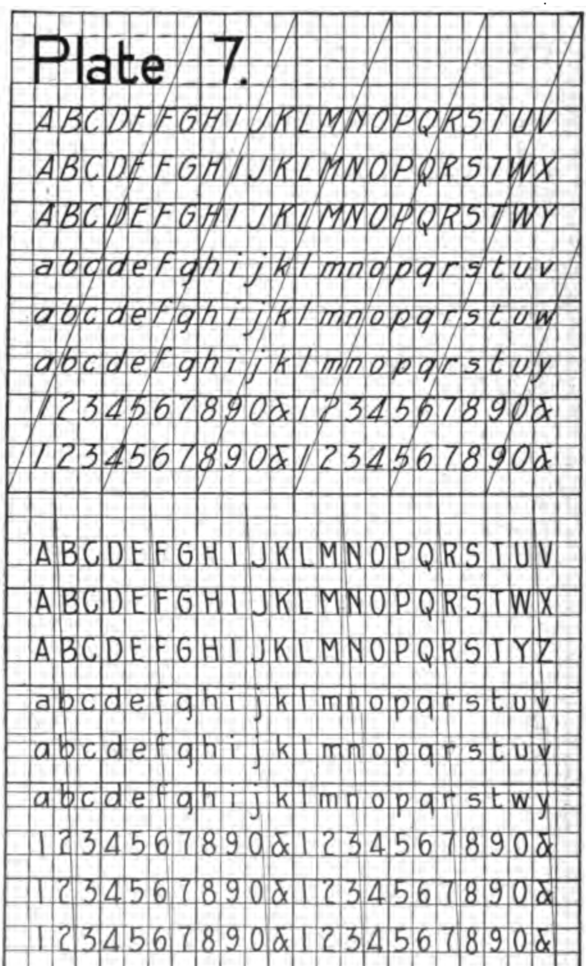

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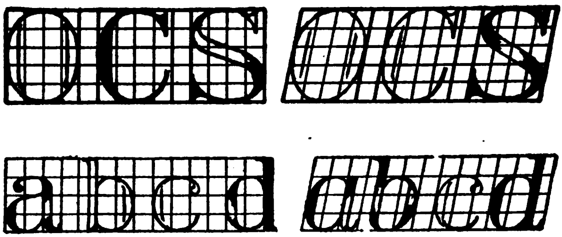
in such work. In designing titles, whether freehand or mechanical, skill in sketching in the outlines, guide lines, axes of symmetry, etc., is of much importance. On the following pages are a few examples of good titles.



DRAWING PENS.—The following pens, arranged in order of fineness, will give sufficient variety for ordinary work.

Gillott's 170, very fine, for very small lettering.

Gillott's 303, extra fine, for small lettering.

Gillott's 404, fine, for small lettering.

Hunt 21, medium, for ordinary lettering.

Hunt 512, Shot Point, for ordinary lettering.

Leonardt 510, E. F. Ball Point, for large lettering and titles.

Hunt 513, Round Point, for large lettering and titles.

Leonardt 516, E. F. Ball Point, for large lettering and titles.

Leonardt 516 F., Ball Point, for very large lettering and titles.

Payzant Pens, K. & E. Co., Nos. 6, 5, 4, 3, 2, 1, for titles.

The following rules should be observed in making letters on drawings free hand.

Use the quill in inking the pen.

Never dip the pen in the ink bottle.

Keep the pen clean.

Ink must not be allowed to dry on the pen and spread the points.

Before using a new pen moisten the points and wipe it dry to insure a free flow of ink.

TOPOGRAPHIC SYMBOLS.—The standard symbols for topographic drawings adopted by the American Railway Engineering Association are given on pages 248 to 251.

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RIGHT-OF-WAY MAP
NEW YORK AND DENVER R.R.
Station 331+55 to Station 542+75
Scale 1 in.=400 ft. January 3, 1915
Office of Chief Engineer
Denver, Colorado.

RIGHT-OF-WAY MAP
NEW YORK AND DENVER R.R.
Station 331+55 to Station 511+10
Scale 1 in =400 ft. January 30, 1915
Office of Chief Engineer
Denver, Colorado.

TOPOGRAPHIC MAP
OF THE
CITY OF BOULDER, COLORADO
Surveyed by the
CLASS IN TOPOGRAPHIC SURVEYING
UNIVERSITY OF COLORADO
FIRST SEMESTER 1914-15
Scale 1 in = 500 ft.

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RIGHT-OF-WAY MAP
NEW YORK AND DENVER R.R.
Station 331+55 to Station 542+75
Scale 1 in.=400 ft. January 3, 1915
Office of Chief Engineer
Denver, Colorado.

RIGHT-OF-WAY MAP
NEW YORK & DENVER R.R.
Station 331+55 to Station 511+10
Scale 1 in.=400 ft. January 1, 1915
Office of Chief Engineer
Denver, Colorado.

Topographic Map
of the
CITY OF BOULDER, COLORADO
Surveyed by the
Class in Topographic Surveying
University of Colorado
First Semester 1914-15
Scale 1 in.=500 ft.

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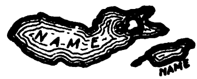
HYDROGRAPHY.

Stream

Springs and Sinks

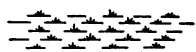
Lakes and Ponds



Falls and Rapids

Water Line

Marsh

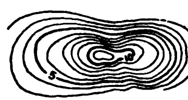


Canals

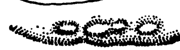
Ditches

RELIEF.

Contour System



Sand



Cliffs

Cut

Embankment

Top of Slope

Bottom of Slope

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* RAILWAYS (TOPOGRAPHICAL MAPS.)

Steam	
Electric	
Street Railways.	

* RAILWAY TRACKS (TRACK MAPS.)

Railway Track or Old Track to Remain	
Old Track to be Taken up	
Proposed Tracks	Red
Proposed (Future) Tracks	Red
Foreign Tracks	Color other than Red or Black with Initials of Road
Alinement { 4° Curve to Right / 2° " " Left }	4°C.R. 2°C.L

BOUNDARY AND SURVEY LINES.

{ Political Divisions; State, County or Township Lines.	Bethel Twp.-Wayne Co., Mich. / Posey Twp.-Adams Co., Ind.
{ Government Surveys, Base, Meridian, Township, Section or Harbor Line	Sec. 18. T. 12 N., R. 1 E., 3rd P.M. / Sec. 13. T. 11 N., R. 1 E., 3rd P.M.
Street, Block or other Property Line	
Survey Lines	Red Preliminary Location
Center Lines	Original (Track or Section) Center Line 19 If Monumented, Show Location and Proper Symbol
Company Property Line	
Fence (on Street Line)	State Kind and Height
Fence (on Company Property Line)	State Kind and Height

* For Railway Track and Yard Studies Use Single or Double Lines.

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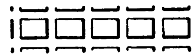
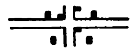
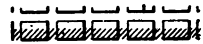
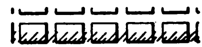
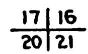
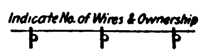
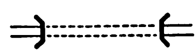
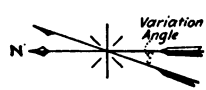
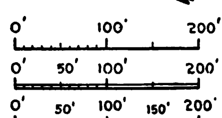
City	
Village	
City Limits	
Fire Limits	
Section Corner	
Section Center	
Triangulation Station or Transit Point	
Bench Mark	B.M.✕1232
Stone Monument	
Iron Monument	

MISCELLANEOUS.

Pole Wire Lines	
Railway Tunnel	
Dimension Lines	
True and Magnetic Meridian	
Graphic Scales	

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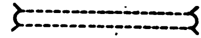
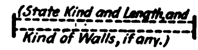
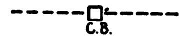
CULVERTS, SEWERS, ETC.

Masonry Arch or Flat Top Culvert


Pipe or Wood Box Culvert or Drain


Catch Basin


Manhole

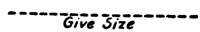

Sump

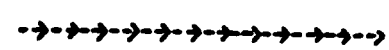

WATER SUPPLY AND PIPE LINES.

Water Tank


Water Column


Track Pan

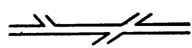

Company Water Pipe


Other Water Pipe


Steam or Gas

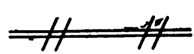

Compressed Air

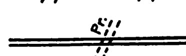

HIGHWAYS AND CROSSINGS.

Public and Main Roads


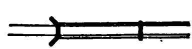
Private and Secondary Roads


Trails


Street and Public Road Crossings


Private Road Crossing


BRIDGES.

Girder


Truss


Trestle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SURVEYING MANUAL

PART II

FIELD AND OFFICE TABLES FOR USE IN SURVEYING.

BY

WILLIAM D. PENCE

AND

MILO S. KETCHUM

Table 1. Logarithms of Numbers.

Table 2. Logarithmic Functions of Angles.

Table 3. Natural Functions of Angles.

Table 4. Squares, Cubes, Square Roots, Cube Roots and Circles.

Table 5. Trigonometric Functions.

Explanation of Tables.

The authors wish to thank the J. B. Lippincott Company for the use of Tables 1 and 2 taken from Suplee's "Five Place Logarithms," and Table 3 taken from Suplee's "Mechanical Engineers' Reference Book"; and the McGraw-Hill Book Company for the use of Tables 4 and 5, taken from Harger and Bonney's "Highway Engineers' Handbook."

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Num. 100 to 139. Log. 000 to 145.

N	L	0	1	2	3	4	5	6	7	8	9
100	00	000	043	087	130	173	217	260	303	346	389
101		432	475	518	561	604	647	689	732	775	817
102		860	903	945	988	*030	*072	*115	*157	*199	*242
103	01	284	326	368	410	452	494	536	578	620	662
104		703	745	787	828	870	912	953	995	*036	*078
105	02	119	160	202	243	284	325	366	407	449	490
106		531	572	612	653	694	735	776	816	857	898
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03	342	383	423	463	503	543	583	623	663	703
109		743	782	822	862	902	941	981	*021	*060	*100
110	04	139	179	218	258	297	336	376	415	454	493
111		532	571	610	650	689	727	766	805	844	883
112		922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05	308	346	385	423	461	500	538	576	614	652
114		690	729	767	805	843	881	918	956	994	*032
115	06	070	108	145	183	221	258	296	333	371	408
116		446	483	521	558	595	633	670	707	744	781
117		819	856	893	930	967	*004	*041	*078	*115	*151
118	07	188	225	262	298	335	372	408	445	482	518
119		555	591	628	664	700	737	773	809	846	882
120		918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08	279	314	350	386	422	458	493	529	565	600
122		636	672	707	743	778	814	849	884	920	955
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09	342	377	412	447	482	517	552	587	621	656
125		691	726	760	795	830	864	899	934	968	*003
126	10	037	072	106	140	175	209	243	278	312	346
127		380	415	449	483	517	551	585	619	653	687
128		721	755	789	823	857	890	924	958	992	*025
129	11	059	093	126	160	193	227	261	294	327	361
130		394	428	461	494	528	561	594	628	661	694
131		727	760	793	826	860	893	926	959	992	*024
132	12	057	090	123	156	189	222	254	287	320	352
133		385	418	450	483	516	548	581	613	646	678
134		710	743	775	808	840	872	905	937	969	*001
135	13	033	066	098	130	162	194	226	258	290	322
136		354	386	418	450	481	513	545	577	609	640
137		672	704	735	767	799	830	862	893	925	956
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14	301	333	364	395	426	457	489	520	551	582
140		613	644	675	706	737	768	799	829	860	891
N	**L**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	44	43
1	4.4	4.3
2	8.8	8.6
3	13.2	12.9
4	17.6	17.2
5	22.0	21.5
6	26.4	25.8
7	30.8	30.1
8	35.2	34.4
9	39.6	38.7

	42	41
1	4.2	4.1
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	40	39
1	4.0	3.9
2	8.0	7.8
3	12.0	11.7
4	16.0	15.6
5	20.0	19.5
6	24.0	23.4
7	28.0	27.3
8	32.0	31.2
9	36.0	35.1

	38	37
1	3.8	3.7
2	7.6	7.4
3	11.4	11.1
4	15.2	14.8
5	19.0	18.5
6	22.8	22.2
7	26.6	25.9
8	30.4	29.6
9	34.2	33.3

	36	35
1	3.6	3.5
2	7.2	7.0
3	10.8	10.5
4	14.4	14.0
5	18.0	17.5
6	21.6	21.0
7	25.2	24.5
8	28.8	28.0
9	32.4	31.5

P. P.

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Num. 140 to 179. Log. 146 to 255.

N	L	0	1	2	3	4	5	6	7	8	9
140	14	613	644	675	706	737	768	799	829	860	891
141		922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15	229	259	290	320	351	381	412	442	473	503
143		534	564	594	625	655	685	715	746	776	806
144		836	866	897	927	957	987	*017	*047	*077	*107
145	16	137	167	197	227	256	286	316	346	376	406
146		435	465	495	524	554	584	613	643	673	702
147		732	761	791	820	850	879	909	938	967	997
148	17	026	056	085	114	143	173	202	231	260	289
149		319	348	377	406	435	464	493	522	551	580
150		609	638	667	696	725	754	782	811	840	869
151		898	926	955	984	*013	*041	*070	*099	*127	*156
152	18	184	213	241	270	298	327	355	384	412	441
153		469	498	526	554	583	611	639	667	696	724
154		752	780	808	837	865	893	921	949	977	*005
155	19	033	061	089	117	145	173	201	229	257	285
156		312	340	368	396	424	451	479	507	535	562
157		590	618	645	673	700	728	756	783	811	838
158		866	893	921	948	976	*003	*030	*058	*085	*112
159	20	140	167	194	222	249	276	303	330	358	385
160		412	439	466	493	520	548	575	602	629	656
161		683	710	737	763	790	817	844	871	898	925
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21	219	245	272	299	325	352	378	405	431	458
164		484	511	537	564	590	617	643	669	696	722
165		748	775	801	827	854	880	906	932	958	985
166	22	011	037	063	089	115	141	167	194	220	246
167		272	298	324	350	376	401	427	453	479	505
168		531	557	583	608	634	660	686	712	737	763
169		789	814	840	866	891	917	943	968	994	*019
170	23	045	070	096	121	147	172	198	223	249	274
171		300	325	350	376	401	426	452	477	502	528
172		553	578	603	629	654	679	704	729	754	779
173		805	830	855	880	905	930	955	980	*005	*030
174	24	055	080	105	130	155	180	204	229	254	279
175		304	329	353	378	403	428	452	477	502	527
176		551	576	601	625	650	674	699	724	748	773
177		797	822	846	871	895	920	944	969	993	*018
178	25	042	066	091	115	139	164	188	212	237	261
179		285	310	334	358	382	406	431	455	479	503
180		527	551	575	600	624	648	672	696	720	744
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	34	33
1	3.4	3.3
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	32	31
1	3.2	3.1
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	30	29
1	3.0	2.9
2	6.0	5.8
3	9.0	8.7
4	12.0	11.6
5	15.0	14.5
6	18.0	17.4
7	21.0	20.3
8	24.0	23.2
9	27.0	26.1

	28	27
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

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Num. 180 to 219. Log. 255 to 342.

N	L	0	1	2	3	4	5	6	7	8	9
180	25	527	551	575	600	624	648	672	696	720	744
181		768	792	816	840	864	888	912	935	959	983
182	26	007	031	055	079	102	126	150	174	198	221
183		245	269	293	316	340	364	387	411	435	458
184		482	505	529	553	576	600	623	647	670	694
185		717	741	764	788	811	834	858	881	905	928
186		951	975	988	*021	*045	*068	*091	*114	*138	*161
187	27	184	207	231	254	277	300	323	346	370	393
188		416	439	462	485	508	531	554	577	600	623
189		646	669	692	715	738	761	784	807	830	852
190		875	898	921	944	967	989	*012	*035	*058	*081
191	28	103	126	149	171	194	217	240	262	285	307
192		330	353	375	398	421	443	466	488	511	533
193		556	578	601	623	646	668	691	713	735	758
194		780	803	825	847	870	892	914	937	959	981
195	29	003	026	048	070	092	115	137	159	181	203
196		226	248	270	292	314	336	358	380	403	425
197		447	469	491	513	535	557	579	601	623	645
198		667	688	710	732	754	776	798	820	842	863
199		885	907	929	951	973	994	*016	*038	*060	*081
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
211		428	449	469	490	510	531	552	572	593	613
212		634	654	675	695	715	736	756	777	797	818
213		838	858	879	899	919	940	960	980	*001	*021
214	33	041	062	082	102	122	143	163	183	203	224
215		244	264	284	304	325	345	365	385	405	425
216		445	465	486	506	526	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	826
218		846	866	885	905	925	945	965	985	*005	*025
219	34	044	064	084	104	124	143	163	183	203	223
220		242	262	282	301	321	341	361	380	400	420
N	**L**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	24	23	22	21	20	19
1	2.4	2.3	2.2	2.1	2.0	1.9
2	4.8	4.6	4.4	4.2	4.0	3.8
3	7.2	6.9	6.6	6.3	6.0	5.7
4	9.6	9.2	8.8	8.4	8.0	7.6
5	12.0	11.5	11.0	10.5	10.0	9.5
6	14.4	13.8	13.2	12.6	12.0	11.4
7	16 8	16.1	15.4	14.7	14.0	13.3
8	19.2	18.4	17.6	16.8	16.0	15.2
9	21.6	20.7	19.8	18.9	18.0	17.1

P. P.

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Num. 220 to 259. Log. 342 to 414.

N	L	0	1	2	3	4	5	6	7	8	9
220	34	242	262	282	301	321	341	361	380	400	420
221		439	459	479	498	518	537	557	577	596	616
222		635	655	674	694	713	733	753	772	792	811
223		830	850	869	889	908	928	947	967	986	*005
224	35	025	044	064	083	102	122	141	160	180	199
225		218	238	257	276	295	315	334	353	372	392
226		411	430	449	468	488	507	526	545	564	583
227		603	622	641	660	679	698	717	736	755	774
228		793	813	832	851	870	889	908	927	946	965
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36	173	192	211	229	248	267	286	305	324	342
231		361	380	399	418	436	455	474	493	511	530
232		549	568	586	605	624	642	661	680	698	717
233		736	754	773	791	810	829	847	866	884	903
234		922	940	959	977	996	*014	*033	*051	*070	*088
235	37	107	125	144	162	181	199	218	236	254	273
236		291	310	328	346	365	383	401	420	438	457
237		475	493	511	530	548	566	585	603	621	639
238		658	676	694	712	731	749	767	785	803	822
239		840	858	876	894	912	931	949	967	985	*003
240	38	021	039	057	075	093	112	130	148	166	184
241		202	220	238	256	274	292	310	328	346	364
242		382	399	417	435	453	471	489	507	525	543
243		561	578	596	614	632	650	668	686	703	721
244		739	757	775	792	810	828	846	863	881	899
245		917	934	952	970	987	*005	*023	*041	*058	*076
246	39	094	111	129	146	164	182	199	217	235	252
247		270	287	305	322	340	358	375	393	410	428
248		445	463	480	498	515	533	550	568	585	602
249		620	637	655	672	690	707	724	742	759	777
250		794	811	829	846	863	881	898	915	933	950
251		967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40	140	157	175	192	209	226	243	261	278	295
253		312	329	346	364	381	398	415	432	449	466
254		483	500	518	535	552	569	586	603	620	637
255		654	671	688	705	722	739	756	773	790	807
256		824	841	858	875	892	909	926	943	960	976
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41	162	179	196	212	229	246	263	280	296	313
259		330	347	363	380	397	414	430	447	464	481
260		497	514	531	547	564	581	597	614	631	647
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

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Num. 260 to 299. Log. 414 to 476.

N	L	0	1	2	3	4	5	6	7	8	9
260	41	497	514	531	547	564	581	597	614	631	647
261		664	681	697	714	731	747	764	780	797	814
262		830	847	863	880	896	913	929	946	963	979
263		996	*012	*029	*045	*062	*078	*095	*111	*127	144
264	42	160	177	193	210	226	243	259	275	292	308
265		325	341	357	374	390	406	423	439	455	472
266		488	504	521	537	553	570	586	602	619	635
267		651	667	684	700	716	732	749	765	781	797
268		813	830	846	862	878	894	911	927	943	959
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43	136	152	169	185	201	217	233	249	265	281
271		297	313	329	345	361	377	393	409	425	441
272		457	473	489	505	521	537	553	569	584	600
273		616	632	648	664	680	696	712	727	743	759
274		775	791	807	823	838	854	870	886	902	917
275		933	949	965	981	996	*012	*028	*044	*059	*075
276	44	091	107	122	138	154	170	185	201	217	232
277		248	264	279	295	311	326	342	358	373	389
278		404	420	436	451	467	483	498	514	529	545
279		560	576	592	607	623	638	654	669	685	700
280		716	731	747	762	778	793	809	824	840	855
281		871	886	902	917	932	948	963	979	994	*010
282	45	025	040	056	071	086	102	117	133	148	163
283		179	194	209	225	240	255	271	286	301	317
284		332	347	362	378	393	408	423	439	454	469
285		484	500	515	530	545	561	576	591	606	621
286		637	652	667	682	697	712	728	743	758	773
287		788	803	818	834	849	864	879	894	909	924
288		939	954	969	984	*000	*015	*030	*045	*060	*075
289	46	090	105	120	135	150	165	180	195	210	225
290		240	255	270	285	300	315	330	345	359	374
291		389	404	419	434	449	464	479	494	509	523
292		538	553	568	583	598	613	627	642	657	672
293		687	702	716	731	746	761	776	790	805	820
294		835	850	864	879	894	909	923	938	953	967
295		982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47	129	144	159	173	188	202	217	232	246	261
297		276	290	305	319	334	349	363	378	392	407
298		422	436	451	465	480	494	509	524	538	553
299		567	582	596	611	625	640	654	669	683	698
300		712	727	741	756	770	784	799	813	828	842
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

	16
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6
7	11.2
8	12.8
9	14.4

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

P. P.

Digitized by Google

Num. 300 to 339. Log. 477 to 531.

N	L	0	1	2	3	4	5	6	7	8	9
300	47	712	727	741	756	770	784	799	813	828	842
301		857	871	885	900	914	929	943	958	972	986
302	48	001	015	029	044	058	073	087	101	116	130
303		144	159	173	187	202	216	230	244	259	273
304		287	302	316	330	344	359	373	387	401	416
305		430	444	458	473	487	501	515	530	544	558
306		572	586	601	615	629	643	657	671	686	700
307		714	728	742	756	770	785	799	813	827	841
308		855	869	883	897	911	926	940	954	968	982
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49	136	150	164	178	192	206	220	234	248	262
311		276	290	304	318	332	346	360	374	388	402
312		415	429	443	457	471	485	499	513	527	541
313		554	568	582	596	610	624	638	651	665	679
314		693	707	721	734	748	762	776	790	803	817
315		831	845	859	872	886	900	914	927	941	955
316		969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50	106	120	133	147	161	174	188	202	215	229
318		243	256	270	284	297	311	325	338	352	365
319		379	393	406	420	433	447	461	474	488	501
320		515	529	542	556	569	583	596	610	623	637
321		651	664	678	691	705	718	732	745	759	772
322		786	799	813	826	840	853	866	880	893	907
323		920	934	947	961	974	987	*001	*014	*028	*041
324	51	055	068	081	095	108	121	135	148	162	175
325		188	202	215	228	242	255	268	282	295	308
326		322	335	348	362	375	388	402	415	428	441
327		455	468	481	495	508	521	534	548	561	574
328		587	601	614	627	640	654	667	680	693	706
329		720	733	746	759	772	786	799	812	825	838
330		851	865	878	891	904	917	930	943	957	970
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52	114	127	140	153	166	179	192	205	218	231
333		244	257	270	284	297	310	323	336	349	362
334		375	388	401	414	427	440	453	466	479	492
335		504	517	530	543	556	569	582	595	608	621
336		634	647	660	673	686	699	711	724	737	750
337		763	776	789	802	815	827	840	853	866	879
338		892	905	917	930	943	956	969	982	994	*007
339	53	020	033	046	058	071	084	097	110	122	135
340		148	161	173	186	199	212	224	237	250	263
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	14	13	12
1	1.4	1.3	1.2
2	2.8	2.6	2.4
3	4.2	3.9	3.6
4	5.6	5.2	4.8
5	7.0	6.5	6.0
6	8.4	7.8	7.2
7	9.8	9.1	8.4
8	11.2	10.4	9.6
9	12.6	11.7	10.8

Digitized by Google

Num. 340 to 379. Log. 531 to 579.

N	L	0	1	2	3	4	5	6	7	8	9
340	53	148	161	173	186	199	212	224	237	250	263
341		275	288	301	314	326	339	352	364	377	390
342		403	415	428	441	453	466	479	491	504	517
343		529	542	555	567	580	593	605	618	631	643
344		656	668	681	694	706	719	732	744	757	769
345		782	794	807	820	832	845	857	870	882	895
346		908	920	933	945	958	970	983	995	*008	*020
347	54	033	045	058	070	083	095	108	120	133	145
348		158	170	183	195	208	220	233	245	258	270
349		283	295	307	320	332	345	357	370	382	394
350		407	419	432	444	456	469	481	494	506	518
351		531	543	555	568	580	593	605	617	630	642
352		654	667	679	691	704	716	728	741	753	765
353		777	790	802	814	827	839	851	864	876	888
354		900	913	925	937	949	962	974	986	998	*011
355	55	023	035	047	060	072	084	096	108	121	133
356		145	157	169	182	194	206	218	230	242	255
357		267	279	291	303	315	328	340	352	364	376
358		388	400	413	425	437	449	461	473	485	497
359		509	522	534	546	558	570	582	594	606	618
360		630	642	654	666	678	691	703	715	727	739
361		751	763	775	787	799	811	823	835	847	859
362		871	883	895	907	919	931	943	955	967	979
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56	110	122	134	146	158	170	182	194	205	217
365		229	241	253	265	277	289	301	312	324	336
366		348	360	372	384	396	407	419	431	443	455
367		467	478	490	502	514	526	538	549	561	573
368		585	597	608	620	632	644	656	667	679	691
369		703	714	726	738	750	761	773	785	797	808
370		820	832	844	855	867	879	891	902	914	926
371		937	949	961	972	984	996	*008	*019	*031	*043
372	57	054	066	078	089	101	113	124	136	148	159
373		171	183	194	206	217	229	241	252	264	276
374		287	299	310	322	334	345	357	368	380	392
375		403	415	426	438	449	461	473	484	496	507
376		519	530	542	553	565	576	588	600	611	623
377		634	646	657	669	680	692	703	715	726	738
378		749	761	772	784	795	807	818	830	841	852
379		864	875	887	898	910	921	933	944	955	967
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

Digitized by Google

Num. 380 to 419. Log. 579 to 623.

N	L	0	1	2	3	4	5	6	7	8	9
380	57	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58	092	104	115	127	138	149	161	172	184	195
382		206	218	229	240	252	263	274	286	297	309
383		320	331	343	354	365	377	388	399	410	422
384		433	444	456	467	478	490	501	512	524	535
385		546	557	569	580	591	602	614	625	636	647
386		659	670	681	692	704	715	726	737	749	760
387		771	782	794	805	816	827	838	850	861	872
388		883	894	906	917	928	939	950	961	973	984
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59	106	118	129	140	151	162	173	184	195	207
391		218	229	240	251	262	273	284	295	306	318
392		329	340	351	362	373	384	395	406	417	428
393		439	450	461	472	483	494	506	517	528	539
394		550	561	572	583	594	605	616	627	638	649
395		660	671	682	693	704	715	726	737	748	759
396		770	780	791	802	813	824	835	846	857	868
397		879	890	901	912	923	934	945	956	966	977
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60	097	108	119	130	141	152	163	173	184	195
400		206	217	228	239	249	260	271	282	293	304
401		314	325	336	347	358	369	379	390	401	412
402		423	433	444	455	466	477	487	498	509	520
403		531	541	552	563	574	584	595	606	617	627
404		638	649	660	670	681	692	703	713	724	735
405		746	756	767	778	788	799	810	821	831	842
406		853	863	874	885	895	906	917	927	938	949
407		959	970	981	991	*002	*013	*023	*034	*045	*055
408	61	066	077	087	098	109	119	130	140	151	162
409		172	183	194	204	215	225	236	247	257	268
410		278	289	300	310	321	331	342	352	363	374
411		384	395	405	416	426	437	448	458	469	479
412		490	500	511	521	532	542	553	563	574	584
413		595	606	616	627	637	648	658	669	679	690
414		700	711	721	731	742	752	763	773	784	794
415		805	815	826	836	847	857	868	878	888	899
416		909	920	930	941	951	962	972	982	993	*003
417	62	014	024	034	045	055	066	076	086	097	107
418		118	128	138	149	159	170	180	190	201	211
419		221	232	242	252	263	273	284	294	304	315
420		325	335	346	356	366	377	387	397	408	418
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

P. P.

Digitized by Google

Num. 420 to 459. Log. 623 to 662.

N	L	0	1	2	3	4	5	6	7	8	9
420	62	325	335	346	356	366	377	387	397	408	418
421		428	439	449	459	469	480	490	500	511	521
422		531	542	552	562	572	583	593	603	613	624
423		634	644	655	665	675	685	696	706	716	726
424		737	747	757	767	778	788	798	808	818	829
425		839	849	859	870	880	890	900	910	921	931
426		941	951	961	972	982	992	*002	*012	*022	*033
427	63	043	053	063	073	083	094	104	114	124	134
428		144	155	165	175	185	195	205	215	225	236
429		246	256	266	276	286	296	306	317	327	337
430		347	357	367	377	387	397	407	417	428	438
431		448	458	468	478	488	498	508	518	528	538
432		548	558	568	579	589	599	609	619	629	639
433		649	659	669	679	689	699	709	719	729	739
434		749	759	769	779	789	799	809	819	829	839
435		849	859	869	879	889	899	909	919	929	939
436		949	959	969	979	988	998	*008	*018	*028	*038
437	64	048	058	068	078	088	098	108	118	128	137
438		147	157	167	177	187	197	207	217	227	237
439		246	256	266	276	286	296	306	316	326	335
440		345	355	365	375	385	395	404	414	424	434
441		444	454	464	473	483	493	503	513	523	532
442		542	552	562	572	582	591	601	611	621	631
443		640	650	660	670	680	689	699	709	719	729
444		738	748	758	768	777	787	797	807	816	826
445		836	846	856	865	875	885	895	904	914	924
446		933	943	953	963	972	982	992	*002	*011	*021
447	65	031	040	050	060	070	079	089	099	108	118
448		128	137	147	157	167	176	186	196	205	215
449		225	234	244	254	263	273	283	292	302	312
450		321	331	341	350	360	369	379	389	398	408
451		418	427	437	447	456	466	475	485	495	504
452		514	523	533	543	552	562	571	581	591	600
453		610	619	629	639	648	658	667	677	686	696
454		706	715	725	734	744	753	763	772	782	792
455		801	811	820	830	839	849	858	868	877	887
456		896	906	916	925	935	944	954	963	973	982
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66	087	096	106	115	124	134	143	153	162	172
459		181	191	200	210	219	229	238	247	257	266
460		276	285	295	304	314	323	332	342	351	361
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Digitized by Google

Num. 460 to 499. Log. 662 to 698.

N	L	0	1	2	3	4	5	6	7	8	9
460	66	276	285	295	304	314	323	332	342	351	361
461		370	380	389	398	408	417	427	436	445	455
462		464	474	483	492	502	511	521	530	539	549
463		558	567	577	586	596	605	614	624	633	642
464		652	661	671	680	689	699	708	717	727	736
465		745	755	764	773	783	792	801	811	820	829
466		839	848	857	867	876	885	894	904	913	922
467		932	941	950	960	969	978	987	997	*006	*015
468	67	025	034	043	052	062	071	080	089	099	108
469		117	127	136	145	154	164	173	182	191	201
470		210	219	228	237	247	256	265	274	284	293
471		302	311	321	330	339	348	357	367	376	385
472		394	403	413	422	431	440	449	459	468	477
473		486	495	504	514	523	532	541	550	560	569
474		578	587	596	605	614	624	633	642	651	660
475		669	679	688	697	706	715	724	733	742	752
476		761	770	779	788	797	806	815	825	834	843
477		852	861	870	879	888	897	906	916	925	934
478		943	952	961	970	979	988	997	*006	*015	*024
479	68	034	043	052	061	070	079	088	097	106	115
480		124	133	142	151	160	169	178	187	196	205
481		215	224	233	242	251	260	269	278	287	296
482		305	314	323	332	341	350	359	368	377	386
483		395	404	413	422	431	440	449	458	467	476
484		485	494	502	511	520	529	538	547	556	565
485		574	583	592	601	610	619	628	637	646	655
486		664	673	681	690	699	708	717	726	735	744
487		753	762	771	780	789	797	806	815	824	833
488		842	851	860	869	878	886	895	904	913	922
489		931	940	949	958	966	975	984	993	*002	*011
490	69	020	028	037	046	055	064	073	082	090	099
491		108	117	126	135	144	152	162	170	179	188
492		197	205	214	223	232	241	249	258	267	276
493		285	294	302	311	320	329	338	346	355	364
494		373	381	390	399	408	417	425	434	443	452
495		461	469	478	487	496	504	513	522	531	539
496		548	557	566	574	583	592	601	609	618	627
497		636	644	653	662	671	679	688	697	705	714
498		723	732	740	749	758	767	775	784	793	801
499		810	819	827	836	845	854	862	871	880	888
500		897	906	914	923	932	940	949	958	966	975
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Digitized by Google

Num. 500 to 539. Log. 698 to 732.

N	L	0	1	2	3	4	5	6	7	8	9
500	69	897	906	914	922	932	940	949	958	966	975
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70	070	079	088	096	105	114	122	131	140	148
503		157	165	174	183	191	200	209	217	226	234
504		243	252	260	269	278	286	295	303	312	321
505		329	338	346	355	364	372	381	389	398	406
506		415	424	432	441	449	458	467	475	484	492
507		501	509	518	526	535	544	552	561	569	578
508		586	595	603	612	621	629	638	646	655	663
509		672	680	689	697	706	714	723	731	740	749
510		757	766	774	783	791	800	808	817	825	834
511		842	851	859	868	876	885	893	902	910	919
512		927	935	944	952	961	969	978	986	995	*003
513	71	012	020	029	037	046	054	063	071	079	088
514		096	105	113	122	130	139	147	155	164	172
515		181	189	198	206	214	223	231	240	248	257
516		265	273	282	290	299	307	315	324	332	341
517		349	357	366	374	383	391	399	408	416	425
518		433	441	450	458	466	475	483	492	500	508
519		517	525	533	542	550	559	567	575	584	592
520		600	609	617	625	634	642	650	659	667	675
521		684	692	700	709	717	725	734	742	750	759
522		767	775	784	792	800	809	817	825	834	842
523		850	858	867	875	883	892	900	908	917	925
524		933	941	950	958	966	975	983	991	999	*008
525	72	016	024	032	041	049	057	066	074	082	090
526		099	107	115	123	132	140	148	156	165	173
527		181	189	198	206	214	222	230	239	247	255
528		263	272	280	288	296	304	313	321	329	337
529		346	354	362	370	378	387	395	403	411	419
530		428	436	444	452	460	469	477	485	493	501
531		509	518	526	534	542	550	558	567	575	583
532		591	599	607	616	624	632	640	648	656	665
533		673	681	689	697	705	713	722	730	738	746
534		754	762	770	779	787	795	803	811	819	827
535		835	843	852	860	868	876	884	892	900	908
536		916	925	933	941	949	957	965	973	981	989
537		997	*006	*014	*022	*030	*038	*046	*054	062	*070
538	73	078	086	094	102	111	119	127	135	143	151
539		159	167	175	183	191	199	207	215	223	231
540		239	247	255	263	272	280	288	296	304	312
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

Digitized by Google

Num. 540 to 579. Log. 732 to 763.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
540	73	239	247	255	263	272	280	288	296	304	312	
541		320	328	336	344	352	360	368	376	384	392	
542		400	408	416	424	432	440	448	456	464	472	
543		480	488	496	504	512	520	528	536	544	552	
544		560	568	576	584	592	600	608	616	624	632	
545		640	648	656	664	672	679	687	695	703	711	
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
548		878	886	894	902	910	918	926	933	941	949	
549		957	965	973	981	989	997	*005	*013	*020	*028	8
550	74	036	044	052	060	068	076	084	092	099	107	1 | 0.8
551		115	123	131	139	147	155	162	170	178	186	2 | 1.6
552		194	202	210	218	225	233	241	249	257	265	3 | 2.4
553		273	280	288	296	304	312	320	327	335	343	4 | 3.2
554		351	359	367	374	382	390	398	406	414	421	5 | 4.0
555		429	437	445	453	461	468	476	484	492	500	6 | 4.8
556		507	515	523	531	539	547	554	562	570	578	7 | 5.6
557		586	593	601	609	617	624	632	640	648	656	8 | 6.4
558		663	671	679	687	695	702	710	718	726	733	9 | 7.2
559		741	749	757	764	772	780	788	796	803	811	
560		819	827	834	842	850	858	865	873	881	889	
561		896	904	912	920	927	935	943	950	958	966	
562		974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75	051	059	066	074	082	089	097	105	113	120	
564		128	136	143	151	159	166	174	182	189	197	
565		205	213	220	228	236	243	251	259	266	274	7
566		282	289	297	305	312	320	328	335	343	351	1 | 0.7
567		358	366	374	381	389	397	404	412	420	427	2 | 1.4
568		435	442	450	458	465	473	481	488	496	504	3 | 2.1
569		511	519	526	534	542	549	557	565	572	580	4 | 2.8
570		587	595	603	610	618	626	633	641	648	656	5 | 3.5
571		664	671	679	686	694	702	709	717	724	732	6 | 4.2
572		740	747	755	762	770	778	785	793	800	808	7 | 4.9
573		815	823	831	838	846	853	861	868	876	884	8 | 5.6
574		891	899	906	914	921	929	937	944	952	959	9 | 6.3
575		967	974	982	989	997	*005	*012	*020	*027	*035	
576	76	042	050	057	065	072	080	087	095	103	110	
577		118	125	133	140	148	155	163	170	178	185	
578		193	200	208	215	223	230	238	245	253	260	
579		268	275	283	290	298	305	313	320	328	335	
580		343	350	358	365	373	380	388	395	403	410	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

Digitized by Google

Num. 580 to 619. Log. 763 to 792.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
580	76	343	350	358	365	373	380	388	395	403	410	8
581		418	425	433	440	448	455	462	470	477	485	1 | 0.8
582		492	500	507	515	522	530	537	545	552	559	2 | 1.5
583		567	574	582	589	597	604	612	619	626	634	3 | 2.4
584		641	649	656	664	671	678	686	693	701	708	4 | 3.2
585		716	723	730	738	745	753	760	768	775	782	5 | 4.0
586		790	797	805	812	819	827	834	842	849	856	6 | 4.8
587		864	871	879	886	893	901	908	916	923	930	7 | 5.6
588		938	945	953	960	967	975	982	989	997	*004	8 | 6.4
589	77	012	019	026	034	041	048	056	063	070	078	9 | 7.2
590		085	093	100	107	115	122	129	137	144	151	
591		159	166	173	181	188	195	203	210	217	225	
592		232	240	247	254	262	269	276	283	291	298	
593		305	313	320	327	335	342	349	357	364	371	
594		379	386	393	401	408	415	422	430	437	444	
595		452	459	466	474	481	488	495	503	510	517	
596		525	532	539	546	554	561	568	576	583	590	
597		597	605	612	619	627	634	641	648	656	663	
598		670	677	685	692	699	706	714	721	728	735	7
599		743	750	757	764	772	779	786	793	801	808	1 | 0.7
600		815	822	830	837	844	851	859	866	873	880	2 | 1.4
601		887	895	902	909	916	924	931	938	945	952	3 | 2.1
602		960	967	974	981	988	996	*003	*010	*017	*025	4 | 2.8
603	78	032	039	046	053	061	068	075	082	089	097	5 | 3.5
604		104	111	118	125	132	140	147	154	161	168	6 | 4.2
605		176	183	190	197	204	211	219	226	233	240	7 | 4.9
606		247	254	262	269	276	283	290	297	305	312	8 | 5.6
607		319	326	333	340	347	355	362	369	376	383	9 | 6.3
608		390	398	405	412	419	426	433	440	447	455	
609		462	469	476	483	490	497	504	512	519	526	
610		533	540	547	554	561	569	576	583	590	597	
611		604	611	618	625	633	640	647	654	661	668	
612		675	682	689	696	704	711	718	725	732	739	
613		746	753	760	767	774	781	789	796	802	810	
614		817	824	831	838	845	852	859	866	873	880	
615		888	895	902	909	916	923	930	937	944	951	
616		958	965	972	979	986	993	*000	*007	*014	*021	
617	79	029	036	043	050	057	064	071	078	085	092	
618		099	106	113	120	127	134	141	148	155	162	
619		169	176	183	190	197	204	211	218	225	232	
620		239	246	253	260	267	274	281	288	295	302	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

Digitized by Google

Num. 620 to 659. Log. 792 to 819.

N	L	0	1	2	3	4	5	6	7	8	9
620	79	239	246	253	260	267	274	281	288	295	302
621		309	316	323	330	337	344	351	358	365	372
622		379	386	393	400	407	414	421	428	435	442
623		449	456	463	470	477	484	491	498	505	511
624		518	525	532	539	546	553	560	567	574	581
625		588	595	602	609	616	623	630	637	644	650
626		657	664	671	678	685	692	699	706	713	720
627		727	734	741	748	754	761	768	775	782	789
628		796	803	810	817	824	831	837	844	851	858
629		865	872	879	886	893	900	906	913	920	927
630		934	941	948	955	962	969	975	982	989	996
631	80	003	010	017	024	030	037	044	051	058	065
632		072	079	085	092	099	106	113	120	127	134
633		140	147	154	161	168	175	182	188	195	202
634		209	216	223	229	236	243	250	257	264	271
635		277	284	291	298	305	312	318	325	332	339
636		346	353	359	366	373	380	387	393	400	407
637		414	421	428	434	441	448	455	462	468	475
638		482	489	496	502	509	516	523	530	536	543
639		550	557	564	570	577	584	591	598	604	611
640		618	625	632	638	645	652	659	665	672	679
641		686	693	699	706	713	720	726	733	740	747
642		754	760	767	774	781	787	794	801	808	814
643		821	828	835	841	848	855	862	868	875	882
644		889	895	902	909	916	922	929	936	943	949
645		956	963	969	976	983	990	996	*003	*010	*017
646	81	023	030	037	043	050	057	064	070	077	084
647		090	097	104	111	117	124	131	137	144	151
648		158	164	171	178	184	191	198	204	211	218
649		224	231	238	245	251	258	265	271	278	285
650		291	298	305	311	318	325	331	338	345	351
651		358	365	371	378	385	391	398	405	411	418
652		425	431	438	445	451	458	465	471	478	485
653		491	498	505	511	518	525	531	538	544	551
654		558	564	571	578	584	591	598	604	611	617
655		624	631	637	644	651	657	664	671	677	684
656		690	697	704	710	717	723	730	737	743	750
657		757	763	770	776	783	790	796	803	809	816
658		823	829	836	842	849	856	862	869	875	882
659		889	895	902	908	915	921	928	935	941	948
660		954	961	968	974	981	987	994	*000	*007	*014
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Digitized by Google

Num. 660 to 699. Log. 819 to 845.

N	L	0	1	2	3	4	5	6	7	8	9
660	81	954	961	968	974	981	987	994	*000	*007	*014
661	82	020	027	033	040	046	053	060	066	073	079
662		086	092	099	105	112	119	125	132	138	145
663		151	158	164	171	178	184	191	197	204	210
664		217	223	230	236	243	249	256	263	269	276
665		282	289	295	302	308	315	321	328	334	341
666		347	354	360	367	373	380	387	393	400	406
667		413	419	426	432	439	445	452	458	465	471
668		478	484	491	497	504	510	517	523	530	536
669		543	549	556	562	569	575	582	588	595	601
670		607	614	620	627	633	640	646	653	659	666
671		672	679	685	692	698	705	711	718	724	730
672		737	743	750	756	763	769	776	782	789	795
673		802	808	814	821	827	834	840	847	853	860
674		866	872	879	885	892	898	905	911	918	924
675		930	937	943	950	956	963	969	975	982	988
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052
677	83	059	065	072	078	085	091	097	104	110	117
678		123	129	136	142	149	155	161	168	174	181
679		187	193	200	206	213	219	225	232	238	245
680		251	257	264	270	276	283	289	296	302	308
681		315	321	327	334	340	347	353	359	366	372
682		378	385	391	398	404	410	417	423	429	436
683		442	448	455	461	467	474	480	487	493	499
684		506	512	518	525	531	537	544	550	556	563
685		569	575	582	588	594	601	607	613	620	626
686		632	639	645	651	658	664	670	677	683	689
687		696	702	708	715	721	727	734	740	746	753
688		759	765	771	778	784	790	797	803	809	816
689		822	828	835	841	847	853	860	866	872	879
690		885	891	897	904	910	916	923	929	935	942
691		948	954	960	967	973	979	985	992	998	*004
692	84	011	017	023	029	036	042	048	055	061	067
693		073	080	086	092	098	105	111	117	123	130
694		136	142	148	155	161	167	173	180	186	192
695		198	205	211	217	223	230	236	242	248	255
696		261	267	273	280	286	292	298	305	311	317
697		323	330	336	342	348	354	361	367	373	379
698		386	392	398	404	410	417	423	429	435	442
699		448	454	460	466	473	479	485	491	497	504
700		510	516	522	528	535	541	547	553	559	566
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Digitized by Google

Num. 700 to 739. Log. 845 to 869.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
700	84	510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	058	
709		065	071	077	083	089	095	101	107	114	120	
710		126	132	138	144	150	156	163	169	175	181	
711		187	193	199	205	211	217	224	230	236	242	
712		248	254	260	266	272	278	285	291	297	303	
713		309	315	321	327	333	339	345	352	358	364	
714		370	376	382	388	394	400	406	412	418	425	
715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	
724		974	980	986	992	998	*004	*010	*016	*022	*028	
725	86	034	040	046	052	058	064	070	076	082	088	
726		094	100	106	112	118	124	130	136	141	147	
727		153	159	165	171	177	183	189	195	201	207	
728		213	219	225	231	237	243	249	255	261	267	
729		273	279	285	291	297	303	308	314	320	326	
730		332	338	344	350	356	362	368	374	380	386	
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	
739		864	870	876	882	888	894	900	906	911	917	
740		923	929	935	941	947	953	958	964	970	976	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Digitized by Google

Num. 740 to 779. Log. 869 to 892.

N	L	0	1	2	3	4	5	6	7	8	9
740	86	923	929	935	941	947	953	958	964	970	976
741		982	988	994	999	*005	*011	*017	*023	*029	*035
742	87	040	046	052	058	064	070	075	081	087	093
743		099	105	111	116	122	128	134	140	146	151
744		157	163	169	175	181	186	192	198	204	210
745		216	221	227	233	239	245	251	256	262	268
746		274	280	286	291	297	303	309	315	320	326
747		332	338	344	349	355	361	367	373	379	384
748		390	396	402	408	413	419	425	431	437	442
749		448	454	460	466	471	477	483	489	495	500
750		506	512	518	523	529	535	541	547	552	558
751		564	570	576	581	587	593	599	604	610	616
752		622	628	633	639	645	651	656	662	668	674
753		679	685	691	697	703	708	714	720	726	731
754		737	743	749	754	760	766	772	777	783	789
755		795	800	806	812	818	823	829	835	841	846
756		852	858	864	869	875	881	887	892	898	904
757		910	915	921	927	933	938	944	950	955	961
758		967	973	978	984	990	996	*001	*007	*013	*018
759	88	024	030	036	041	047	053	058	064	070	076
760		081	087	093	098	104	110	116	121	127	133
761		138	144	150	156	161	167	173	178	184	190
762		195	201	207	213	218	224	230	235	241	247
763		252	258	264	270	275	281	287	292	298	304
764		309	315	321	326	332	338	343	349	355	360
765		366	372	377	383	389	395	400	406	412	417
766		423	429	434	440	446	451	457	463	468	474
767		480	485	491	497	502	508	513	519	525	530
768		536	542	547	553	559	564	570	576	581	587
769		593	598	604	610	615	621	627	632	638	643
770		649	655	660	666	672	677	683	689	694	700
771		705	711	717	722	728	734	739	745	750	756
772		762	767	773	779	784	790	795	801	807	812
773		818	824	829	835	840	846	852	857	863	868
774		874	880	885	891	897	902	908	913	919	925
775		930	936	941	947	953	958	964	969	975	981
776		986	992	997	*003	*009	*014	*020	*025	*031	*037
777	89	042	048	053	059	064	070	076	081	087	092
778		098	104	109	115	120	126	131	137	143	148
779		154	159	165	170	176	182	187	193	198	204
780		209	215	221	226	232	237	243	248	254	260
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Digitized by Google

Num. 780 to 819. Log. 892 to 913.

N	L	0	1	2	3	4	5	6	7	8	9
780	89	209	215	221	226	232	237	243	248	254	260
781		265	271	276	282	287	293	298	304	310	315
782		321	326	332	337	343	348	354	360	365	371
783		376	382	387	393	398	404	409	415	421	426
784		432	437	443	448	454	459	465	470	476	481
785		487	492	498	504	509	515	520	526	531	537
786		542	548	553	559	564	570	575	581	586	592
787		597	603	609	614	620	625	631	636	642	647
788		653	658	664	669	675	680	686	691	697	702
789		708	713	719	724	730	735	741	746	752	757
790		763	768	774	779	785	790	796	801	807	812
791		818	823	829	834	840	845	851	856	862	867
792		873	878	883	889	894	900	905	911	916	922
793		927	933	938	944	949	955	960	966	971	977
794		982	988	993	998	*004	*009	*015	*020	*026	*031
795	90	037	042	048	053	059	064	069	075	080	086
796		091	097	102	108	113	119	124	129	135	140
797		146	151	157	162	168	173	179	184	189	195
798		200	206	211	217	222	227	233	238	244	249
799		255	260	266	271	276	282	287	293	298	304
800		309	314	320	325	331	336	342	347	352	358
801		363	369	374	380	385	390	396	401	407	412
802		417	423	428	434	439	445	450	455	461	466
803		472	477	482	488	493	499	504	509	515	520
804		526	531	536	542	547	553	558	563	569	574
805		580	585	590	596	601	607	612	617	623	628
806		634	639	644	650	655	660	666	671	677	682
807		687	693	698	703	709	714	720	725	730	736
808		741	747	752	757	763	768	773	779	784	789
809		795	800	806	811	816	822	827	832	838	843
810		849	854	859	865	870	875	881	886	891	897
811		902	907	913	918	924	929	934	940	945	950
812		956	961	966	972	977	982	988	993	998	*004
813	91	009	014	020	025	030	036	041	046	052	057
814		062	068	073	078	084	089	094	100	105	110
815		116	121	126	132	137	142	148	153	158	164
816		169	174	180	185	190	196	201	206	212	217
817		222	228	233	238	243	249	254	259	265	270
818		275	281	286	291	297	302	307	312	318	323
819		328	334	339	344	350	355	360	365	371	376
820		381	387	392	397	403	408	413	418	424	429
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Digitized by Google

Num. 820 to 859. Log. 913 to 934.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
820	91	381	387	392	397	403	408	413	418	424	429	
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	
831		960	965	971	976	981	986	991	997	*002	*007	
832	92	012	018	023	028	033	038	044	049	054	059	
833		065	070	075	080	085	091	096	101	106	111	
834		117	122	127	132	137	143	148	153	158	163	
835		169	174	179	184	189	195	200	205	210	215	
836		221	226	231	236	241	247	252	257	262	267	
837		273	278	283	288	293	298	304	309	314	319	
838		324	330	335	340	345	350	355	361	366	371	
839		376	381	387	392	397	402	407	412	418	423	
840		428	433	438	443	449	454	459	464	469	474	
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	
859		399	404	409	414	420	425	430	435	440	445	
860		450	455	460	465	470	475	480	485	490	495	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.:

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Digitized by Google

Num. 860 to 899. Log. 934 to 954.

N	L	0	1	2	3	4	5	6	7	8	9
860	93	450	455	460	465	470	475	480	485	490	495
861		500	505	510	515	520	526	531	536	541	546
862		551	556	561	566	571	576	581	586	591	596
863		601	606	611	616	621	626	631	636	641	646
864		651	656	661	666	671	676	682	687	692	697
865		702	707	712	717	722	727	732	737	742	747
866		752	757	762	767	772	777	782	787	792	797
867		802	807	812	817	822	827	832	837	842	847
868		852	857	862	867	872	877	882	887	892	897
869		902	907	912	917	922	927	932	937	942	947
870		952	957	962	967	972	977	982	987	992	997
871	94	002	007	012	017	022	027	032	037	042	047
872		052	057	062	067	072	077	082	086	091	096
873		101	106	111	116	121	126	131	136	141	146
874		151	156	161	166	171	176	181	186	191	196
875		201	206	211	216	221	226	231	236	240	245
876		250	255	260	265	270	275	280	285	290	295
877		300	305	310	315	320	325	330	335	340	345
878		349	354	359	364	369	374	379	384	389	394
879		399	404	409	414	419	424	429	433	438	443
880		448	453	458	463	468	473	478	483	488	493
881		498	503	507	512	517	522	527	532	537	542
882		547	552	557	562	567	571	576	581	586	591
883		596	601	606	611	616	621	626	630	635	640
884		645	650	655	660	665	670	675	680	685	689
885		694	699	704	709	714	719	724	729	734	738
886		743	748	753	758	763	768	773	778	783	787
887		792	797	802	807	812	817	822	827	832	836
888		841	846	851	856	861	866	871	876	880	885
889		890	895	900	905	910	915	919	924	929	934
890		939	944	949	954	959	963	968	973	978	983
891		988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95	036	041	046	051	056	061	066	071	075	080
893		085	090	095	100	105	109	114	119	124	129
894		134	139	143	148	153	158	163	168	173	177
895		182	187	192	197	202	207	211	216	221	226
896		231	236	240	245	250	255	260	265	270	274
897		279	284	289	294	299	303	308	313	318	323
898		328	332	337	342	347	352	357	361	366	371
899		376	381	386	390	395	400	405	410	415	419
900		424	429	434	439	444	448	453	458	463	468
N	**L**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Digitized by Google

Num. 900 to 939. Log. 954 to 973.

N	L	0	1	2	3	4	5	6	7	8	9
900	95	424	429	434	439	444	448	453	458	463	468
901		472	477	482	487	492	497	501	506	511	516
902		521	525	530	535	540	545	550	554	559	564
903		569	574	578	583	588	593	598	602	607	612
904		617	622	626	631	636	641	646	650	655	660
905		665	670	674	679	684	689	694	698	703	708
906		713	718	722	727	732	737	742	746	751	756
907		761	766	770	775	780	785	789	794	799	804
908		809	813	818	823	828	832	837	842	847	852
909		856	861	866	871	875	880	885	890	895	899
910		904	909	914	918	923	928	933	938	942	947
911		952	957	961	966	971	976	980	985	990	995
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042
913	96	047	052	057	061	066	071	076	080	085	090
914		095	099	104	109	114	118	123	128	133	137
915		142	147	152	156	161	166	171	175	180	185
916		190	194	199	204	209	213	218	223	227	232
917		237	242	246	251	256	261	265	270	275	280
918		284	289	294	298	303	308	313	317	322	327
919		332	336	341	346	350	355	360	365	369	374
920		379	384	388	393	398	402	407	412	417	421
921		426	431	435	440	445	450	454	459	464	468
922		473	478	483	487	492	497	501	506	511	515
923		520	525	530	534	539	544	548	553	558	562
924		567	572	577	581	586	591	595	600	605	609
925		614	619	624	628	633	638	642	647	652	656
926		661	666	670	675	680	685	689	694	699	703
927		708	713	717	722	727	731	736	741	745	750
928		755	759	764	769	774	778	783	788	792	797
929		802	806	811	816	820	825	830	834	839	844
930		848	853	858	862	867	872	876	881	886	890
931		895	900	904	909	914	918	923	928	932	937
932		942	946	951	956	960	965	970	974	979	984
933		988	993	997	*002	*007	*011	*016	*021	*025	*030
934	97	035	039	044	049	053	058	063	067	072	077
935		081	086	090	095	100	104	109	114	118	123
936		128	132	137	142	146	151	155	160	165	169
937		174	179	183	188	192	197	202	206	211	216
938		220	225	230	234	239	243	248	253	257	262
939		267	271	276	280	285	290	294	299	304	308
940		313	317	322	327	331	336	340	345	350	354
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Digitized by Google

Num. 940 to 979. Log. 973 to 991.

N	L	0	1	2	3	4	5	6	7	8	9
940	97	313	317	322	327	331	336	340	345	350	354
941		359	364	368	373	377	382	387	391	396	400
942		405	410	414	419	424	428	433	437	442	447
943		451	456	460	465	470	474	479	483	488	493
944		497	502	506	511	516	520	525	529	534	539
945		543	548	552	557	562	566	571	575	580	585
946		589	594	598	603	607	612	617	621	626	630
947		635	640	644	649	653	658	663	667	672	676
948		681	685	690	695	699	704	708	713	717	722
949		727	731	736	740	745	749	754	759	763	768
950		772	777	782	786	791	795	800	804	809	813
951		818	823	827	832	836	841	845	850	855	859
952		864	868	873	877	882	886	891	896	900	905
953		909	914	918	923	928	932	937	941	946	950
954		955	959	964	968	973	978	982	987	991	996
955	98	000	005	009	014	019	023	028	032	037	041
956		046	050	055	059	064	068	073	078	082	087
957		091	096	100	105	109	114	118	123	127	132
958		137	141	146	150	155	159	164	168	173	177
959		182	186	191	195	200	204	209	214	218	223
960		227	232	236	241	245	250	254	259	263	268
961		272	277	281	286	290	295	299	304	308	313
962		318	322	327	331	336	340	345	349	354	358
963		363	367	372	376	381	385	390	394	399	403
964		408	412	417	421	426	430	435	439	444	448
965		453	457	462	466	471	475	480	484	489	493
966		498	502	507	511	516	520	525	529	534	538
967		543	547	552	556	561	565	570	574	579	583
968		588	592	597	601	605	610	614	619	623	628
969		632	637	641	646	650	655	659	664	668	673
970		677	682	686	691	695	700	704	709	713	717
971		722	726	731	735	740	744	749	753	758	762
972		767	771	776	780	784	789	793	798	802	807
973		811	816	820	825	829	834	838	843	847	851
974		856	860	865	869	874	878	883	887	892	896
975		900	905	909	914	918	923	927	932	936	941
976		945	949	954	958	963	967	972	976	981	985
977		989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99	034	038	043	047	052	056	061	065	069	074
979		078	083	087	092	096	100	105	109	114	118
980		123	127	131	136	140	145	149	154	158	162
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Digitized by Google

Num. 980 to 1000. Log. 991 to 999.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
980	99	123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	
983		255	260	264	269	273	277	282	286	291	295	
984		300	304	308	313	317	322	326	330	335	339	
985		344	348	352	357	361	366	370	374	379	383	
986		388	392	396	401	405	410	414	419	423	427	
987		432	436	441	445	449	454	458	463	467	471	
988		476	480	484	489	493	498	502	506	511	515	
989		520	524	528	533	537	542	546	550	555	559	4
990		564	568	572	577	581	585	590	594	599	603	1 | 0.4
991		607	612	616	621	625	629	634	638	642	647	2 | 0.8
992		651	656	660	664	669	673	677	682	686	691	3 | 1.2
993		695	699	704	708	712	717	721	726	730	734	4 | 1.6
994		739	743	747	752	756	760	765	769	774	778	5 | 2.0
995		782	787	791	795	800	804	808	813	817	822	6 | 2.4
996		826	830	835	839	843	848	852	856	861	865	7 | 2.8
997		870	874	878	883	887	891	896	900	904	909	8 | 3.2
998		913	917	922	926	930	935	939	944	948	952	9 | 3.6
999		957	961	965	970	974	978	983	987	991	996	
1000	000	000	043	087	130	174	217	260	304	347	391	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

Logarithms of Important Numbers.

Number.	Logarithm.
$\pi = 3.141\ 593$	$0.497\ 150$
$\frac{4}{3}\pi = 4.188\ 790$	$0.622\ 089$
$\frac{1}{6}\pi = 0.523\ 599$	$\bar{1}.718\ 999$
$\frac{1}{\pi} = 0.318\ 310$	$\bar{1}.502\ 850$
$\pi^2 = 9.869\ 604$	$0.994\ 300$
$\frac{1}{\pi^2} = 0.101\ 321$	$\bar{1}.005\ 700$
$\sqrt{\pi} = 1.772\ 454$	$0.248\ 575$
$\frac{1}{\sqrt{\pi}} = 0.564\ 190$	$\bar{1}.751\ 425$
$\sqrt[3]{\pi} = 1.464\ 592$	$0.165\ 717$
$\frac{1}{\sqrt[3]{\pi}} = 0.682\ 784$	$\bar{1}.834\ 283$
$\sqrt[3]{\frac{6}{\pi}} = 1.240\ 701$	$0.093\ 667$

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0° **Logarithms.** **179°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	Inf. Neg.	Infinite.	Inf. Neg.	Infinite.	10.00000	10.00000	60
1	6.46373	13.53627	6.46373	13.53627	00000	00000	59
2	76476	23524	76476	23524	00000	00000	58
3	94085	05915	94085	05915	00000	00000	57
4	7.06579	12.93421	7.06579	12.93421	00000	00000	56
5	7.16270	12.83730	7.16270	12.83730	10.00000	10.00000	55
6	24188	75812	24188	75812	00000	00000	54
7	30882	69118	30882	69118	00000	00000	53
8	36682	63318	36682	63318	00000	00000	52
9	41797	58203	41797	58203	00000	00000	51
10	7.46373	12.53627	7.46373	12.53627	10.00000	10.00000	50
11	50512	49488	50512	49488	00000	00000	49
12	54291	45709	54291	45709	00000	00000	48
13	57767	42233	57767	42233	00000	00000	47
14	60985	39015	60986	39014	00000	00000	46
15	7.63982	12.36018	7.63982	12.36018	10.00000	10.00000	45
16	66784	33216	66785	33215	00000	00000	44
17	69417	30583	69418	30582	00001	9.99999	43
18	71900	28100	71900	28100	00001	99999	42
19	74248	25752	74248	25752	00001	99999	41
20	7.76475	12.23525	7.76476	12.23524	10.00001	9.99999	40
21	78594	21406	78595	21405	00001	99999	39
22	80615	19385	80615	19385	00001	99999	38
23	82545	17455	82546	17454	00001	99999	37
24	84393	15607	84394	15606	00001	99999	36
25	7.86166	12.13834	7.86167	12.13833	10.00001	9.99999	35
26	87870	12130	87871	12129	00001	99999	34
27	89509	10491	89510	10490	00001	99999	33
28	91088	08912	91089	08911	00001	99999	32
29	92612	07388	92613	07387	00002	99998	31
30	7.94084	12.05916	7.94086	12.05914	10.00002	9.99998	30
31	95508	04492	95510	04490	00002	99998	29
32	96887	03113	96889	03111	00002	99998	28
33	98223	01777	98225	01775	00002	99998	27
34	99520	00480	99522	00478	00002	99998	26
35	8.00779	11.99221	8.00781	11.99219	10.00002	9.99998	25
36	02002	97998	02004	97996	00002	99998	24
37	03192	96808	03194	96806	00003	99997	23
38	04350	95650	04353	95647	00003	99997	22
39	05478	94522	05481	94519	00003	99997	21
40	8.06578	11.93422	8.06581	11.93419	10.00003	9.99997	20
41	07650	92350	07653	92347	00003	99997	19
42	08696	91304	08700	91300	00003	99997	18
43	09718	90282	09722	90278	00003	99997	17
44	10717	89283	10720	89280	00004	99996	16
45	8.11693	11.88307	8.11696	11.88304	10.00004	9.99996	15
46	12647	87353	12651	87349	00004	99996	14
47	13581	86419	13585	86415	00004	99996	13
48	14495	85505	14500	85500	00004	99996	12
49	15391	84609	15395	84605	00004	99996	11
50	8.16268	11.83732	8.16273	11.83727	10.00005	9.99995	10
51	17128	82872	17133	82867	00005	99995	9
52	17971	82029	17976	82024	00005	99995	8
53	18798	81202	18804	81196	00005	99995	7
54	19610	80390	19616	80384	00005	99995	6
55	8.20407	11.79593	8.20413	11.79587	10.00006	9.99994	5
56	21189	78811	21195	78805	00006	99994	4
57	21958	78042	21964	78036	00006	99994	3
58	22713	77287	22720	77280	00006	99994	2
59	23456	76544	23462	76538	00006	99994	1
60	24186	75814	24192	75808	00007	99993	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

90° **89°**

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1° **Logarithms.** 178°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.24186	11.75814	8.24192	11.75808	10.00007	9.99993	60
1	24903	75097	24910	75090	00007	99993	59
2	25609	74391	25616	74384	00007	99993	58
3	26304	73696	26312	73688	00007	99993	57
4	26988	73012	26996	73004	00008	99992	56
5	8.27661	11.72339	8.27669	11.72331	10.00008	9.99992	55
6	28324	71676	28332	71668	00008	99992	54
7	28977	71023	28986	71014	00008	99992	53
8	29621	70379	29629	70371	00008	99992	52
9	30255	69745	30263	69737	00009	99991	51
10	8.30879	11.69121	8.30888	11.69112	10.00009	9.99991	50
11	31495	68505	31505	68495	00009	99991	49
12	32103	67897	32112	67888	00010	99990	48
13	32702	67298	32711	67289	00010	99990	47
14	33292	66708	33302	66698	00010	99990	46
15	8.33875	11.66125	8.33886	11.66114	10.00010	9.99990	45
16	34450	65550	34461	65539	00011	99989	44
17	35018	64982	35029	64971	00011	99989	43
18	35578	64422	35590	64410	00011	99989	42
19	36131	63869	36143	63857	00011	99989	41
20	8.36678	11.63322	8.36689	11.63311	10.00012	9.99988	40
21	37217	62783	37229	62771	00012	99988	39
22	37750	62250	37762	62238	00012	99988	38
23	38276	61724	38289	61711	00013	99987	37
24	38796	61204	38809	61191	00013	99987	36
25	8.39310	11.60690	8.39323	11.60677	10.00013	9.99987	35
26	39818	60182	39832	60168	00014	99986	34
27	40320	59680	40334	59666	00014	99986	33
28	40816	59184	40830	59170	00014	99986	32
29	41307	58693	41321	58679	00015	99985	31
30	8.41792	11.58208	8.41807	11.58193	10.00015	9.99985	30
31	42272	57728	42287	57713	00015	99985	29
32	42746	57254	42762	57238	00016	99984	28
33	43216	56784	43232	56768	00016	99984	27
34	43680	56320	43696	56304	00016	99984	26
35	8.44139	11.55861	8.44156	11.55844	10.00017	9.99983	25
36	44594	55406	44611	55389	00017	99983	24
37	45044	54956	45061	54939	00017	99983	23
38	45489	54511	45507	54493	00018	99982	22
39	45930	54070	45948	54052	00018	99982	21
40	8.46366	11.53634	8.46385	11.53615	10.00018	9.99982	20
41	46799	53201	46817	53183	00019	99981	19
42	47226	52774	47245	52755	00019	99981	18
43	47650	52350	47669	52331	00019	99981	17
44	48069	51931	48089	51911	00020	99980	16
45	8.48485	11.51515	8.48505	11.51495	10.00020	9.99980	15
46	48896	51104	48917	51083	00021	99979	14
47	49304	50696	49325	50675	00021	99979	13
48	49708	50292	49729	50271	00021	99979	12
49	50108	49892	50130	49870	00022	99978	11
50	8.50504	11.49496	8.50527	11.49473	10.00022	9.99978	10
51	50897	49103	50920	49080	00023	99977	9
52	51287	48713	51310	48690	00023	99977	8
53	51673	48327	51696	48304	00023	99977	7
54	52055	47945	52079	47921	00024	99976	6
55	8.52434	11.47566	8.52459	11.47541	10.00024	9.99976	5
56	52810	47190	52835	47165	00025	99975	4
57	53183	46817	53208	46792	00025	99975	3
58	53552	46448	53578	46422	00026	99974	2
59	53919	46081	53945	46055	00026	99974	1
60	54282	45718	54308	45692	00026	99974	0
M.	**Cosine.**	**Secant.**	**Cotangent.**	**Tangent.**	**Cosecant.**	**Sine.**	**M.**

91° 88°

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2° **Logarithms.** **177°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.54282	11.45718	8.54308	11.45692	10.00026	9.99974	60
1	54642	45358	54669	45331	00027	99973	59
2	54999	45001	55027	44973	00027	99973	58
3	55354	44646	55382	44618	00028	99972	57
4	55705	44295	55734	44266	00028	99972	56
5	8.56054	11.43946	8.56083	11.43917	10.00029	9.99971	55
6	56400	43600	56429	43571	00029	99971	54
7	56743	43257	56773	43227	00030	99970	53
8	57084	42916	57114	42886	00030	99970	52
9	57421	42579	57452	42548	00031	99969	51
10	8.57757	11.42243	8.57788	11.42212	10.00031	9.99969	50
11	58089	41911	58121	41879	00032	99968	49
12	58419	41581	58451	41549	00032	99968	48
13	58747	41253	58779	41221	00033	99967	47
14	59072	40928	59105	40895	00033	99967	46
15	8.59395	11.40605	8.59428	11.40572	10.00033	9.99967	45
16	59715	40285	59749	40251	00034	99966	44
17	60033	39967	60068	39932	00034	99966	43
18	60349	39651	60384	39616	00035	99965	42
19	60662	39338	60698	39302	00036	99964	41
20	8.60973	11.39027	8.61009	11.38991	10.00036	9.99964	40
21	61282	38718	61319	38681	00037	99963	39
22	61589	38411	61626	38374	00037	99963	38
23	61894	38106	61931	38069	00038	99962	37
24	62196	37804	62234	37766	00038	99962	36
25	8.62497	11.37503	8.62535	11.37465	10.00039	9.99961	35
26	62795	37205	62834	37166	00039	99961	34
27	63091	36909	63131	36869	00040	99960	33
28	63385	36615	63426	36574	00040	99960	32
29	63678	36322	63718	36282	00041	99959	31
30	8.63968	11.36032	8.64009	11.35991	10.00041	9.99959	30
31	64256	35744	64298	35702	00042	99958	29
32	64543	35457	64585	35415	00042	99958	28
33	64827	35173	64870	35130	00043	99957	27
34	65110	34890	65154	34846	00044	99956	26
35	8.65391	11.34609	8.65435	11.34565	10.00044	9.99956	25
36	65670	34330	65715	34285	00045	99955	24
37	65947	34053	65993	34007	00045	99955	23
38	66223	33777	66269	33731	00046	99954	22
39	66497	33503	66543	33457	00046	99954	21
40	8.66769	11.33231	8.66816	11.33184	10.00047	9.99953	20
41	67039	32961	67087	32913	00048	99952	19
42	67308	32692	67356	32644	00048	99952	18
43	67575	32425	67624	32376	00049	99951	17
44	67841	32159	67890	32110	00049	99951	16
45	8.68104	11.31896	8.68154	11.31846	10.00050	9.99950	15
46	68367	31633	68417	31583	00051	99949	14
47	68627	31373	68678	31322	00051	99949	13
48	68886	31114	68938	31062	00052	99948	12
49	69144	30856	69196	30804	00052	99948	11
50	8.69400	11.30600	8.69453	11.30547	10.00053	9.99947	10
51	69654	30346	69708	30292	00054	99946	9
52	69907	30093	69962	30038	00054	99946	8
53	70159	29841	70214	29786	00055	99945	7
54	70409	29591	70465	29535	00056	99944	6
55	8.70658	11.29342	8.70714	11.29286	10.00056	9.99944	5
56	70905	29095	70962	29038	00057	99943	4
57	71151	28849	71208	28792	00058	99942	3
58	71395	28605	71453	28547	00058	99942	2
59	71638	28362	71697	28303	00059	99941	1
60	71880	28120	71940	28060	00060	99940	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

92° **87°**

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3°			Logarithms.				176°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.71880	11.28120	8.71940	11.28060	10.00060	9.99940	60
1	72120	27880	72181	27819	00060	99940	59
2	72359	27641	72420	27580	00061	99939	58
3	72597	27403	72659	27341	00062	99938	57
4	72834	27166	72896	27104	00062	99938	56
5	8.73069	11.26931	8.73132	11.26868	10.00063	9.99937	55
6	73303	26697	73366	26634	00064	99936	54
7	73535	26465	73600	26400	00064	99936	53
8	73767	26233	73832	26168	00065	99935	52
9	73997	26003	74063	25937	00066	99934	51
10	8.74226	11.25774	8.74292	11.25708	10.00066	9.99934	50
11	74454	25546	74521	25479	00067	99933	49
12	74680	25320	74748	25252	00068	99932	48
13	74906	25094	74974	25026	00068	99932	47
14	75130	24870	75199	24801	00069	99931	46
15	8.75353	11.24647	8.75423	11.24577	10.00070	9.99930	45
16	75575	24425	75645	24355	00071	99929	44
17	75795	24205	75867	24133	00071	99929	43
18	76015	23985	76087	23913	00072	99928	42
19	76234	23766	76306	23694	00073	99927	41
20	8.76451	11.23549	8.76525	11.23475	10.00074	9.99926	40
21	76667	23333	76742	23258	00074	99926	39
22	76883	23117	76958	23042	00075	99925	38
23	77097	22903	77173	22827	00076	99924	37
24	77310	22690	77387	22613	00077	99923	36
25	8.77522	11.22478	8.77600	11.22400	10.00077	9.99923	35
26	77733	22267	77811	22189	00078	99922	34
27	77943	22057	78022	21978	00079	99921	33
28	78152	21848	78232	21768	00080	99920	32
29	78360	21640	78441	21559	00080	99920	31
30	8.78568	11.21432	8.78649	11.21351	10.00081	9.99919	30
31	78774	21226	78855	21145	00082	99918	29
32	78979	21021	79061	20939	00083	99917	28
33	79183	20817	79266	20734	00083	99917	27
34	79386	20614	79470	20530	00084	99916	26
35	8.79588	11.20412	8.79673	11.20327	10.00085	9.99915	25
36	79789	20211	79875	20125	00086	99914	24
37	79990	20010	80076	19924	00087	99913	23
38	80189	19811	80277	19723	00087	99913	22
39	80388	19612	80476	19524	00088	99912	21
40	8.80585	11.19415	8.80674	11.19326	10.00089	9.99911	20
41	80782	19218	80872	19128	00090	99910	19
42	80978	19022	81068	18932	00091	99909	18
43	81173	18827	81264	18736	00091	99909	17
44	81367	18633	81459	18541	00092	99908	16
45	8.81560	11.18440	8.81653	11.18347	10.00093	9.99907	15
46	81752	18248	81846	18154	00094	99906	14
47	81944	18056	82038	17962	00095	99905	13
48	82134	17866	82230	17770	00096	99904	12
49	82324	17676	82420	17580	00096	99904	11
50	8.82513	11.17487	8.82610	11.17390	10.00097	9.99903	10
51	82701	17299	82799	17201	00098	99902	9
52	82888	17112	82987	17013	00099	99901	8
53	83075	16925	83175	16825	00100	99900	7
54	83261	16739	83361	16639	00101	99899	6
55	8.83446	11.16554	8.83547	11.16453	10.00102	9.99898	5
56	83630	16370	83732	16268	00102	99898	4
57	83813	16187	83916	16084	00103	99897	3
58	83996	16004	84100	15900	00104	99896	2
59	84177	15823	84282	15718	00105	99895	1
60	84358	15642	84464	15536	00106	99894	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
93°							86°

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4° **Logarithms.** 175°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.84358	11.15642	8.84464	11.15536	10.00106	9.99894	60
1	84539	15461	84646	15354	00107	99893	59
2	84718	15282	84826	15174	00108	99892	58
3	84897	15103	85006	14994	00109	99891	57
4	85075	14925	85185	14815	00109	99891	56
5	8.85252	11.14748	8.85363	11.14637	10.00110	9.99890	55
6	85429	14571	85540	14460	00111	99889	54
7	85605	14395	85717	14283	00112	99888	53
8	85780	14220	85893	14107	00113	99887	52
9	85955	14045	86069	13931	00114	99886	51
10	8.86128	11.13872	8.86243	11.13757	10.00115	9.99885	50
11	86301	13699	86417	13583	00116	99884	49
12	86474	13526	86591	13409	00117	99883	48
13	86645	13355	86763	13237	00118	99882	47
14	86816	13184	86935	13065	00119	99881	46
15	8.86987	11.13013	8.87106	11.12894	10.00120	9.99880	45
16	87156	12844	87277	12723	00121	99879	44
17	87325	12675	87447	12553	00121	99879	43
18	87494	12506	87616	12384	00122	99878	42
19	87661	12339	87785	12215	00123	99877	41
20	8.87829	11.12171	8.87953	11.12047	10.00124	9.99876	40
21	87995	12005	88120	11880	00125	99875	39
22	88161	11839	88287	11713	00126	99874	38
23	88326	11674	88453	11547	00127	99873	37
24	88490	11510	88618	11382	00128	99872	36
25	8.88654	11.11346	8.88783	11.11217	10.00129	9.99871	35
26	88817	11183	88948	11052	00130	99870	34
27	88980	11020	89111	10889	00131	99869	33
28	89142	10858	89274	10726	00132	99868	32
29	89304	10696	89437	10563	00133	99867	31
30	8.89464	11.10536	8.89598	11.10402	10.00134	9.99866	30
31	89625	10375	89760	10240	00135	99865	29
32	89784	10216	89920	10080	00136	99864	28
33	89943	10057	90080	09920	00137	99863	27
34	90102	09898	90240	09760	00138	99862	26
35	8.90260	11.09740	8.90399	11.09601	10.00139	9.99861	25
36	90417	09583	90557	09443	00140	99860	24
37	90574	09426	90715	09285	00141	99859	23
38	90730	09270	90872	09128	00142	99858	22
39	90885	09115	91029	08971	00143	99857	21
40	8.91040	11.08960	8.91185	11.08815	10.00144	9.99856	20
41	91195	08805	91340	08660	00145	99855	19
42	91349	08651	91495	08505	00146	99854	18
43	91502	08498	91650	08350	00147	99853	17
44	91655	08345	91803	08197	00148	99852	16
45	8.91807	11.08193	8.91957	11.08043	10.00149	9.99851	15
46	91959	08041	92110	07890	00150	99850	14
47	92110	07890	92262	07738	00152	99848	13
48	92261	07739	92414	07586	00153	99847	12
49	92411	07589	92565	07435	00154	99846	11
50	8.92561	11.07439	8.92716	11.07284	10.00155	9.99845	10
51	92710	07290	92866	07134	00156	99844	9
52	92859	07141	93016	06984	00157	99843	8
53	93007	06993	93165	06835	00158	99842	7
54	93154	06846	93313	06687	00159	99841	6
55	8.93301	11.06699	8.93462	11.06538	10.00160	9.99840	5
56	93448	06552	93609	06391	00161	99839	4
57	93594	06406	93756	06244	00162	99838	3
58	93740	06260	93903	06097	00163	99837	2
59	93885	06115	94049	05951	00164	99836	1
60	94030	05970	94195	05805	00166	99834	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

94° 85°

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5° **Logarithms.** 174°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.94030	11.05970	8.94195	11.05805	10.00166	9.99834	60
1	94174	05826	94340	05660	00167	99833	59
2	94317	05683	94485	05515	00168	99832	58
3	94461	05539	94630	05370	00169	99831	57
4	94603	05397	94773	05227	00170	99830	56
5	8.94746	11.05254	8.94917	11.05083	10.00171	9.99829	55
6	94887	05113	95060	04940	00172	99828	54
7	95029	04971	95202	04798	00173	99827	53
8	95170	04830	95344	04656	00175	99825	52
9	95310	04690	95486	04514	00176	99824	51
10	8.95450	11.04550	8.95627	11.04373	10.00177	9.99823	50
11	95589	04411	95767	04233	00178	99822	49
12	95728	04272	95908	04092	00179	99821	48
13	95867	04133	96047	03953	00180	99820	47
14	96005	03995	96187	03813	00181	99819	46
15	8.96143	11.03857	8.96325	11.03675	10.00183	9.99817	45
16	96280	03720	96464	03536	00184	99816	44
17	96417	03583	96602	03398	00185	99815	43
18	96553	03447	96739	03261	00186	99814	42
19	96689	03311	96877	03123	00187	99813	41
20	8.96825	11.03175	8.97013	11.02987	10.00188	9.99812	40
21	96960	03040	97150	02850	00190	99810	39
22	97095	02905	97285	02715	00191	99809	38
23	97229	02771	97421	02579	00192	99808	37
24	97363	02637	97556	02444	00193	99807	36
25	8.97496	11.02504	8.97691	11.02309	10.00194	9.99806	35
26	97629	02371	97825	02175	00196	99804	34
27	97762	02238	97959	02041	00197	99803	33
28	97894	02106	98092	01908	00198	99802	32
29	98026	01974	98225	01775	00199	99801	31
30	8.98157	11.01843	8.98358	11.01642	10.00200	9.99800	30
31	98288	01712	98490	01510	00202	99798	29
32	98419	01581	98622	01378	00203	99797	28
33	98549	01451	98753	01247	00204	99796	27
34	98679	01321	98884	01116	00205	99795	26
35	8.98808	11.01192	8.99015	11.00985	10.00207	9.99793	25
36	98937	01063	99145	00855	00208	99792	24
37	99066	00934	99275	00725	00209	99791	23
38	99194	00806	99405	00595	00210	99790	22
39	99322	00678	99534	00466	00212	99788	21
40	8.99450	11.00550	8.99662	11.00338	10.00213	9.99787	20
41	99577	00423	99791	00209	00214	99786	19
42	99704	00296	99919	00081	00215	99785	18
43	99830	00170	9.00046	10.99954	00217	99783	17
44	99956	00044	00174	99826	00218	99782	16
45	9.00082	10.99918	9.00301	10.99699	10.00219	9.99781	15
46	00207	99793	00427	99573	00220	99780	14
47	00332	99668	00553	99447	00222	99778	13
48	00456	99544	00679	99321	00223	99777	12
49	00581	99419	00805	99195	00224	99776	11
50	9.00704	10.99296	9.00930	10.99070	10.00225	9.99775	10
51	00828	99172	01055	98945	00227	99773	9
52	00951	99049	01179	98821	00228	99772	8
53	01074	98926	01303	98697	00229	99771	7
54	01196	98804	01427	98573	00231	99769	6
55	9.01318	10.98682	9.01550	10.98450	10.00232	9.99768	5
56	01440	98560	01673	98327	00233	99767	4
57	01561	98439	01796	98204	00235	99765	3
58	01682	98318	01918	98082	00236	99764	2
59	01803	98197	02040	97960	00237	99763	1
60	01923	98077	02162	97838	00239	99761	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

95° 84°

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6° **Logarithms.** 173°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.01923	10.98077	9.02162	10.97838	10.00239	9.99761	60
1	02043	97957	02283	97717	00240	99760	59
2	02163	97837	02404	97596	00241	99759	58
3	02283	97717	02525	97475	00243	99757	57
4	02402	97598	02645	97355	00244	99756	56
5	9.02520	10.97480	9.02766	10.97234	10.00245	9.99755	55
6	02639	97361	02885	97115	00247	99753	54
7	02757	97243	03005	96995	00248	99752	53
8	02874	97126	03124	96876	00249	99751	52
9	02992	97008	03242	96758	00251	99749	51
10	9.03109	10.96891	9.03361	10.96639	10.00252	9.99748	50
11	03226	96774	03479	96521	00253	99747	49
12	03342	96658	03597	96403	00255	99745	48
13	03458	96542	03714	96286	00256	99744	47
14	03574	96426	03832	96168	00258	99742	46
15	9.03690	10.96310	9.03948	10.96052	10.00259	9.99741	45
16	03805	96195	04065	95935	00260	99740	44
17	03920	96080	04181	95819	00262	99738	43
18	04034	95966	04297	95703	00263	99737	42
19	04149	95851	04413	95587	00264	99736	41
20	9.04262	10.95738	9.04528	10.95472	10.00266	9.99734	40
21	04376	95624	04643	95357	00267	99733	39
22	04490	95510	04758	95242	00269	99731	38
23	04603	95397	04873	95127	00270	99730	37
24	04715	95285	04987	95013	00272	99728	36
25	9.04828	10.95172	9.05101	10.94899	10.00273	9.99727	35
26	04940	95060	05214	94786	00274	99726	34
27	05052	94948	05328	94672	00276	99724	33
28	05164	94836	05441	94559	00277	99723	32
29	05275	94725	05553	94447	00279	99721	31
30	9.05386	10.94614	9.05666	10.94334	10.00280	9.99720	30
31	05497	94503	05778	94222	00282	99718	29
32	05607	94393	05890	94110	00283	99717	28
33	05717	94283	06002	93998	00284	99716	27
34	05827	94173	06113	93887	00286	99714	26
35	9.05937	10.94063	9.06224	10.93776	10.00287	9.99713	25
36	06046	93954	06335	93665	00289	99711	24
37	06155	93845	06445	93555	00290	99710	23
38	06264	93736	06556	93444	00292	99708	22
39	06372	93628	06666	93334	00293	99707	21
40	9.06481	10.93519	9.06775	10.93225	10.00295	9.99705	20
41	06589	93411	06885	93115	00296	99704	19
42	06696	93304	06994	93006	00298	99702	18
43	06804	93196	07103	92897	00299	99701	17
44	06911	93089	07211	92789	00301	99699	16
45	9.07018	10.92982	9.07320	10.92680	10.00302	9.99698	15
46	07124	92876	07428	92572	00304	99696	14
47	07231	92769	07536	92464	00305	99695	13
48	07337	92663	07643	92357	00307	99693	12
49	07442	92558	07751	92249	00308	99692	11
50	9.07548	10.92452	9.07858	10.92142	10.00310	9.99690	10
51	07653	92347	07964	92036	00311	99689	9
52	07758	92242	08071	91929	00313	99687	8
53	07863	92137	08177	91823	00314	99686	7
54	07968	92032	08283	91717	00316	99684	6
55	9.08072	10.91928	9.08389	10.91611	10.00317	9.99683	5
56	08176	91824	08495	91505	00319	99681	4
57	08280	91720	08600	91400	00320	99680	3
58	08383	91617	08705	91295	00322	99678	2
59	08486	91514	08810	91190	00323	99677	1
60	08589	91411	08914	91086	00325	99675	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

96° 83°

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7° **Logarithms.** 172°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.08589	10.91411	9.08914	10.91086	10.00325	9.99675	60
1	08692	91308	09019	90981	00326	99674	59
2	08795	91205	09123	90877	00328	99672	58
3	08897	91103	09227	90773	00330	99670	57
4	08999	91001	09330	90670	00331	99669	56
5	9.09101	10.90899	9.09434	10.90566	10.00333	9.99667	55
6	09202	90798	09537	90463	00334	99666	54
7	09304	90696	09640	90360	00336	99664	53
8	09405	90595	09742	90258	00337	99663	52
9	09506	90494	09845	90155	00339	99661	51
10	9.09606	10.90394	9.09947	10.90053	10.00341	9.99659	50
11	09707	90293	10049	89951	00342	99658	49
12	09807	90193	10150	89850	00344	99656	48
13	09907	90093	10252	89748	00345	99655	47
14	10006	89994	10353	89647	00347	99653	46
15	9.10106	10.89894	9.10454	10.89546	10.00349	9.99651	45
16	10205	89795	10555	89445	00350	99650	44
17	10304	89696	10656	89344	00352	99648	43
18	10402	89598	10756	89244	00353	99647	42
19	10501	89499	10856	89144	00355	99645	41
20	9.10599	10.89401	9.10956	10.89044	10.00357	9.99643	40
21	10697	89303	11056	88944	00358	99642	39
22	10795	89205	11155	88845	00360	99640	38
23	10893	89107	11254	88746	00362	99638	37
24	10990	89010	11353	88647	00363	99637	36
25	9.11087	10.88913	9.11452	10.88548	10.00365	9.99635	35
26	11184	88816	11551	88449	00367	99633	34
27	11281	88719	11649	88351	00368	99632	33
28	11377	88623	11747	88253	00370	99630	32
29	11474	88526	11845	88155	00371	99629	31
30	9.11570	10.88430	9.11943	10.88057	10.00373	9.99627	30
31	11666	88334	12040	87960	00375	99625	29
32	11761	88239	12138	87862	00376	99624	28
33	11857	88143	12235	87765	00378	99622	27
34	11952	88048	12332	87668	00380	99620	26
35	9.12047	10.87953	9.12428	10.87572	10.00382	9.99618	25
36	12142	87858	12525	87475	00383	99617	24
37	12236	87764	12621	87379	00385	99615	23
38	12331	87669	12717	87283	00387	99613	22
39	12425	87575	12813	87187	00388	99612	21
40	9.12519	10.87481	9.12909	10.87091	10.00390	9.99610	20
41	12612	87388	13004	86996	00392	99608	19
42	12706	87294	13099	86901	00393	99607	18
43	12799	87201	13194	86806	00395	99605	17
44	12892	87108	13289	86711	00397	99603	16
45	9.12985	10.87015	9.13384	10.86616	10.00399	9.99601	15
46	13078	86922	13478	86522	00400	99600	14
47	13171	86829	13573	86427	00402	99598	13
48	13263	86737	13667	86333	00404	99596	12
49	13355	86645	13761	86239	00405	99595	11
50	9.13447	10.86553	9.13854	10.86146	10.00407	9.99593	10
51	13539	86461	13948	86052	00409	99591	9
52	13630	86370	14041	85959	00411	99589	8
53	13722	86278	14134	85866	00412	99588	7
54	13813	86187	14227	85773	00414	99586	6
55	9.13904	10.86096	9.14320	10.85680	10.00416	9.99584	5
56	13994	86006	14412	85588	00418	99582	4
57	14085	85915	14504	85496	00419	99581	3
58	14175	85825	14597	85403	00421	99579	2
59	14266	85734	14688	85312	00423	99577	1
60	14356	85644	14780	85220	00425	99575	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

97° 82°

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8° **Logarithms.** **171°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.14356	10.85644	9.14780	10.85220	10.00425	9.99575	60
1	14445	85555	14872	85128	00426	99574	59
2	14535	85465	14963	85037	00428	99572	58
3	14624	85376	15054	84946	00430	99570	57
4	14714	85286	15145	84855	00432	99568	56
5	9.14803	10.85197	9.15236	10.84764	10.00434	9.99566	55
6	14891	85109	15327	84673	00435	99565	54
7	14980	85020	15417	84583	00437	99563	53
8	15069	84931	15508	84492	00439	99561	52
9	15157	84843	15598	84402	00441	99559	51
10	9.15245	10.84755	9.15688	10.84312	10.00443	9.99557	50
11	15333	84667	15777	84223	00444	99556	49
12	15421	84579	15867	84133	00446	99554	48
13	15508	84492	15956	84044	00448	99552	47
14	15596	84404	16046	83954	00450	99550	46
15	9.15683	10.84317	9.16135	10.83865	10.00452	9.99548	45
16	15770	84230	16224	83776	00454	99546	44
17	15857	84143	16312	83688	00455	99545	43
18	15944	84056	16401	83599	00457	99543	42
19	16030	83970	16489	83511	00459	99541	41
20	9.16116	10.83884	9.16577	10.83423	10.00461	9.99539	40
21	16203	83797	16665	83335	00463	99537	39
22	16289	83711	16753	83247	00465	99535	38
23	16374	83626	16841	83159	00467	99533	37
24	16460	83540	16928	83072	00468	99532	36
25	9.16545	10.83455	9.17016	10.82984	10.00470	9.99530	35
26	16631	83369	17103	82897	00472	99528	34
27	16716	83284	17190	82810	00474	99526	33
28	16801	83199	17277	82723	00476	99524	32
29	16886	83114	17363	82637	00478	99522	31
30	9.16970	10.83030	9.17450	10.82550	10.00480	9.99520	30
31	17055	82945	17536	82464	00482	99518	29
32	17139	82861	17622	82378	00483	99517	28
33	17223	82777	17708	82292	00485	99515	27
34	17307	82693	17794	82206	00487	99513	26
35	9.17391	10.82609	9.17880	10.82120	10.00489	9.99511	25
36	17474	82526	17965	82035	00491	99509	24
37	17558	82442	18051	81949	00493	99507	23
38	17641	82359	18136	81864	00495	99505	22
39	17724	82276	18221	81779	00497	99503	21
40	9.17807	10.82193	9.18306	10.81694	10.00499	9.99501	20
41	17890	82110	18391	81609	00501	99499	19
42	17973	82027	18475	81525	00503	99497	18
43	18055	81945	18560	81440	00505	99495	17
44	18137	81863	18644	81356	00506	99494	16
45	9.18220	10.81780	9.18728	10.81272	10.00508	9.99492	15
46	18302	81698	18812	81188	00510	99490	14
47	18383	81617	18896	81104	00512	99488	13
48	18465	81535	18979	81021	00514	99486	12
49	18547	81453	19063	80937	00516	99484	11
50	9.18628	10.81372	9.19146	10.80854	10.00518	9.99482	10
51	18709	81291	19229	80771	00520	99480	9
52	18790	81210	19312	80688	00522	99478	8
53	18871	81129	19395	80605	00524	99476	7
54	18952	81048	19478	80522	00526	99474	6
55	9.19033	10.80967	9.19561	10.80439	10.00528	9.99472	5
56	19113	80887	19643	80357	00530	99470	4
57	19193	80807	19725	80275	00532	99468	3
58	19273	80727	19807	80193	00534	99466	2
59	19353	80647	19889	80111	00536	99464	1
60	19433	80567	19971	80029	00538	99462	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

98° **81°**

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9° **Logarithms.** 170°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.19433	10.80567	9.19971	10.80029	10.00538	9.99462	60
1	19513	80487	20053	79947	00540	99460	59
2	19592	80408	20134	79866	00542	99458	58
3	19672	80328	20216	79784	00544	99456	57
4	19751	80249	20297	79703	00546	99454	56
5	9.19830	10.80170	9.20378	10.79622	10.00548	9.99452	55
6	19909	80091	20459	79541	00550	99450	54
7	19988	80012	20540	79460	00552	99448	53
8	20067	79933	20621	79379	00554	99446	52
9	20145	79855	20701	79299	00556	99444	51
10	9.20223	10.79777	9.20782	10.79218	10.00558	9.99442	50
11	20302	79698	20862	79138	00560	99440	49
12	20380	79620	20942	79058	00562	99438	48
13	20458	79542	21022	78978	00564	99436	47
14	20535	79465	21102	78898	00566	99434	46
15	9.20613	10.79387	9.21182	10.78818	10.00568	9.99432	45
16	20691	79309	21261	78739	00571	99429	44
17	20768	79232	21341	78659	00573	99427	43
18	20845	79155	21420	78580	00575	99425	42
19	20922	79078	21499	78501	00577	99423	41
20	9.20999	10.79001	9.21578	10.78422	10.00579	9.99421	40
21	21076	78924	21657	78343	00581	99419	39
22	21153	78847	21736	78264	00583	99417	38
23	21229	78771	21814	78186	00585	99415	37
24	21306	78694	21893	78107	00587	99413	36
25	9.21382	10.78618	9.21971	10.78029	10.00589	9.99411	35
26	21458	78542	22049	77951	00591	99409	34
27	21534	78466	22127	77873	00593	99407	33
28	21610	78390	22205	77795	00596	99404	32
29	21685	78315	22283	77717	00598	99402	31
30	9.21761	10.78239	9.22361	10.77639	10.00600	9.99400	30
31	21836	78164	22438	77562	00602	99398	29
32	21912	78088	22516	77484	00604	99396	28
33	21987	78013	22593	77407	00606	99394	27
34	22062	77938	22670	77330	00608	99392	26
35	9.22137	10.77863	9.22747	10.77253	10.00610	9.99390	25
36	22211	77789	22824	77176	00612	99388	24
37	22286	77714	22901	77099	00615	99385	23
38	22361	77639	22977	77023	00617	99383	22
39	22435	77565	23054	76946	00619	99381	21
40	9.22509	10.77491	9.23130	10.76870	10.00621	9.99379	20
41	22583	77417	23206	76794	00623	99377	19
42	22657	77343	23283	76717	00625	99375	18
43	22731	77269	23359	76641	00628	99372	17
44	22805	77195	23435	76565	00630	99370	16
45	9.22878	10.77122	9.23510	10.76490	10.00632	9.99368	15
46	22952	77048	23586	76414	00634	99366	14
47	23025	76975	23661	76339	00636	99364	13
48	23098	76902	23737	76263	00638	99362	12
49	23171	76829	23812	76188	00641	99359	11
50	9.23244	10.76756	9.23887	10.76113	10.00643	9.99357	10
51	23317	76683	23962	76038	00645	99355	9
52	23390	76610	24037	75963	00647	99353	8
53	23462	76538	24112	75888	00649	99351	7
54	23535	76465	24186	75814	00652	99348	6
55	9.23607	10.76393	9.24261	10.75739	10.00654	9.99346	5
56	23679	76321	24335	75665	00656	99344	4
57	23752	76248	24410	75590	00658	99342	3
58	23823	76177	24484	75516	00660	99340	2
59	23895	76105	24558	75442	00663	99337	1
60	23967	76033	24632	75368	00665	99335	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

99° 80°

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10° **Logarithms.** **169°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.23967	10.76033	9.24632	10.75368	10.00665	9.99335	60
1	24039	75961	24706	75294	00667	99333	59
2	24110	75890	24779	75221	00669	99331	58
3	24181	75819	24853	75147	00672	99328	57
4	24253	75747	24926	75074	00674	99326	56
5	9.24324	10.75676	9.25000	10.75000	10.00676	9.99324	55
6	24395	75605	25073	74927	00678	99322	54
7	24466	75534	25146	74854	00681	99319	53
8	24536	75464	25219	74781	00683	99317	52
9	24607	75393	25292	74708	00685	99315	51
10	9.24677	10.75323	9.25365	10.74635	10.00687	9.99313	50
11	24748	75252	25437	74563	00690	99310	49
12	24818	75182	25510	74490	00692	99308	48
13	24888	75112	25582	74418	00694	99306	47
14	24958	75042	25655	74345	00696	99304	46
15	9.25028	10.74972	9.25727	10.74273	10.00699	9.99301	45
16	25098	74902	25799	74201	00701	99299	44
17	25168	74832	25871	74129	00703	99297	43
18	25237	74763	25943	74057	00706	99294	42
19	25307	74693	26015	73985	00708	99292	41
20	9.25376	10.74624	9.26086	10.73914	10.00710	9.99290	40
21	25445	74555	26158	73842	00712	99288	39
22	25514	74486	26229	73771	00715	99285	38
23	25583	74417	26301	73699	00717	99283	37
24	25652	74348	26372	73628	00719	99281	36
25	9.25721	10.74279	9.26443	10.73557	10.00722	9.99278	35
26	25790	74210	26514	73486	00724	99276	34
27	25858	74142	26585	73415	00726	99274	33
28	25927	74073	26655	73345	00729	99271	32
29	25995	74005	26726	73274	00731	99269	31
30	9.26063	10.73937	9.26797	10.73203	10.00733	9.99267	30
31	26131	73869	26867	73133	00736	99264	29
32	26199	73801	26937	73063	00738	99262	28
33	26267	73733	27008	72992	00740	99260	27
34	26335	73665	27078	72922	00743	99257	26
35	9.26403	10.73597	9.27148	10.72852	10.00745	9.99255	25
36	26470	73530	27218	72782	00748	99252	24
37	26538	73462	27288	72712	00750	99250	23
38	26605	73395	27357	72643	00752	99248	22
39	26672	73328	27427	72573	00755	99245	21
40	9.26739	10.73261	9.27496	10.72504	10.00757	9.99243	20
41	26806	73194	27566	72434	00759	99241	19
42	26873	73127	27635	72365	00762	99238	18
43	26940	73060	27704	72296	00764	99236	17
44	27007	72993	27773	72227	00767	99233	16
45	9.27073	10.72927	9.27842	10.72158	10.00769	9.99231	15
46	27140	72860	27911	72089	00771	99229	14
47	27206	72794	27980	72020	00774	99226	13
48	27273	72727	28049	71951	00776	99224	12
49	27339	72661	28117	71883	00779	99221	11
50	9.27405	10.72595	9.28186	10.71814	10.00781	9.99219	10
51	27471	72529	28254	71746	00783	99217	9
52	27537	72463	28323	71677	00786	99214	8
53	27602	72398	28391	71609	00788	99212	7
54	27668	72332	28459	71541	00791	99209	6
55	9.27734	10.72266	9.28527	10.71473	10.00793	9.99207	5
56	27799	72201	28595	71405	00796	99204	4
57	27864	72136	28662	71338	00798	99202	3
58	27930	72070	28730	71270	00800	99200	2
59	27995	72005	28798	71202	00803	99197	1
60	28060	71940	28865	71135	00805	99195	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

100° **79°**

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11° **Logarithms.** 168°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.28060	10.71940	9.28865	10.71135	10.00805	9.99195	60
1	28125	71875	28933	71067	00808	99192	59
2	28190	71810	29000	71000	00810	99190	58
3	28254	71746	29067	70933	00813	99187	57
4	28319	71681	29134	70866	00815	99185	56
5	9.28384	10.71616	9.29201	10.70799	10.00818	9.99182	55
6	28448	71552	29268	70732	00820	99180	54
7	28512	71488	29335	70665	00823	99177	53
8	28577	71423	29402	70598	00825	99175	52
9	28641	71359	29468	70532	00828	99172	51
10	9.28705	10.71295	9.29535	10.70465	10.00830	9.99170	50
11	28769	71231	29601	70399	00833	99167	49
12	28833	71167	29668	70332	00835	99165	48
13	28896	71104	29734	70266	00838	99162	47
14	28960	71040	29800	70200	00840	99160	46
15	9.29024	10.70976	9.29866	10.70134	10.00843	9.99157	45
16	29087	70913	29932	70068	00845	99155	44
17	29150	70850	29998	70002	00848	99152	43
18	29214	70786	30064	69936	00850	99150	42
19	29277	70723	30130	69870	00853	99147	41
20	9.29340	10.70660	9.30195	10.69805	10.00855	9.99145	40
21	29403	70597	30261	69739	00858	99142	39
22	29466	70534	30326	69674	00860	99140	38
23	29529	70471	30391	69609	00863	99137	37
24	29591	70409	30457	69543	00865	99135	36
25	9.29654	10.70346	9.30522	10.69478	10.00868	9.99132	35
26	29716	70284	30587	69413	00870	99130	34
27	29779	70221	30652	69348	00873	99127	33
28	29841	70159	30717	69283	00876	99124	32
29	29903	70097	30782	69218	00878	99122	31
30	9.29966	10.70034	9.30846	10.69154	10.00881	9.99119	30
31	30028	69972	30911	69089	00883	99117	29
32	30090	69910	30975	69025	00886	99114	28
33	30151	69849	31040	68960	00888	99112	27
34	30213	69787	31104	68896	00891	99109	26
35	9.30275	10.69725	9.31168	10.68832	10.00894	9.99106	25
36	30336	69664	31233	68767	00896	99104	24
37	30398	69602	31297	68703	00899	99101	23
38	30459	69541	31361	68639	00901	99099	22
39	30521	69479	31425	68575	00904	99096	21
40	9.30582	10.69418	9.31489	10.68511	10.00907	9.99093	20
41	30643	69357	31552	68448	00909	99091	19
42	30704	69296	31616	68384	00912	99088	18
43	30765	69235	31679	68321	00914	99086	17
44	30826	69174	31743	68257	00917	99083	16
45	9.30887	10.69113	9.31806	10.68194	10.00920	9.99080	15
46	30947	69053	31870	68130	00922	99078	14
47	31008	68992	31933	68067	00925	99075	13
48	31068	68932	31996	68004	00928	99072	12
49	31129	68871	32059	67941	00930	99070	11
50	9.31189	10.68811	9.32122	10.67878	10.00933	9.99067	10
51	31250	68750	32185	67815	00936	99064	9
52	31310	68690	32248	67752	00938	99062	8
53	31370	68630	32311	67689	00941	99059	7
54	31430	68570	32373	67627	00944	99056	6
55	9.31490	10.68510	9.32436	10.67564	10.00946	9.99054	5
56	31549	68451	32498	67502	00949	99051	4
57	31609	68391	32561	67439	00952	99048	3
58	31669	68331	32623	67377	00954	99046	2
59	31728	68272	32685	67315	00957	99043	1
60	31788	68212	32747	67253	00960	99040	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

101° 78°

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12° **Logarithms.** 167°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.31788	10.68212	9.32747	10.67253	10.00960	9.99040	60
1	31847	68153	32810	67190	00962	99038	59
2	31907	68093	32872	67128	00965	99035	58
3	31966	68034	32933	67067	00968	99032	57
4	32025	67975	32995	67005	00970	99030	56
5	9.32084	10.67916	9.33057	10.66943	10.00973	9.99027	55
6	32143	67857	33119	66881	00976	99024	54
7	32202	67798	33180	66820	00978	99022	53
8	32261	67739	33242	66758	00981	99019	52
9	32319	67681	33303	66697	00984	99016	51
10	9.32378	10.67622	9.33365	10.66635	10.00987	9.99013	50
11	32437	67563	33426	66574	00989	99011	49
12	32495	67505	33487	66513	00992	99008	48
13	32553	67447	33548	66452	00995	99005	47
14	32612	67388	33609	66391	00998	99002	46
15	9.32670	10.67330	9.33670	10.66330	10.01000	9.99000	45
16	32728	67272	33731	66269	01003	98997	44
17	32786	67214	33792	66208	01006	98994	43
18	32844	67156	33853	66147	01009	98991	42
19	32902	67098	33913	66087	01011	98989	41
20	9.32960	10.67040	9.33974	10.66026	10.01014	9.98986	40
21	33018	66982	34034	65966	01017	98983	39
22	33075	66925	34095	65905	01020	98980	38
23	33133	66867	34155	65845	01022	98978	37
24	33190	66810	34215	65785	01025	98975	36
25	9.33248	10.66752	9.34276	10.65724	10.01028	9.98972	35
26	33305	66695	34336	65664	01031	98969	34
27	33362	66638	34396	65604	01033	98967	33
28	33420	66580	34456	65544	01036	98964	32
29	33477	66523	34516	65484	01039	98961	31
30	9.33534	10.66466	9.34576	10.65424	10.01042	9.98958	30
31	33591	66409	34635	65365	01045	98955	29
32	33647	66353	34695	65305	01047	98953	28
33	33704	66296	34755	65245	01050	98950	27
34	33761	66239	34814	65186	01053	98947	26
35	9.33818	10.66182	9.34874	10.65126	10.01056	9.98944	25
36	33874	66126	34933	65067	01059	98941	24
37	33931	66069	34992	65008	01062	98938	23
38	33987	66013	35051	64949	01064	98936	22
39	34043	65957	35111	64889	01067	98933	21
40	9.34100	10.65900	9.35170	10.64830	10.01070	9.98930	20
41	34156	65844	35229	64771	01073	98927	19
42	34212	65788	35288	64712	01076	98924	18
43	34268	65732	35347	64653	01079	98921	17
44	34324	65676	35405	64595	01081	98919	16
45	9.34380	10.65620	9.35464	10.64536	10.01084	9.98916	15
46	34436	65564	35523	64477	01087	98913	14
47	34491	65509	35581	64419	01090	98910	13
48	34547	65453	35640	64360	01093	98907	12
49	34602	65398	35698	64302	01096	98904	11
50	9.34658	10.65342	9.35757	10.64243	10.01099	9.98901	10
51	34713	65287	35815	64185	01102	98898	9
52	34769	65231	35873	64127	01104	98896	8
53	34824	65176	35931	64069	01107	98893	7
54	34879	65121	35989	64011	01110	98890	6
55	9.34934	10.65066	9.36047	10.63953	10.01113	9.98887	5
56	34989	65011	36105	63895	01116	98884	4
57	35044	64956	36163	63837	01119	98881	3
58	35099	64901	36221	63779	01122	98878	2
59	35154	64846	36279	63721	01125	98875	1
60	35209	64791	36336	63664	01128	98872	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

102° 77°

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13° **Logarithms.** **166°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.35209	10.64791	9.36336	10.63664	10.01128	9.98872	60
1	35263	64737	36394	63606	01131	98869	59
2	35318	64682	36452	63548	01133	98867	58
3	35373	64627	36509	63491	01136	98864	57
4	35427	64573	36566	63434	01139	98861	56
5	9.35481	10.64519	9.36624	10.63376	10.01142	9.98858	55
6	35536	64464	36681	63319	01145	98855	54
7	35590	64410	36738	63262	01148	98852	53
8	35644	64356	36795	63205	01151	98849	52
9	35698	64302	36852	63148	01154	98846	51
10	9.35752	10.64248	9.36909	10.63091	10.01157	9.98843	50
11	35806	64194	36966	63034	01160	98840	49
12	35860	64140	37023	62977	01163	98837	48
13	35914	64086	37080	62920	01166	98834	47
14	35968	64032	37137	62863	01169	98831	46
15	9.36022	10.63978	9.37193	10.62807	10.01172	9.98828	45
16	36075	63925	37250	62750	01175	98825	44
17	36129	63871	37306	62694	01178	98822	43
18	36182	63818	37363	62637	01181	98819	42
19	36236	63764	37419	62581	01184	98816	41
20	9.36289	10.63711	9.37476	10.62524	10.01187	9.98813	40
21	36342	63658	37532	62468	01190	98810	39
22	36395	63605	37588	62412	01193	98807	38
23	36449	63551	37644	62356	01196	98804	37
24	36502	63498	37700	62300	01199	98801	36
25	9.36555	10.63445	9.37756	10.62244	10.01202	9.98798	35
26	36608	63392	37812	62188	01205	98795	34
27	36660	63340	37868	62132	01208	98792	33
28	36713	63287	37924	62076	01211	98789	32
29	36766	63234	37980	62020	01214	98786	31
30	9.36819	10.63181	9.38035	10.61965	10.01217	9.98783	30
31	36871	63129	38091	61909	01220	98780	29
32	36924	63076	38147	61853	01223	98777	28
33	36976	63024	38202	61798	01226	98774	27
34	37028	62972	38257	61743	01229	98771	26
35	9.37081	10.62919	9.38313	10.61687	10.01232	9.98768	25
36	37133	62867	38368	61632	01235	98765	24
37	37185	62815	38423	61577	01238	98762	23
38	37237	62763	38479	61521	01241	98759	22
39	37289	62711	38534	61466	01244	98756	21
40	9.37341	10.62659	9.38589	10.61411	10.01247	9.98753	20
41	37393	62607	38644	61356	01250	98750	19
42	37445	62555	38699	61301	01254	98746	18
43	37497	62503	38754	61246	01257	98743	17
44	37549	62451	38808	61192	01260	98740	16
45	9.37600	10.62400	9.38863	10.61137	10.01263	9.98737	15
46	37652	62348	38918	61082	01266	98734	14
47	37703	62297	38972	61028	01269	98731	13
48	37755	62245	39027	60973	01272	98728	12
49	37806	62194	39082	60918	01275	98725	11
50	9.37858	10.62142	9.39136	10.60864	10.01278	9.98722	10
51	37909	62091	39190	60810	01281	98719	9
52	37960	62040	39245	60755	01285	98715	8
53	38011	61989	39299	60701	01288	98712	7
51	38062	61938	39353	60647	01291	98709	6
55	9.38113	10.61887	9.39407	10.60593	10.01294	9.98706	5
56	38164	61836	39461	60539	01297	98703	4
57	38215	61785	39515	60485	01300	98700	3
58	38266	61734	39569	60431	01303	98697	2
59	38317	61683	39623	60377	01306	98694	1
60	38368	61632	39677	60323	01310	98690	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

103° **76°**

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14° **Logarithms.** **165°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.38368	10.61632	9.39677	10.60323	10.01310	9.98690	60
1	38418	61582	39731	60269	01313	98687	59
2	38469	61531	39785	60215	01316	98684	58
3	38519	61481	39838	60162	01319	98681	57
4	38570	61430	39892	60108	01322	98678	56
5	9.38620	10.61380	9.39945	10.60055	10.01325	9.98675	55
6	38670	61330	39999	60001	01329	98671	54
7	38721	61279	40052	59948	01332	98668	53
8	38771	61229	40106	59894	01335	98665	52
9	38821	61179	40159	59841	01338	98662	51
10	9.38871	10.61129	9.40212	10.59788	10.01341	9.98659	50
11	38921	61079	40266	59734	01344	98656	49
12	38971	61029	40319	59681	01348	98652	48
13	39021	60979	40372	59628	01351	98649	47
14	39071	60929	40425	59575	01354	98646	46
15	9.39121	10.60879	9.40478	10.59522	10.01357	9.98643	45
16	39170	60830	40531	59469	01360	98640	44
17	39220	60780	40584	59416	01364	98636	43
18	39270	60730	40636	59364	01367	98633	42
19	39319	60681	40689	59311	01370	98630	41
20	9.39369	10.60631	9.40742	10.59258	10.01373	9.98627	40
21	39418	60582	40795	59205	01377	98623	39
22	39467	60533	40847	59153	01380	98620	38
23	39517	60483	40900	59100	01383	98617	37
24	39566	60434	40952	59048	01386	98614	36
25	9.39615	10.60385	9.41005	10.58995	10.01390	9.98610	35
26	39664	60336	41057	58943	01393	98607	34
27	39713	60287	41109	58891	01396	98604	33
28	39762	60238	41161	58839	01399	98601	32
29	39811	60189	41214	58786	01403	98597	31
30	9.39860	10.60140	9.41266	10.58734	10.01406	9.98594	30
31	39909	60091	41318	58682	01409	98591	29
32	39958	60042	41370	58630	01412	98588	28
33	40006	59994	41422	58578	01416	98584	27
34	40055	59945	41474	58526	01419	98581	26
35	9.40103	10.59897	9.41526	10.58474	10.01422	9.98578	25
36	40152	59848	41578	58422	01426	98574	24
37	40200	59800	41629	58371	01429	98571	23
38	40249	59751	41681	58319	01432	98568	22
39	40297	59703	41733	58267	01435	98565	21
40	9.40346	10.59654	9.41784	10.58216	10.01439	9.98561	20
41	40394	59606	41836	58164	01442	98558	19
42	40442	59558	41887	58113	01445	98555	18
43	40490	59510	41939	58061	01449	98551	17
44	40538	59462	41990	58010	01452	98548	16
45	9.40586	10.59414	9.42041	10.57959	10.01455	9.98545	15
46	40634	59366	42093	57907	01459	98541	14
47	40682	59318	42144	57856	01462	98538	13
48	40730	59270	42195	57805	01465	98535	12
49	40778	59222	42246	57754	01469	98531	11
50	9.40825	10.59175	9.42297	10.57703	10.01472	9.98528	10
51	40873	59127	42348	57652	01475	98525	9
52	40921	59079	42399	57601	01479	98521	8
53	40968	59032	42450	57550	01482	98518	7
54	41016	58984	42501	57499	01485	98515	6
55	9.41063	10.58937	9.42552	10.57448	10.01489	9.98511	5
56	41111	58889	42603	57397	01492	98508	4
57	41158	58842	42653	57347	01495	98505	3
58	41205	58795	42704	57296	01499	98501	2
59	41252	58748	42755	57245	01502	98498	1
60	41300	58700	42805	57195	01506	98494	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

104° **75°**

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15° **Logarithms.** 164°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.41300	10.58700	9.42805	10.57195	10.01506	9.98494	60
1	41347	58653	42856	57144	01509	98491	59
2	41394	58606	42906	57094	01512	98488	58
3	41441	58559	42957	57043	01516	98484	57
4	41488	58512	43007	56993	01519	98481	56
5	9.41535	10.58465	9.43057	10.56943	10.01523	9.98477	55
6	41582	58418	43108	56892	01526	98474	54
7	41628	58372	43158	56842	01529	98471	53
8	41675	58325	43208	56792	01533	98467	52
9	41722	58278	43258	56742	01536	98464	51
10	9.41768	10.58232	9.43308	10.56692	10.01540	9.98460	50
11	41815	58185	43358	56642	01543	98457	49
12	41861	58139	43408	56592	01547	98453	48
13	41908	58092	43458	56542	01550	98450	47
14	41954	58046	43508	56492	01553	98447	46
15	9.42001	10.57999	9.43558	10.56442	10.01557	9.98443	45
16	42047	57953	43607	56393	01560	98440	44
17	42093	57907	43657	56343	01564	98436	43
18	42140	57860	43707	56293	01567	98433	42
19	42186	57814	43756	56244	01571	98429	41
20	9.42232	10.57768	9.43806	10.56194	10.01574	9.98426	40
21	42278	57722	43855	56145	01578	98422	39
22	42324	57676	43905	56095	01581	98419	38
23	42370	57630	43954	56046	01585	98415	37
24	42416	57584	44004	55996	01588	98412	36
25	9.42461	10.57539	9.44053	10.55947	10.01591	9.98409	35
26	42507	57493	44102	55898	01595	98405	34
27	42553	57447	44151	55849	01598	98402	33
28	42599	57401	44201	55799	01602	98398	32
29	42644	57356	44250	55750	01605	98395	31
30	9.42690	10.57310	9.44299	10.55701	10.01609	9.98391	30
31	42735	57265	44348	55652	01612	98388	29
32	42781	57219	44397	55603	01616	98384	28
33	42826	57174	44446	55554	01619	98381	27
34	42872	57128	44495	55505	01623	98377	26
35	9.42917	10.57083	9.44544	10.55456	10.01627	9.98373	25
36	42962	57038	44592	55408	01630	98370	24
37	43008	56992	44641	55359	01634	98366	23
38	43053	56947	44690	55310	01637	98363	22
39	43098	56902	44738	55262	01641	98359	21
40	9.43143	10.56857	9.44787	10.55213	10.01644	9.98356	20
41	43188	56812	44836	55164	01648	98352	19
42	43233	56767	44884	55116	01651	98349	18
43	43278	56722	44933	55067	01655	98345	17
44	43323	56677	44981	55019	01658	98342	16
45	9.43367	10.56633	9.45029	10.54971	10.01662	9.98338	15
46	43412	56588	45078	54922	01666	98334	14
47	43457	56543	45126	54874	01669	98331	13
48	43502	56498	45174	54826	01673	98327	12
49	43546	56454	45222	54778	01676	98324	11
50	9.43591	10.56409	9.45271	10.54729	10.01680	9.98320	10
51	43635	56365	45319	54681	01683	98317	9
52	43680	56320	45367	54633	01687	98313	8
53	43724	56276	45415	54585	01691	98309	7
54	43769	56231	45463	54537	01694	98306	6
55	9.43813	10.56187	9.45511	10.54489	10.01698	9.98302	5
56	43857	56143	45559	54441	01701	98299	4
57	43901	56099	45606	54394	01705	98295	3
58	43946	56054	45654	54346	01709	98291	2
59	43990	56010	45702	54298	01712	98288	1
60	44034	55966	45750	54250	01716	98284	0
M.	**Cosine.**	**Secant.**	**Cotangent.**	**Tangent.**	**Cosecant.**	**Sine.**	**M.**

105° 74°

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16° **Logarithms.** **163°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.44034	10.55966	9.45750	10.54250	10.01716	9.98284	60
1	44078	55922	45797	54203	01719	98281	59
2	44122	55878	45845	54155	01723	98277	58
3	44166	55834	45892	54108	01727	98273	57
4	44210	55790	45940	54060	01730	98270	56
5	9.44253	10.55747	9.45987	10.54013	10.01734	9.98266	55
6	44297	55703	46035	53965	01738	98262	54
7	44341	55659	46082	53918	01741	98259	53
8	44385	55615	46130	53870	01745	98255	52
9	44428	55572	46177	53823	01749	98251	51
10	9.44472	10.55528	9.46224	10.53776	10.01752	9.98248	50
11	44516	55484	46271	53729	01756	98244	49
12	44559	55441	46319	53681	01760	98240	48
13	44602	55398	46366	53634	01763	98237	47
14	44646	55354	46413	53587	01767	98233	46
15	9.44689	10.55311	9.46460	10.53540	10.01771	9.98229	45
16	44733	55267	46507	53493	01774	98226	44
17	44776	55224	46554	53446	01778	98222	43
18	44819	55181	46601	53399	01782	98218	42
19	44862	55138	46648	53352	01785	98215	41
20	9.44905	10.55095	9.46694	10.53306	10.01789	9.98211	40
21	44948	55052	46741	53259	01793	98207	39
22	44992	55008	46788	53212	01796	98204	38
23	45035	54965	46835	53165	01800	98200	37
24	45077	54923	46881	53119	01804	98196	36
25	9.45120	10.54880	9.46928	10.53072	10.01808	9.98192	35
26	45163	54837	46975	53025	01811	98189	34
27	45206	54794	47021	52979	01815	98185	33
28	45249	54751	47068	52932	01819	98181	32
29	45292	54708	47114	52886	01823	98177	31
30	9.45334	10.54666	9.47160	10.52840	10.01826	9.98174	30
31	45377	54623	47207	52793	01830	98170	29
32	45419	54581	47253	52747	01834	98166	28
33	45462	54538	47299	52701	01838	98162	27
34	45504	54496	47346	52654	01841	98159	26
35	9.45547	10.54453	9.47392	10.52608	10.01845	9.98155	25
36	45589	54411	47438	52562	01849	98151	24
37	45632	54368	47484	52516	01853	98147	23
38	45674	54326	47530	52470	01856	98144	22
39	45716	54284	47576	52424	01860	98140	21
40	9.45758	10.54242	9.47622	10.52378	10.01864	9.98136	20
41	45801	54199	47668	52332	01868	98132	19
42	45843	54157	47714	52286	01871	98129	18
43	45885	54115	47760	52240	01875	98125	17
44	45927	54073	47806	52194	01879	98121	16
45	9.45969	10.54031	9.47852	10.52148	10.01883	9.98117	15
46	46011	53989	47897	52103	01887	98113	14
47	46053	53947	47943	52057	01890	98110	13
48	46095	53905	47989	52011	01894	98106	12
49	46136	53864	48035	51965	01898	98102	11
50	9.46178	10.53822	9.48080	10.51920	10.01902	9.98098	10
51	46220	53780	48126	51874	01906	98094	9
52	46262	53738	48171	51829	01910	98090	8
53	46303	53697	48217	51783	01913	98087	7
54	46345	53655	48262	51738	01917	98083	6
55	9.46386	10.53614	9.48307	10.51693	10.01921	9.98079	5
56	46428	53572	48353	51647	01925	98075	4
57	46469	53531	48398	51602	01929	98071	3
58	46511	53489	48443	51557	01933	98067	2
59	46552	53448	48489	51511	01937	98063	1
60	46594	53406	48534	51466	01940	98060	0
M.	**Cosine.**	**Secant.**	**Cotangent.**	**Tangent.**	**Cosecant.**	**Sine.**	**M.**

106° **73°**

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17°			Logarithms.				162°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.46594	10.53406	9.48534	10.51466	10.01940	9.98060	60
1	46635	53365	48579	51421	01944	98056	59
2	46676	53324	48624	51376	01948	98052	58
3	46717	53283	48669	51331	01952	98048	57
4	46758	53242	48714	51286	01956	98044	56
5	9.46800	10.53200	9.48759	10.51241	10.01960	9.98040	55
6	46841	53159	48804	51196	01964	98036	54
7	46882	53118	48849	51151	01968	98032	53
8	46923	53077	48894	51106	01971	98029	52
9	46964	53036	48939	51061	01975	98025	51
10	9.47005	10.52995	9.48984	10.51016	10.01979	9.98021	50
11	47045	52955	49029	50971	01983	98017	49
12	47086	52914	49073	50927	01987	98013	48
13	47127	52873	49118	50882	01991	98009	47
14	47168	52832	49163	50837	01995	98005	46
15	9.47209	10.52791	9.49207	10.50793	10.01999	9.98001	45
16	47249	52751	49252	50748	02003	97997	44
17	47290	52710	49296	50704	02007	97993	43
18	47330	52670	49341	50659	02011	97989	42
19	47371	52629	49385	50615	02014	97986	41
20	9.47411	10.52589	9.49430	10.50570	10.02018	9.97982	40
21	47452	52548	49474	50526	02022	97978	39
22	47492	52508	49519	50481	02026	97974	38
23	47533	52467	49563	50437	02030	97970	37
24	47573	52427	49607	50393	02034	97966	36
25	9.47613	10.52387	9.49652	10.50348	10.02038	9.97962	35
26	47654	52346	49696	50304	02042	97958	34
27	47694	52306	49740	50260	02046	97954	33
28	47734	52266	49784	50216	02050	97950	32
29	47774	52226	49828	50172	02054	97946	31
30	9.47814	10.52186	9.49872	10.50128	10.02058	9.97942	30
31	47854	52146	49916	50084	02062	97938	29
32	47894	52106	49960	50040	02066	97934	28
33	47934	52066	50004	49996	02070	97930	27
34	47974	52026	50048	49952	02074	97926	26
35	9.48014	10.51986	9.50092	10.49908	10.02078	9.97922	25
36	48054	51946	50136	49864	02082	97918	24
37	48094	51906	50180	49820	02086	97914	23
38	48133	51867	50223	49777	02090	97910	22
39	48173	51827	50267	49733	02094	97906	21
40	9.48213	10.51787	9.50311	10.49689	10.02098	9.97902	20
41	48252	51748	50355	49645	02102	97898	19
42	48292	51708	50398	49602	02106	97894	18
43	48332	51668	50442	49558	02110	97890	17
44	48371	51629	50485	49515	02114	97886	16
45	9.48411	10.51589	9.50529	10.49471	10.02118	9.97882	15
46	48450	51550	50572	49428	02122	97878	14
47	48490	51510	50616	49384	02126	97874	13
48	48529	51471	50659	49341	02130	97870	12
49	48568	51432	50703	49297	02134	97866	11
50	9.48607	10.51393	9.50746	10.49254	10.02139	9.97861	10
51	48647	51353	50789	49211	02143	97857	9
52	48686	51314	50833	49167	02147	97853	8
53	48725	51275	50876	49124	02151	97849	7
54	48764	51236	50919	49081	02155	97845	6
55	9.48803	10.51197	9.50962	10.49038	10.02159	9.97841	5
56	48842	51158	51005	48995	02163	97837	4
57	48881	51119	51048	48952	02167	97833	3
58	48920	51080	51092	48908	02171	97829	2
59	48959	51041	51135	48865	02175	97825	1
60	48998	51002	51178	48822	02179	97821	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
107°							72°

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18° **Logarithms.** **161°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.48998	10.51002	9.51178	10.48822	10.02179	9.97821	60
1	49037	50963	51221	48779	02183	97817	59
2	49076	50924	51264	48736	02188	97812	58
3	49115	50885	51306	48694	02192	97808	57
4	49153	50847	51349	48651	02196	97804	56
5	9.49192	10.50808	9.51392	10.48608	10.02200	9.97800	55
6	49231	50769	51435	48565	02204	97796	54
7	49269	50731	51478	48522	02208	97792	53
8	49308	50692	51520	48480	02212	97788	52
9	49347	50653	51563	48437	02216	97784	51
10	9.49385	10.50615	9.51606	10.48394	10.02221	9.97779	50
11	49424	50576	51648	48352	02225	97775	49
12	49462	50538	51691	48309	02229	97771	48
13	49500	50500	51734	48266	02233	97767	47
14	49539	50461	51776	48224	02237	97763	46
15	9.49577	10.50423	9.51819	10.48181	10.02241	9.97759	45
16	49615	50385	51861	48139	02246	97754	44
17	49654	50346	51903	48097	02250	97750	43
18	49692	50308	51946	48054	02254	97746	42
19	49730	50270	51988	48012	02258	97742	41
20	9.49768	10.50232	9.52031	10.47969	10.02262	9.97738	40
21	49806	50194	52073	47927	02266	97734	39
22	49844	50156	52115	47885	02271	97729	38
23	49882	50118	52157	47843	02275	97725	37
24	49920	50080	52200	47800	02279	97721	36
25	9.49958	10.50042	9.52242	10.47758	10.02283	9.97717	35
26	49996	50004	52284	47716	02287	97713	34
27	50034	49966	52326	47674	02292	97708	33
28	50072	49928	52368	47632	02296	97704	32
29	50110	49890	52410	47590	02300	97700	31
30	9.50148	10.49852	9.52452	10.47548	10.02304	9.97696	30
31	50185	49815	52494	47506	02309	97691	29
32	50223	49777	52536	47464	02313	97687	28
33	50261	49739	52578	47422	02317	97683	27
34	50298	49702	52620	47380	02321	97679	26
35	9.50336	10.49664	9.52661	10.47339	10.02326	9.97674	25
36	50374	49626	52703	47297	02330	97670	24
37	50411	49589	52745	47255	02334	97666	23
38	50449	49551	52787	47213	02338	97662	22
39	50486	49514	52829	47171	02343	97657	21
40	9.50523	10.49477	9.52870	10.47130	10.02347	9.97653	20
41	50561	49439	52912	47088	02351	97649	19
42	50598	49402	52953	47047	02355	97645	18
43	50635	49365	52995	47005	02360	97640	17
44	50673	49327	53037	46963	02364	97636	16
45	9.50710	10.49290	9.53078	10.46922	10.02368	9.97632	15
46	50747	49253	53120	46880	02372	97628	14
47	50784	49216	53161	46839	02377	97623	13
48	50821	49179	53202	46798	02381	97619	12
49	50858	49142	53244	46756	02385	97615	11
50	9.50896	10.49104	9.53285	10.46715	10.02390	9.97610	10
51	50933	49067	53327	46673	02394	97606	9
52	50970	49030	53368	46632	02398	97602	8
53	51007	48993	53409	46591	02403	97597	7
54	51043	48957	53450	46550	02407	97593	6
55	9.51080	10.48920	9.53492	10.46508	10.02411	9.97589	5
56	51117	48883	53533	46467	02416	97584	4
57	51154	48846	53574	46426	02420	97580	3
58	51191	48809	53615	46385	02424	97576	2
59	51227	48773	53656	46344	02429	97571	1
60	51264	48736	53697	46303	02433	97567	0
M.	**Cosine.**	**Secant.**	**Cotangent.**	**Tangent.**	**Cosecant.**	**Sine.**	**M.**

108° **71°**

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19° **Logarithms.** **160°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.51264	10.48736	9.53697	10.46303	10.02433	9.97567	60
1	51301	48699	53738	46262	02437	97563	59
2	51338	48662	53779	46221	02442	97558	58
3	51374	48626	53820	46180	02446	97554	57
4	51411	48589	53861	46139	02450	97550	56
5	9.51447	10.48553	9.53902	10.46098	10.02455	9.97545	55
6	51484	48516	53943	46057	02459	97541	54
7	51520	48480	53984	46016	02464	97536	53
8	51557	48443	54025	45975	02468	97532	52
9	51593	48407	54065	45935	02472	97528	51
10	9.51629	10.48371	9.54106	10.45894	10.02477	9.97523	50
11	51666	48334	54147	45853	02481	97519	49
12	51702	48298	54187	45813	02485	97515	48
13	51738	48262	54228	45772	02490	97510	47
14	51774	48226	54269	45731	02494	97506	46
15	9.51811	10.48189	9.54309	10.45691	10.02499	9.97501	45
16	51847	48153	54350	45650	02503	97497	44
17	51883	48117	54390	45610	02508	97492	43
18	51919	48081	54431	45569	02512	97488	42
19	51955	48045	54471	45529	02516	97484	41
20	9.51991	10.48009	9.54512	10.45488	10.02521	9.97479	40
21	52027	47973	54552	45448	02525	97475	39
22	52063	47937	54593	45407	02530	97470	38
23	52099	47901	54633	45367	02534	97466	37
24	52135	47865	54673	45327	02539	97461	36
25	9.52171	10.47829	9.54714	10.45286	10.02543	9.97457	35
26	52207	47793	54754	45246	02547	97453	34
27	52242	47758	54794	45206	02552	97448	33
28	52278	47722	54835	45165	02556	97444	32
29	52314	47686	54875	45125	02561	97439	31
30	9.52350	10.47650	9.54915	10.45085	10.02565	9.97435	30
31	52385	47615	54955	45045	02570	97430	29
32	52421	47579	54995	45005	02574	97426	28
33	52456	47544	55035	44965	02579	97421	27
34	52492	47508	55075	44925	02583	97417	26
35	9.52527	10.47473	9.55115	10.44885	10.02588	9.97412	25
36	52563	47437	55155	44845	02592	97408	24
37	52598	47402	55195	44805	02597	97403	23
38	52634	47366	55235	44765	02601	97399	22
39	52669	47331	55275	44725	02606	97394	21
40	9.52705	10.47295	9.55315	10.44685	10.02610	9.97390	20
41	52740	47260	55355	44645	02615	97385	19
42	52775	47225	55395	44605	02619	97381	18
43	52811	47189	55434	44566	02624	97376	17
44	52846	47154	55474	44526	02628	97372	16
45	9.52881	10.47119	9.55514	10.44486	10.02633	9.97367	15
46	52916	47084	55554	44446	02637	97363	14
47	52951	47049	55593	44407	02642	97358	13
48	52986	47014	55633	44367	02647	97353	12
49	53021	46979	55673	44327	02651	97349	11
50	9.53056	10.46944	9.55712	10.44288	10.02656	9.97344	10
51	53092	46908	55752	44248	02660	97340	9
52	53126	46874	55791	44209	02665	97335	8
53	53161	46839	55831	44169	02669	97331	7
54	53196	46804	55870	44130	02674	97326	6
55	9.53231	10.46769	9.55910	10.44090	10.02678	9.97322	5
56	53266	46734	55949	44051	02683	97317	4
57	53301	46699	55989	44011	02688	97312	3
58	53336	46664	56028	43972	02692	97308	2
59	53370	46630	56067	43933	02697	97303	1
60	53405	46595	56107	43893	02701	97299	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

109° **70°**

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20°			Logarithms.				159°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.53405	10.46595	9.56107	10.43893	10.02701	9.97299	60
1	53440	46560	56146	43854	02706	97294	59
2	53475	46525	56185	43815	02711	97289	58
3	53509	46491	56224	43776	02715	97285	57
4	53544	46456	56264	43736	02720	97280	56
5	9.53578	10.46422	9.56303	10.43697	10.02724	9.97276	55
6	53613	46387	56342	43658	02729	97271	54
7	53647	46353	56381	43619	02734	97266	53
8	53682	46318	56420	43580	02738	97262	52
9	53716	46284	56459	43541	02743	97257	51
10	9.53751	10.46249	9.56498	10.43502	10.02748	9.97252	50
11	53785	46215	56537	43463	02752	97248	49
12	53819	46181	56576	43424	02757	97243	48
13	53854	46146	56615	43385	02762	97238	47
14	53888	46112	56654	43346	02766	97234	46
15	9.53922	10.46078	9.56693	10.43307	10.02771	9.97229	45
16	53957	46043	56732	43268	02776	97224	44
17	53991	46009	56771	43229	02780	97220	43
18	54025	45975	56810	43190	02785	97215	42
19	54059	45941	56849	43151	02790	97210	41
20	9.54093	10.45907	9.56887	10.43113	10.02794	9.97206	40
21	54127	45873	56926	43074	02799	97201	39
22	54161	45839	56965	43035	02804	97196	38
23	54195	45805	57004	42996	02808	97192	37
24	54229	45771	57042	42958	02813	97187	36
25	9.54263	10.45737	9.57081	10.42919	10.02818	9.97182	35
26	54297	45703	57120	42880	02822	97178	34
27	54331	45669	57158	42842	02827	97173	33
28	54365	45635	57197	42803	02832	97168	32
29	54399	45601	57235	42765	02837	97163	31
30	9.54433	10.45567	9.57274	10.42726	10.02841	9.97159	30
31	54466	45534	57312	42688	02846	97154	29
32	54500	45500	57351	42649	02851	97149	28
33	54534	45466	57389	42611	02855	97145	27
34	54567	45433	57428	42572	02860	97140	26
35	9.54601	10.45399	9.57466	10.42534	10.02865	9.97135	25
36	54635	45365	57504	42496	02870	97130	24
37	54668	45332	57543	42457	02874	97126	23
38	54702	45298	57581	42419	02879	97121	22
39	54735	45265	57619	42381	02884	97116	21
40	9.54769	10.45231	9.57658	10.42342	10.02889	9.97111	20
41	54802	45198	57696	42304	02893	97107	19
42	54836	45164	57734	42266	02898	97102	18
43	54869	45131	57772	42228	02903	97097	17
44	54903	45097	57810	42190	02908	97092	16
45	9.54936	10.45064	9.57849	10.42151	10.02913	9.97087	15
46	54969	45031	57887	42113	02917	97083	14
47	55003	44997	57925	42075	02922	97078	13
48	55036	44964	57963	42037	02927	97073	12
49	55069	44931	58001	41999	02932	97068	11
50	9.55102	10.44898	9.58039	10.41961	10.02937	9.97063	10
51	55136	44864	58077	41923	02941	97059	9
52	55169	44831	58115	41885	02946	97054	8
53	55202	44798	58153	41847	02951	97049	7
54	55235	44765	58191	41809	02956	97044	6
55	9.55268	10.44732	9.58229	10.41771	10.02961	9.97039	5
56	55301	44699	58267	41733	02965	97035	4
57	55334	44666	58304	41696	02970	97030	3
58	55367	44633	58342	41658	02975	97025	2
59	55400	44600	58380	41620	02980	97020	1
60	55433	44567	58418	41582	02985	97015	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
110°							69°

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21° **Logarithms.** **158°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.55433	10.44567	9.58418	10.41582	10.02985	9.97015	60
1	55466	44534	58455	41545	02990	97010	59
2	55499	44501	58493	41507	02995	97005	58
3	55532	44468	58531	41469	02999	97001	57
4	55564	44436	58569	41431	03004	96996	56
5	9.55597	10.44403	9.58606	10.41394	10.03009	9.96991	55
6	55630	44370	58644	41356	03014	96986	54
7	55663	44337	58681	41319	03019	96981	53
8	55695	44305	58719	41281	03024	96976	52
9	55728	44272	58757	41243	03029	96971	51
10	9.55761	10.44239	9.58794	10.41206	10.03034	9.96966	50
11	55793	44207	58832	41168	03038	96962	49
12	55826	44174	58869	41131	03043	96957	48
13	55858	44142	58907	41093	03048	96952	47
14	55891	44109	58944	41056	03053	96947	46
15	9.55923	10.44077	9.58981	10.41019	10.03058	9.96942	45
16	55956	44044	59019	40981	03063	96937	44
17	55988	44012	59056	40944	03068	96932	43
18	56021	43979	59094	40906	03073	96927	42
19	56053	43947	59131	40869	03078	96922	41
20	9.56085	10.43915	9.59168	10.40832	10.03083	9.96917	40
21	56118	43882	59205	40795	03088	96912	39
22	56150	43850	59243	40757	03093	96907	38
23	56182	43818	59280	40720	03097	96903	37
24	56215	43785	59317	40683	03102	96898	36
25	9.56247	10.43753	9.59354	10.40646	10.03107	9.96893	35
26	56279	43721	59391	40609	03112	96888	34
27	56311	43689	59429	40571	03117	96883	33
28	56343	43657	59466	40534	03122	96878	32
29	56375	43625	59503	40497	03127	96873	31
30	9.56408	10.43592	9.59540	10.40460	10.03132	9.96868	30
31	56440	43560	59577	40423	03137	96863	29
32	56472	43528	59614	40386	03142	96858	28
33	56504	43496	59651	40349	03147	96853	27
34	56536	43464	59688	40312	03152	96848	26
35	9.56568	10.43432	9.59725	10.40275	10.03157	9.96843	25
36	56599	43401	59762	40238	03162	96838	24
37	56631	43369	59799	40201	03167	96833	23
38	56663	43337	59835	40165	03172	96828	22
39	56695	43305	59872	40128	03177	96823	21
40	9.56727	10.43273	9.59909	10.40091	10.03182	9.96818	20
41	56759	43241	59946	40054	03187	96813	19
42	56790	43210	59983	40017	03192	96808	18
43	56822	43178	60019	39981	03197	96803	17
44	56854	43146	60056	39944	03202	96798	16
45	9.56886	10.43114	9.60093	10.39907	10.03207	9.96793	15
46	56917	43083	60130	39870	03212	96788	14
47	56949	43051	60166	39834	03217	96783	13
48	56980	43020	60203	39797	03222	96778	12
49	57012	42988	60240	39760	03228	96772	11
50	9.57044	10.42956	9.60276	10.39724	10.03233	9.96767	10
51	57075	42925	60313	39687	03238	96762	9
52	57107	42893	60349	39651	03243	96757	8
53	57138	42862	60386	39614	03248	96752	7
54	57169	42831	60422	39578	03253	96747	6
55	9.57201	10.42799	9.60459	10.39541	10.03258	9.96742	5
56	57232	42768	60495	39505	03263	96737	4
57	57264	42736	60532	39468	03268	96732	3
58	57295	42705	60568	39432	03273	96727	2
59	57326	42674	60605	39395	03278	96722	1
60	57358	42642	60641	39359	03283	96717	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

111° **68°**

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22° **Logarithms.** 157°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.57358	10.42642	9.60641	10.39359	10.03283	9.96717	60
1	57389	42611	60677	39323	03289	96711	59
2	57420	42580	60714	39286	03294	96706	58
3	57451	42549	60750	39250	03299	96701	57
4	57482	42518	60786	39214	03304	96696	56
5	9.57514	10.42486	9.60823	10.39177	10.03309	9.96691	55
6	57545	42455	60859	39141	03314	96686	54
7	57576	42424	60895	39105	03319	96681	53
8	57607	42393	60931	39069	03324	96676	52
9	57638	42362	60967	39033	03330	96670	51
10	9.57669	10.42331	9.61004	10.38996	10.03335	9.96665	50
11	57700	42300	61040	38960	03340	96660	49
12	57731	42269	61076	38924	03345	96655	48
13	57762	42238	61112	38888	03350	96650	47
14	57793	42207	61148	38852	03355	96645	46
15	9.57824	10.42176	9.61184	10.38816	10.03360	9.96640	45
16	57855	42145	61220	38780	03366	96634	44
17	57885	42115	61256	38744	03371	96629	43
18	57916	42084	61292	38708	03376	96624	42
19	57947	42053	61328	38672	03381	96619	41
20	9.57978	10.42022	9.61364	10.38636	10.03386	9.96614	40
21	58008	41992	61400	38600	03392	96608	39
22	58039	41961	61436	38564	03397	96603	38
23	58070	41930	61472	38528	03402	96598	37
24	58101	41899	61508	38492	03407	96593	36
25	9.58131	10.41869	9.61544	10.38456	10.03412	9.96588	35
26	58162	41838	61579	38421	03418	96582	34
27	58192	41808	61615	38385	03423	96577	33
28	58223	41777	61651	38349	03428	96572	32
29	58253	41747	61687	38313	03433	96567	31
30	9.58284	10.41716	9.61722	10.38278	10.03438	9.96562	30
31	58314	41686	61758	38242	03444	96556	29
32	58345	41655	61794	38206	03449	96551	28
33	58375	41625	61830	38170	03454	96546	27
34	58406	41594	61865	38135	03459	96541	26
35	9.58436	10.41564	9.61901	10.38099	10.03465	9.96535	25
36	58467	41533	61936	38064	03470	96530	24
37	58497	41503	61972	38028	03475	96525	23
38	58527	41473	62008	37992	03480	96520	22
39	58557	41443	62043	37957	03486	96514	21
40	9.58588	10.41412	9.62079	10.37921	10.03491	9.96509	20
41	58618	41382	62114	37886	03496	96504	19
42	58648	41352	62150	37850	03502	96498	18
43	58678	41322	62185	37815	03507	96493	17
44	58709	41291	62221	37779	03512	96488	16
45	9.58739	10.41261	9.62256	10.37744	10.03517	9.96483	15
46	58769	41231	62292	37708	03523	96477	14
47	58799	41201	62327	37673	03528	96472	13
48	58829	41171	62362	37638	03533	96467	12
49	58859	41141	62398	37602	03539	96461	11
50	9.58889	10.41111	9.62433	10.37567	10.03544	9.96456	10
51	58919	41081	62468	37532	03549	96451	9
52	58949	41051	62504	37496	03555	96445	8
53	58979	41021	62539	37461	03560	96440	7
54	59009	40991	62574	37426	03565	96435	6
55	9.59039	10.40961	9.62609	10.37391	10.03571	9.96429	5
56	59069	40931	62645	37355	03576	96424	4
57	59098	40902	62680	37320	03581	96419	3
58	59128	40872	62715	37285	03587	96413	2
59	59158	40842	62750	37250	03592	96408	1
60	59188	40812	62785	37215	03597	96403	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

23°			Logarithms.				156°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.59188	10.40812	9.62785	10.37215	10.03597	9.96403	60
1	59218	40782	62820	37180	03603	96397	59
2	59247	40753	62855	37145	03608	96392	58
3	59277	40723	62890	37110	03613	96387	57
4	59307	40693	62926	37074	03619	96381	56
5	9.59336	10.40664	9.62961	10.37039	10.03624	9.96376	55
6	59366	40634	62996	37004	03630	96370	54
7	59396	40604	63031	36969	03635	96365	53
8	59425	40575	63066	36934	03640	96360	52
9	59455	40545	63101	36899	03646	96354	51
10	9.59484	10.40516	9.63135	10.36865	10.03651	9.96349	50
11	59514	40486	63170	36830	03657	96343	49
12	59543	40457	63205	36795	03662	96338	48
13	59573	40427	63240	36760	03667	96333	47
14	59602	40398	63275	36725	03673	96327	46
15	9.59632	10.40368	9.63310	10.36690	10.03678	9.96322	45
16	59661	40339	63345	36655	03684	96316	44
17	59690	40310	63379	36621	03689	96311	43
18	59720	40280	63414	36586	03695	96305	42
19	59749	40251	63449	36551	03700	96300	41
20	9.59778	10.40222	9.63484	10.36516	10.03706	9.96294	40
21	59808	40192	63519	36481	03711	96289	39
22	59837	40163	63553	36447	03716	96284	38
23	59866	40134	63588	36412	03722	96278	37
24	59895	40105	63623	36377	03727	96273	36
25	9.59924	10.40076	9.63657	10.36343	10.03733	9.96267	35
26	59954	40046	63692	36308	03738	96262	34
27	59983	40017	63726	36274	03744	96256	33
28	60012	39988	63761	36239	03749	96251	32
29	60041	39959	63796	36204	03755	96245	31
30	9.60070	10.39930	9.63830	10.36170	10.03760	9.96240	30
31	60099	39901	63865	36135	03766	96234	29
32	60128	39872	63899	36101	03771	96229	28
33	60157	39843	63934	36066	03777	96223	27
34	60186	39814	63968	36032	03782	96218	26
35	9.60215	10.39785	9.64003	10.35997	10.03788	9.96212	25
36	60244	39756	64037	35963	03793	96207	24
37	60273	39727	64072	35928	03799	96201	23
38	60302	39698	64106	35894	03804	96196	22
39	60331	39669	64140	35860	03810	96190	21
40	9.60359	10.39641	9.64175	10.35825	10.03815	9.96185	20
41	60388	39612	64209	35791	03821	96179	19
42	60417	39583	64243	35757	03826	96174	18
43	60446	39554	64278	35722	03832	96168	17
44	60474	39526	64312	35688	03838	96162	16
45	9.60503	10.39497	9.64346	10.35654	10.03843	9.96157	15
46	60532	39468	64381	35619	03849	96151	14
47	60561	39439	64415	35585	03854	96146	13
48	60589	39411	64449	35551	03860	96140	12
49	60618	39382	64483	35517	03865	96135	11
50	9.60646	10.39354	9.64517	10.35483	10.03871	9.96129	10
51	60675	39325	64552	35448	03877	96123	9
52	60704	39296	64586	35414	03882	96118	8
53	60732	39268	64620	35380	03888	96112	7
54	60761	39239	64654	35346	03893	96107	6
55	9.60789	10.39211	9.64688	10.35312	10.03899	9.96101	5
56	60818	39182	64722	35278	03905	96095	4
57	60846	39154	64756	35244	03910	96090	3
58	60875	39125	64790	35210	03916	96084	2
59	60903	39097	64824	35176	03921	96079	1
60	60931	39069	64858	35142	03927	96073	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
113°							66°

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24° **Logarithms.** 155°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.60931	10.39069	9.64858	10.35142	10.03927	9.96073	60
1	60960	39040	64892	35108	03933	96067	59
2	60988	39012	64926	35074	03938	96062	58
3	61016	38984	64960	35040	03944	96056	57
4	61045	38955	64994	35006	03950	96050	56
5	9.61073	10.38927	9.65028	10.34972	10.03955	9.96045	55
6	61101	38899	65062	34938	03961	96039	54
7	61129	38871	65096	34904	03966	96034	53
8	61158	38842	65130	34870	03972	96028	52
9	61186	38814	65164	34836	03978	96022	51
10	9.61214	10.38786	9.65197	10.34803	10.03983	9.96017	50
11	61242	38758	65231	34769	03989	96011	49
12	61270	38730	65265	34735	03995	96005	48
13	61298	38702	65299	34701	04000	96000	47
14	61326	38674	65333	34667	04006	95994	46
15	9.61354	10.38646	9.65366	10.34634	10.04012	9.95988	45
16	61382	38618	65400	34600	04018	95982	44
17	61411	38589	65434	34566	04023	95977	43
18	61438	38562	65467	34533	04029	95971	42
19	61466	38534	65501	34499	04035	95965	41
20	9.61494	10.38506	9.65535	10.34465	10.04040	9.95960	40
21	61522	38478	65568	34432	04046	95954	39
22	61550	38450	65602	34398	04052	95948	38
23	61578	38422	65636	34364	04058	95942	37
24	61606	38394	65669	34331	04063	95937	36
25	9.61634	10.38366	9.65703	10.34297	10.04069	9.95931	35
26	61662	38338	65736	34264	04075	95925	34
27	61689	38311	65770	34230	04080	95920	33
28	61717	38283	65803	34197	04086	95914	32
29	61745	38255	65837	34163	04092	95908	31
30	9.61773	10.38227	9.65870	10.34130	10.04098	9.95902	30
31	61800	38200	65904	34096	04103	95897	29
32	61828	38172	65937	34063	04109	95891	28
33	61856	38144	65971	34029	04115	95885	27
34	61883	38117	66004	33996	04121	95879	26
35	9.61911	10.38089	9.66038	10.33962	10.04127	9.95873	25
36	61939	38061	66071	33929	04132	95868	24
37	61966	38034	66104	33896	04138	95862	23
38	61994	38006	66138	33862	04144	95856	22
39	62021	37979	66171	33829	04150	95850	21
40	9.62049	10.37951	9.66204	10.33796	10.04156	9.95844	20
41	62076	37924	66238	33762	04161	95839	19
42	62104	37896	66271	33729	04167	95833	18
43	62131	37869	66304	33696	04173	95827	17
44	62159	37841	66337	33663	04179	95821	16
45	9.62186	10.37814	9.66371	10.33629	10.04185	9.95815	15
46	62214	37786	66404	33596	04190	95810	14
47	62241	37759	66437	33563	04196	95804	13
48	62268	37732	66470	33530	04202	95798	12
49	62296	37704	66503	33497	04208	95792	11
50	9.62323	10.37677	9.66537	10.33463	10.04214	9.95786	10
51	62350	37650	66570	33430	04220	95780	9
52	62377	37623	66603	33397	04225	95775	8
53	62405	37595	66636	33364	04231	95769	7
54	62432	37568	66669	33331	04237	95763	6
55	9.62459	10.37541	9.66702	10.33298	10.04243	9.95757	5
56	62486	37514	66735	33265	04249	95751	4
57	62513	37487	66768	33232	04255	95745	3
58	62541	37459	66801	33199	04261	95739	2
59	62568	37432	66834	33166	04267	95733	1
60	62595	37405	66867	33133	04272	95728	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

114° 65°

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25° **Logarithms.** **154°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.62595	10.37405	9.66867	10.33133	10.04272	9.95728	60
1	62622	37378	66900	33100	04278	95722	59
2	62649	37351	66933	33067	04284	95716	58
3	62676	37324	66966	33034	04290	95710	57
4	62703	37297	66999	33001	04296	95704	56
5	9.62730	10.37270	9.67032	10.32968	10.04302	9.95698	55
6	62757	37243	67065	32935	04308	95692	54
7	62784	37216	67098	32902	04314	95686	53
8	62811	37189	67131	32869	04320	95680	52
9	62838	37162	67163	32837	04326	95674	51
10	9.62865	10.37135	9.67196	10.32804	10.04332	9.95668	50
11	62892	37108	67229	32771	04337	95663	49
12	62918	37082	67262	32738	04343	95657	48
13	62945	37055	67295	32705	04349	95651	47
14	62972	37028	67327	32673	04355	95645	46
15	9.62999	10.37001	9.67360	10.32640	10.04361	9.95639	45
16	63026	36974	67393	32607	04367	95633	44
17	63052	36948	67426	32574	04373	95627	43
18	63079	36921	67458	32542	04379	95621	42
19	63106	36894	67491	32509	04385	95615	41
20	9.63133	10.36867	9.67524	10.32476	10.04391	9.95609	40
21	63159	36841	67556	32444	04397	95603	39
22	63186	36814	67589	32411	04403	95597	38
23	63213	36787	67622	32378	04409	95591	37
24	63239	36761	67654	32346	04415	95585	36
25	9.63266	10.36734	9.67687	10.32313	10.04421	9.95579	35
26	63292	36708	67719	32281	04427	95573	34
27	63319	36681	67752	32248	04433	95567	33
28	63345	36655	67785	32215	04439	95561	32
29	63372	36628	67817	32183	04445	95555	31
30	9.63398	10.36602	9.67850	10.32150	10.04451	9.95549	30
31	63425	36575	67882	32118	04457	95543	29
32	63451	36549	67915	32085	04463	95537	28
33	63478	36522	67947	32053	04469	95531	27
34	63504	36496	67980	32020	04475	95525	26
35	9.63531	10.36469	9.68012	10.31988	10.04481	9.95519	25
36	63557	36443	68044	31956	04487	95513	24
37	63583	36417	68077	31923	04493	95507	23
38	63610	36390	68109	31891	04500	95500	22
39	63636	36364	68142	31858	04506	95494	21
40	9.63662	10.36338	9.68174	10.31826	10.04512	9.95488	20
41	63689	36311	68206	31794	04518	95482	19
42	63715	36285	68239	31761	04524	95476	18
43	63741	36259	68271	31729	04530	95470	17
44	63767	36233	68303	31697	04536	95464	16
45	9.63794	10.36206	9.68336	10.31664	10.04542	9.95458	15
46	63820	36180	68368	31632	04548	95452	14
47	63846	36154	68400	31600	04554	95446	13
48	63872	36128	68432	31568	04560	95440	12
49	63898	36102	68465	31535	04566	95434	11
50	9.63924	10.36076	9.68497	10.31503	10.04573	9.95427	10
51	63950	36050	68529	31471	04579	95421	9
52	63976	36024	68561	31439	04585	95415	8
53	64002	35998	68593	31407	04591	95409	7
54	64028	35972	68626	31374	04597	95403	6
55	9.64054	10.35946	9.68658	10.31342	10.04603	9.95397	5
56	64080	35920	68690	31310	04609	95391	4
57	64106	35894	68722	31278	04616	95384	3
58	64132	35868	68754	31246	04622	95378	2
59	64158	35842	68786	31214	04628	95372	1
60	64184	35816	68818	31182	04634	95366	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

115° **64°**

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26° **Logarithms.** **153°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.64184	10.35816	9.68818	10.31182	10.04634	9.95366	60
1	64210	35790	68850	31150	04640	95360	59
2	64236	35764	68882	31118	04646	95354	58
3	64262	35738	68914	31086	04652	95348	57
4	64288	35712	68946	31054	04659	95341	56
5	9.64313	10.35687	9.68978	10.31022	10.04665	9.95335	55
6	64339	35661	69010	30990	04671	95329	54
7	64365	35635	69042	30958	04677	95323	53
8	64391	35609	69074	30926	04683	95317	52
9	64417	35583	69106	30894	04690	95310	51
10	9.64442	10.35558	9.69138	10.30862	10.04696	9.95304	50
11	64468	35532	69170	30830	04702	95298	49
12	64494	35506	69202	30798	04708	95292	48
13	64519	35481	69234	30766	04714	95286	47
14	64545	35455	69266	30734	04721	95279	46
15	9.64571	10.35429	9.69298	10.30702	10.04727	9.95273	45
16	64596	35404	69329	30671	04733	95267	44
17	64622	35378	69361	30639	04739	95261	43
18	64647	35353	69393	30607	04746	95254	42
19	64673	35327	69425	30575	04752	95248	41
20	9.64698	10.35302	9.69457	10.30543	10.04758	9.95242	40
21	64724	35276	69488	30512	04764	95236	39
22	64749	35251	69520	30480	04771	95229	38
23	64775	35225	69552	30448	04777	95223	37
24	64800	35200	69584	30416	04783	95217	36
25	9.64826	10.35174	9.69615	10.30385	10.04789	9.95211	35
26	64851	35149	69647	30353	04796	95204	34
27	64877	35123	69679	30321	04802	95198	33
28	64902	35098	69710	30290	04808	95192	32
29	64927	35073	69742	30258	04815	95185	31
30	9.64953	10.35047	9.69774	10.30226	10.04821	9.95179	30
31	64978	35022	69805	30195	04827	95173	29
32	65003	34997	69837	30163	04833	95167	28
33	65029	34971	69868	30132	04840	95160	27
34	65054	34946	69900	30100	04846	95154	26
35	9.65079	10.34921	9.69932	10.30068	10.04852	9.95148	25
36	65104	34896	69963	30037	04859	95141	24
37	65130	34870	69995	30005	04865	95135	23
38	65155	34845	70026	29974	04871	95129	22
39	65180	34820	70058	29942	04878	95122	21
40	9.65205	10.34795	9.70089	10.29911	10.04884	9.95116	20
41	65230	34770	70121	29879	04890	95110	19
42	65255	34745	70152	29848	04897	95103	18
43	65281	34719	70184	29816	04903	95097	17
44	65306	34694	70215	29785	04910	95090	16
45	9.65331	10.34669	9.70247	10.29753	10.04916	9.95084	15
46	65356	34644	70278	29722	04922	95078	14
47	65381	34619	70309	29691	04929	95071	13
48	65406	34594	70341	29659	04935	95065	12
49	65431	34569	70372	29628	04941	95059	11
50	9.65456	10.34544	9.70404	10.29596	10.04948	9.95052	10
51	65481	34519	70435	29565	04954	95046	9
52	65506	34494	70466	29534	04961	95039	8
53	65531	34469	70498	29502	04967	95033	7
54	65556	34444	70529	29471	04973	95027	6
55	9.65580	10.34420	9.70560	10.29440	10.04980	9.95020	5
56	65605	34395	70592	29408	04986	95014	4
57	65630	34370	70623	29377	04993	95007	3
58	65665	34345	70654	29346	04999	95001	2
59	65680	34320	70685	29315	05005	94995	1
60	65705	34295	70717	29283	05012	94988	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

116° **63°**

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27° **Logarithms.** **152°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.65705	10.34295	9.70717	10.29283	10.05012	9.94988	60
1	65729	34271	70748	29252	05018	94982	59
2	65754	34246	70779	29221	05025	94975	58
3	65779	34221	70810	29190	05031	94969	57
4	65804	34196	70841	29159	05038	94962	56
5	9.65828	10.34172	9.70873	10.29127	10.05044	9.94956	55
6	65853	34147	70904	29096	05051	94949	54
7	65878	34122	70935	29065	05057	94943	53
8	65902	34098	70966	29034	05064	94936	52
9	65927	34073	70997	29003	05070	94930	51
10	9.65952	10.34048	9.71028	10.28972	10.05077	9.94923	50
11	65976	34024	71059	28941	05083	94917	49
12	66001	33999	71090	28910	05089	94911	48
13	66025	33975	71121	28879	05096	94904	47
14	66050	33950	71153	28847	05102	94898	46
15	9.66075	10.33925	9.71184	10.28816	10.05109	9.94891	45
16	66099	33901	71215	28785	05115	94885	44
17	66124	33876	71246	28754	05122	94878	43
18	66148	33852	71277	28723	05129	94871	42
19	66173	33827	71308	28692	05135	94865	41
20	9.66197	10.33803	9.71339	10.28661	10.05142	9.94858	40
21	66221	33779	71370	28630	05148	94852	39
22	66246	33754	71401	28599	05155	94845	38
23	66270	33730	71431	28569	05161	94839	37
24	66295	33705	71462	28538	05168	94832	36
25	9.66319	10.33681	9.71493	10.28507	10.05174	9.94826	35
26	66343	33657	71524	28476	05181	94819	34
27	66368	33632	71555	28445	05187	94813	33
28	66392	33608	71586	28414	05194	94806	32
29	66416	33584	71617	28383	05201	94799	31
30	9.66441	10.33559	9.71648	10.28352	10.05207	9.94793	30
31	66465	33535	71679	28321	05214	94786	29
32	66489	33511	71709	28291	05220	94780	28
33	66513	33487	71740	28260	05227	94773	27
34	66537	33463	71771	28229	05233	94767	26
35	9.66562	10.33438	9.71802	10.28198	10.05240	9.94760	25
36	66586	33414	71833	28167	05247	94753	24
37	66610	33390	71863	28137	05253	94747	23
38	66634	33366	71894	28106	05260	94740	22
39	66658	33342	71925	28075	05266	94734	21
40	9.66682	10.33318	9.71955	10.28045	10.05273	9.94727	20
41	66706	33294	71986	28014	05280	94720	19
42	66731	33269	72017	27983	05286	94714	18
43	66755	33245	72048	27952	05293	94707	17
44	66779	33221	72078	27922	05300	94700	16
45	9.66803	10.33197	9.72109	10.27891	10.05306	9.94694	15
46	66827	33173	72140	27860	05313	94687	14
47	66851	33149	72170	27830	05320	94680	13
48	66875	33125	72201	27799	05326	94674	12
49	66899	33101	72231	27769	05333	94667	11
50	9.66922	10.33078	9.72262	10.27738	10.05340	9.94660	10
51	66946	33054	72293	27707	05346	94654	9
52	66970	33030	72323	27677	05353	94647	8
53	66994	33006	72354	27646	05360	94640	7
54	67018	32982	72384	27616	05366	94634	6
55	9.67042	10.32958	9.72415	10.27585	10.05373	9.94627	5
56	67066	32934	72445	27555	05380	94620	4
57	67090	32910	72476	27524	05386	94614	3
58	67113	32887	72506	27494	05393	94607	2
59	67137	32863	72537	27463	05400	94600	1
60	67161	32839	72567	27433	05407	94593	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

117° **62°**

Digitized by Google

28° **Logarithms.** **151°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.67161	10.32839	9.72567	10.27433	10.05407	9.94593	60
1	67185	32815	72598	27402	05413	94587	59
2	67208	32792	72628	27372	05420	94580	58
3	67232	32768	72659	27341	05427	94573	57
4	67256	32744	72689	27311	05433	94567	56
5	9.67280	10.32720	9.72720	10.27280	10.05440	9.94560	55
6	67303	32697	72750	27250	05447	94553	54
7	67327	32673	72780	27220	05454	94546	53
8	67350	32650	72811	27189	05460	94540	52
9	67374	32626	72841	27159	05467	94533	51
10	9.67398	10.32602	9.72872	10.27128	10.05474	9.94526	50
11	67421	32579	72902	27098	05481	94519	49
12	67445	32555	72932	27068	05487	94513	48
13	67468	32532	72963	27037	05494	94506	47
14	67492	32508	72993	27007	05501	94499	46
15	9.67515	10.32485	9.73023	10.26977	10.05508	9.94492	45
16	67539	32461	73054	26946	05515	94485	44
17	67562	32438	73084	26916	05521	94479	43
18	67586	32414	73114	26886	05528	94472	42
19	67609	32391	73144	26856	05535	94465	41
20	9.67633	10.32367	9.73175	10.26825	10.05542	9.94458	40
21	67656	32344	73205	26795	05549	94451	39
22	67680	32320	73235	26765	05555	94445	38
23	67703	32297	73265	26735	05562	94438	37
24	67726	32274	73295	26705	05569	94431	36
25	9.67750	10.32250	9.73326	10.26674	10.05576	9.94424	35
26	67773	32227	73356	26644	05583	94417	34
27	67796	32204	73386	26614	05590	94410	33
28	67820	32180	73416	26584	05596	94404	32
29	67843	32157	73446	26554	05603	94397	31
30	9.67866	10.32134	9.73476	10.26524	10.05610	9.94390	30
31	67890	32110	73507	26493	05617	94383	29
32	67913	32087	73537	26463	05624	94376	28
33	67936	32064	73567	26433	05631	94369	27
34	67959	32041	73597	26403	05638	94362	26
35	9.67982	10.32018	9.73627	10.26373	10.05645	9.94355	25
36	68006	31994	73657	26343	05651	94349	24
37	68029	31971	73687	26313	05658	94342	23
38	68052	31948	73717	26283	05665	94335	22
39	68075	31925	73747	26253	05672	94328	21
40	9.68098	10.31902	9.73777	10.26223	10.05679	9.94321	20
41	68121	31879	73807	26193	05686	94314	19
42	68144	31856	73837	26163	05693	94307	18
43	68167	31833	73867	26133	05700	94300	17
44	68190	31810	73897	26103	05707	94293	16
45	9.68213	10.31787	9.73927	10.26073	10.05714	9.94286	15
46	68237	31763	73957	26043	05721	94279	14
47	68260	31740	73987	26013	05727	94273	13
48	68283	31717	74017	25983	05734	94266	12
49	68305	31695	74047	25953	05741	94259	11
50	9.68328	10.31672	9.74077	10.25923	10.05748	9.94252	10
51	68351	31649	74107	25893	05755	94245	9
52	68374	31626	74137	25863	05762	94238	8
53	68397	31603	74166	25834	05769	94231	7
54	68420	31580	74196	25804	05776	94224	6
55	9.68443	10.31557	9.74226	10.25774	10.05783	9.94217	5
56	68466	31534	74256	25744	05790	94210	4
57	68489	31511	74286	25714	05797	94203	3
58	68512	31488	74316	25684	05804	94196	2
59	68534	31466	74345	25655	05811	94189	1
60	68557	31443	74375	25625	05818	94182	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

118° **61°**

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29° **Logarithms.** **150°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.68557	10.31443	9.74375	10.25625	10.05818	9.94182	60
1	68580	31420	74405	25595	05825	94175	59
2	68603	31397	74435	25565	05832	94168	58
3	68625	31375	74465	25535	05839	94161	57
4	68648	31352	74494	25506	05846	94154	56
5	9.68671	10.31329	9.74524	10.25476	10.05853	9.94147	55
6	68694	31306	74554	25446	05860	94140	54
7	68716	31284	74583	25417	05867	94133	53
8	68739	31261	74613	25387	05874	94126	52
9	68762	31238	74643	25357	05881	94119	51
10	9.68784	10.31216	9.74673	10.25327	10.05888	9.94112	50
11	68807	31193	74702	25298	05895	94105	49
12	68829	31171	74732	25268	05902	94098	48
13	68852	31148	74762	25238	05910	94090	47
14	68875	31125	74791	25209	05917	94083	46
15	9.68897	10.31103	9.74821	10.25179	10.05924	9.94076	45
16	68920	31080	74851	25149	05931	94069	44
17	68942	31058	74880	25120	05938	94062	43
18	68965	31035	74910	25090	05945	94055	42
19	68987	31013	74939	25061	05952	94048	41
20	9.69010	10.30990	9.74969	10.25031	10.05959	9.94041	40
21	69032	30968	74998	25002	05966	94034	39
22	69055	30945	75028	24972	05973	94027	38
23	69077	30923	75058	24942	05980	94020	37
24	69100	30900	75087	24913	05988	94012	36
25	9.69122	10.30878	9.75117	10.24883	10.05995	9.94005	35
26	69144	30856	75146	24854	06002	93998	34
27	69167	30833	75176	24824	06009	93991	33
28	69189	30811	75205	24795	06016	93984	32
29	69212	30788	75235	24765	06023	93977	31
30	9.69234	10.30766	9.75264	10.24736	10.06030	9.93970	30
31	69256	30744	75294	24706	06037	93963	29
32	69279	30721	75323	24677	06045	93955	28
33	69301	30699	75353	24647	06052	93948	27
34	69323	30677	75382	24618	06059	93941	26
35	9.69345	10.30655	9.75411	10.24589	10.06066	9.93934	25
36	69368	30632	75441	24559	06073	93927	24
37	69390	30610	75470	24530	06080	93920	23
38	69412	30588	75500	24500	06088	93912	22
39	69434	30566	75529	24471	06095	93905	21
40	9.69456	10.30544	9.75558	10.24442	10.06102	9.93898	20
41	69479	30521	75588	24412	06109	93891	19
42	69501	30499	75617	24383	06116	93884	18
43	69523	30477	75647	24353	06124	93876	17
44	69545	30455	75676	24324	06131	93869	16
45	9.69567	10.30433	9.75705	10.24295	10.06138	9.93862	15
46	69589	30411	75735	24265	06145	93855	14
47	69611	30389	75764	24236	06153	93847	13
48	69633	30367	75793	24207	06160	93840	12
49	69655	30345	75822	24178	06167	93833	11
50	9.69677	10.30323	9.75852	10.24148	10.06174	9.93826	10
51	69699	30301	75881	24119	06181	93819	9
52	69721	30279	75910	24090	06189	93811	8
53	69743	30257	75939	24061	06196	93804	7
54	69765	30235	75969	24031	06203	93797	6
55	9.69787	10.30213	9.75998	10.24002	10.06211	9.93789	5
56	69809	30191	76027	23973	06218	93782	4
57	69831	30169	76056	23944	06225	93775	3
58	69853	30147	76086	23914	06232	93768	2
59	69875	30125	76115	23885	06240	93760	1
60	69897	30103	76144	23856	06247	93753	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

119° **60°**

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30° **Logarithms.** 149°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.69897	10.30103	9.76144	10.23856	10.06247	9.93753	60
1	69919	30081	76173	23827	06254	93746	59
2	69941	30059	76202	23798	06262	93738	58
3	69963	30037	76231	23769	06269	93731	57
4	69984	30016	76261	23739	06276	93724	56
5	9.70006	10.29994	9.76290	10.23710	10.06283	9.93717	55
6	70028	29972	76319	23681	06291	93709	54
7	70050	29950	76348	23652	06298	93702	53
8	70072	29928	76377	23623	06305	93695	52
9	70093	29907	76406	23594	06313	93687	51
10	9.70115	10.29885	9.76435	10.23565	10.06320	9.93680	50
11	70137	29863	76464	23536	06327	93673	49
12	70159	29841	76493	23507	06335	93665	48
13	70180	29820	76522	23478	06342	93658	47
14	70202	29798	76551	23449	06350	93650	46
15	9.70224	10.29776	9.76580	10.23420	10.06357	9.93643	45
16	70245	29755	76609	23391	06364	93636	44
17	70267	29733	76639	23361	06372	93628	43
18	70288	29712	76668	23332	06379	93621	42
19	70310	29690	76697	23303	06386	93614	41
20	9.70332	10.29668	9.76725	10.23275	10.06394	9.93606	40
21	70353	29647	76754	23246	06401	93599	39
22	70375	29625	76783	23217	06409	93591	38
23	70396	29604	76812	23188	06416	93584	37
24	70418	29582	76841	23159	06423	93577	36
25	9.70439	10.29561	9.76870	10.23130	10.06431	9.93569	35
26	70461	29539	76899	23101	06438	93562	34
27	70482	29518	76928	23072	06446	93554	33
28	70504	29496	76957	23043	06453	93547	32
29	70525	29475	76986	23014	06461	93539	31
30	9.70547	10.29453	9.77015	10.22985	10.06468	9.93532	30
31	70568	29432	77044	22956	06475	93525	29
32	70590	29410	77073	22927	06483	93517	28
33	70611	29389	77101	22899	06490	93510	27
34	70633	29367	77130	22870	06498	93502	26
35	9.70654	10.29346	9.77159	10.22841	10.06505	9.93495	25
36	70675	29325	77188	22812	06513	93487	24
37	70697	29303	77217	22783	06520	93480	23
38	70718	29282	77246	22754	06528	93472	22
39	70739	29261	77274	22726	06535	93465	21
40	9.70761	10.29239	9.77303	10.22697	10.06543	9.93457	20
41	70782	29218	77332	22668	06550	93450	19
42	70803	29197	77361	22639	06558	93442	18
43	70824	29176	77390	22610	06565	93435	17
44	70846	29154	77418	22582	06573	93427	16
45	9.70867	10.29133	9.77447	10.22553	10.06580	9.93420	15
46	70888	29112	77476	22524	06588	93412	14
47	70909	29091	77505	22495	06595	93405	13
48	70931	29069	77533	22467	06603	93397	12
49	70952	29048	77562	22438	06610	93390	11
50	9.70973	10.29027	9.77591	10.22409	10.06618	9.93382	10
51	70994	29006	77619	22381	06625	93375	9
52	71015	28985	77648	22352	06633	93367	8
53	71036	28964	77677	22323	06640	93360	7
54	71058	28942	77706	22294	06648	93352	6
55	9.71079	10.28921	9.77734	10.22266	10.06656	9.93344	5
56	71100	28900	77763	22237	06663	93337	4
57	71121	28879	77791	22209	06671	93329	3
58	71142	28858	77820	22180	06678	93322	2
59	71163	28837	77849	22151	06686	93314	1
60	71184	28816	77877	22123	06693	93307	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

120° 59°

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31° **Logarithms.** **148°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.71184	10.28816	9.77877	10.22123	10.06693	9.93307	60
1	71205	28795	77906	22094	06701	93299	59
2	71226	28774	77935	22065	06709	93291	58
3	71247	28753	77963	22037	06716	93284	57
4	71268	28732	77992	22008	06724	93276	56
5	9.71289	10.28711	9.78020	10.21980	10.06731	9.93269	55
6	71310	28690	78049	21951	06739	93261	54
7	71331	28669	78077	21923	06747	93253	53
8	71352	28648	78106	21894	06754	93246	52
9	71373	28627	78135	21865	06762	93238	51
10	9.71393	10.28607	9.78163	10.21837	10.06770	9.93230	50
11	71414	28586	78192	21808	06777	93223	49
12	71435	28565	78220	21780	06785	93215	48
13	71456	28544	78249	21751	06793	93207	47
14	71477	28523	78277	21723	06800	93200	46
15	9.71498	10.28502	9.78306	10.21694	10.06808	9.93192	45
16	71519	28481	78334	21666	06816	93184	44
17	71539	28461	78363	21637	06823	93177	43
18	71560	28440	78391	21609	06831	93169	42
19	71581	28419	78419	21581	06839	93161	41
20	9.71602	10.28398	9.78448	10.21552	10.06846	9.93154	40
21	71622	28378	78476	21524	06854	93146	39
22	71643	28357	78505	21495	06862	93138	38
23	71664	28336	78533	21467	06869	93131	37
24	71685	28315	78562	21438	06877	93123	36
25	9.71705	10.28295	9.78590	10.21410	10.06885	9.93115	35
26	71726	28274	78618	21382	06892	93108	34
27	71747	28253	78647	21353	06900	93100	33
28	71767	28233	78675	21325	06908	93092	32
29	71788	28212	78704	21296	06916	93084	31
30	9.71809	10.28191	9.78732	10.21268	10.06923	9.93077	30
31	71829	28171	78760	21240	06931	93069	29
32	71850	28150	78789	21211	06939	93061	28
33	71870	28130	78817	21183	06947	93053	27
34	71891	28109	78845	21155	06954	93046	26
35	9.71911	10.28089	9.78874	10.21126	10.06962	9.93038	25
36	71932	28068	78902	21098	06970	93030	24
37	71952	28048	78930	21070	06978	93022	23
38	71973	28027	78959	21041	06986	93014	22
39	71994	28006	78987	21013	06993	93007	21
40	9.72014	10.27986	9.79015	10.20985	10.07001	9.92999	20
41	72034	27966	79043	20957	07009	92991	19
42	72055	27945	79072	20928	07017	92983	18
43	72075	27925	79100	20900	07024	92976	17
44	72096	27904	79128	20872	07032	92968	16
45	9.72116	10.27884	9.79156	10.20844	10.07040	9.92960	15
46	72137	27863	79185	20815	07048	92952	14
47	72157	27843	79213	20787	07056	92944	13
48	72177	27823	79241	20759	07064	92936	12
49	72198	27802	79269	20731	07071	92929	11
50	9.72218	10.27782	9.79297	10.20703	10.07079	9.92921	10
51	72238	27762	79326	20674	07087	92913	9
52	72259	27741	79354	20646	07095	92905	8
53	72279	27721	79382	20618	07103	92897	7
54	72299	27701	79410	20590	07111	92889	6
55	9.72320	10.27680	9.79438	10.20562	10.07119	9.92881	5
56	72340	27660	79466	20534	07126	92874	4
57	72360	27640	79495	20505	07134	92866	3
58	72381	27619	79523	20477	07142	92858	2
59	72401	27599	79551	20449	07150	92850	1
60	72421	27579	79579	20421	07158	92842	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

121° **58°**

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32° **Logarithms.** **147°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.72421	10.27579	9.79579	10.20421	10.07158	9.92842	60
1	72441	27559	79607	20393	07166	92834	59
2	72461	27539	79635	20365	07174	92826	58
3	72482	27518	79663	20337	07182	92818	57
4	72502	27498	79691	20309	07190	92810	56
5	9.72522	10.27478	9.79719	10.20281	10.07197	9.92803	55
6	72542	27458	79747	20253	07205	92795	54
7	72562	27438	79776	20224	07213	92787	53
8	72582	27418	79804	20196	07221	92779	52
9	72602	27398	79832	20168	07229	92771	51
10	9.72622	10.27378	9.79860	10.20140	10.07237	9.92763	50
11	72643	27357	79888	20112	07245	92755	49
12	72663	27337	79916	20084	07253	92747	48
13	72683	27317	79944	20056	07261	92739	47
14	72703	27297	79972	20028	07269	92731	46
15	9.72723	10.27277	9.80000	10.20000	10.07277	9.92723	45
16	72743	27257	80028	19972	07285	92715	44
17	72763	27237	80056	19944	07293	92707	43
18	72783	27217	80084	19916	07301	92699	42
19	72803	27197	80112	19888	07309	92691	41
20	9.72823	10.27177	9.80140	10.19860	10.07317	9.92683	40
21	72843	27157	80168	19832	07325	92675	39
22	72863	27137	80195	19805	07333	92667	38
23	72883	27117	80223	19777	07341	92659	37
24	72902	27098	80251	19749	07349	92651	36
25	9.72922	10.27078	9.80279	10.19721	10.07357	9.92643	35
26	72942	27058	80307	19693	07365	92635	34
27	72962	27038	80335	19665	07373	92627	33
28	72982	27018	80363	19637	07381	92619	32
29	73002	26998	80391	19609	07389	92611	31
30	9.73022	10.26978	9.80419	10.19581	10.07397	9.92603	30
31	73041	26959	80447	19553	07405	92595	29
32	73061	26939	80474	19526	07413	92587	28
33	73081	26919	80502	19498	07421	92579	27
34	73101	26899	80530	19470	07429	92571	26
35	9.73121	10.26879	9.80558	10.19442	10.07437	9.92563	25
36	73140	26860	80586	19414	07445	92555	24
37	73160	26840	80614	19386	07454	92546	23
38	73180	26820	80642	19358	07462	92538	22
39	73200	26800	80669	19331	07470	92530	21
40	9.73219	10.26781	9.80697	10.19303	10.07478	9.92522	20
41	73239	26761	80725	19275	07486	92514	19
42	73259	26741	80753	19247	07494	92506	18
43	73278	26722	80781	19219	07502	92498	17
44	73298	26702	80808	19192	07510	92490	16
45	9.73318	10.26682	9.80836	10.19164	10.07518	9.92482	15
46	73337	26663	80864	19136	07527	92473	14
47	73357	26643	80892	19108	07535	92465	13
48	73377	26623	80919	19081	07543	92457	12
49	73396	26604	80947	19053	07551	92449	11
50	9.73416	10.26584	9.80975	10.19025	10.07559	9.92441	10
51	73435	26565	81003	18997	07567	92433	9
52	73455	26545	81030	18970	07575	92425	8
53	73474	26526	81058	18942	07584	92416	7
54	73494	26506	81086	18914	07592	92408	6
55	9.73513	10.26487	9.81113	10.18887	10.07600	9.92400	5
56	73533	26467	81141	18859	07608	92392	4
57	73552	26448	81169	18831	07616	92384	3
58	73572	26428	81196	18804	07624	92376	2
59	73591	26409	81224	18776	07633	92367	1
60	73611	26389	81252	18748	07641	92359	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

122° **57°**

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33° **Logarithms.** **146°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.73611	10.26389	9.81252	10.18748	10.07641	9.92359	60
1	73630	26370	81279	18721	07649	92351	59
2	73650	26350	81307	18693	07657	92343	58
3	73669	26331	81335	18665	07665	92335	57
4	73689	26311	81362	18638	07674	92326	56
5	9.73708	10.26292	9.81390	10.18610	10.07682	9.92318	55
6	73727	26273	81418	18582	07690	92310	54
7	73747	26253	81445	18555	07698	92302	53
8	73766	26234	81473	18527	07707	92293	52
9	73785	26215	81500	18500	07715	92285	51
10	9.73805	10.26195	9.81528	10.18472	10.07723	9.92277	50
11	73824	26176	81556	18444	07731	92269	49
12	73843	26157	81583	18417	07740	92260	48
13	73863	26137	81611	18389	07748	92252	47
14	73882	26118	81638	18362	07756	92244	46
15	9.73901	10.26099	9.81666	10.18334	10.07765	9.92235	45
16	73921	26079	81693	18307	07773	92227	44
17	73940	26060	81721	18279	07781	92219	43
18	73959	26041	81748	18252	07789	92211	42
19	73978	26022	81776	18224	07798	92202	41
20	9.73997	10.26003	9.81803	10.18197	10.07806	9.92194	40
21	74017	25983	81831	18169	07814	92186	39
22	74036	25964	81858	18142	07823	92177	38
23	74055	25945	81886	18114	07831	92169	37
24	74074	25926	81913	18087	07839	92161	36
25	9.74093	10.25907	9.81941	10.18059	10.07848	9.92152	35
26	74113	25887	81968	18032	07856	92144	34
27	74132	25868	81996	18004	07864	92136	33
28	74151	25849	82023	17977	07873	92127	32
29	74170	25830	82051	17949	07881	92119	31
30	9.74189	10.25811	9.82078	10.17922	10.07889	9.92111	30
31	74208	25792	82106	17894	07898	92102	29
32	74227	25773	82133	17867	07906	92094	28
33	74246	25754	82161	17839	07914	92086	27
34	74265	25735	82188	17812	07923	92077	26
35	9.74284	10.25716	9.82215	10.17785	10.07931	9.92069	25
36	74303	25697	82243	17757	07940	92060	24
37	74322	25678	82270	17730	07948	92052	23
38	74341	25659	82298	17702	07956	92044	22
39	74360	25640	82325	17675	07965	92035	21
40	9.74379	10.25621	9.82352	10.17648	10.07973	9.92027	20
41	74398	25602	82380	17620	07982	92018	19
42	74417	25583	82407	17593	07990	92010	18
43	74436	25564	82435	17565	07998	92002	17
44	74455	25545	82462	17538	08007	91993	16
45	9.74474	10.25526	9.82489	10.17511	10.08015	9.91985	15
46	74493	25507	82517	17483	08024	91976	14
47	74512	25488	82544	17456	08032	91968	13
48	74531	25469	82571	17429	08041	91959	12
49	74549	25451	82599	17401	08049	91951	11
50	9.74568	10.25432	9.82626	10.17374	10.08058	9.91942	10
51	74587	25413	82653	17347	08066	91934	9
52	74606	25394	82681	17319	08075	91925	8
53	74625	25375	82708	17292	08083	91917	7
54	74644	25356	82735	17265	08092	91908	6
55	9.74662	10.25338	9.82762	10.17238	10.08100	9.91900	5
56	74681	25319	82790	17210	08109	91891	4
57	74700	25300	82817	17183	08117	91883	3
58	74719	25281	82844	17156	08126	91874	2
59	74737	25263	82871	17129	08134	91866	1
60	74756	25244	82899	17101	08143	91857	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

123° **56°**

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34° **Logarithms.** 145°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.74756	10.25244	9.82899	10.17101	10.08143	9.91857	60
1	74775	25225	82926	17074	08151	91849	59
2	74794	25206	82953	17047	08160	91840	58
3	74812	25188	82980	17020	08168	91832	57
4	74831	25169	83008	16992	08177	91823	56
5	9.74850	10.25150	9.83035	10.16965	10.08185	9.91815	55
6	74868	25132	83062	16938	08194	91806	54
7	74887	25113	83089	16911	08202	91798	53
8	74906	25094	83117	16883	08211	91789	52
9	74924	25076	83144	16856	08219	91781	51
10	9.74943	10.25057	9.83171	10.16829	10.08228	9.91772	50
11	74961	25039	83198	16802	08237	91763	49
12	74980	25020	83225	16775	08245	91755	48
13	74999	25001	83252	16748	08254	91746	47
14	75017	24983	83280	16720	08262	91738	46
15	9.75036	10.24964	9.83307	10.16693	10.08271	9.91729	45
16	75054	24946	83334	16666	08280	91720	44
17	75073	24927	83361	16639	08288	91712	43
18	75091	24909	83388	16612	08297	91703	42
19	75110	24890	83415	16585	08305	91695	41
20	9.75128	10.24872	9.83442	10.16558	10.08314	9.91686	40
21	75147	24853	83470	16530	08323	91677	39
22	75165	24835	83497	16503	08331	91669	38
23	75184	24816	83524	16476	08340	91660	37
24	75202	24798	83551	16449	08349	91651	36
25	9.75221	10.24779	9.83578	10.16422	10.08357	9.91643	35
26	75239	24761	83605	16395	08366	91634	34
27	75258	24742	83632	16368	08375	91625	33
28	75276	24724	83659	16341	08383	91617	32
29	75294	24706	83686	16314	08392	91608	31
30	9.75313	10.24687	9.83713	10.16287	10.08401	9.91599	30
31	75331	24669	83740	16260	08409	91591	29
32	75350	24650	83768	16232	08418	91582	28
33	75368	24632	83795	16205	08427	91573	27
34	75386	24614	83822	16178	08435	91565	26
35	9.75405	10.24595	9.83849	10.16151	10.08444	9.91556	25
36	75423	24577	83876	16124	08453	91547	24
37	75441	24559	83903	16097	08462	91538	23
38	75459	24541	83930	16070	08470	91530	22
39	75478	24522	83957	16043	08479	91521	21
40	9.75496	10.24504	9.83984	10.16016	10.08488	9.91512	20
41	75514	24486	84011	15989	08496	91504	19
42	75533	24467	84038	15962	08505	91495	18
43	75551	24449	84065	15935	08514	91486	17
44	75569	24431	84092	15908	08523	91477	16
45	9.75587	10.24413	9.84119	10.15881	10.08531	9.91469	15
46	75605	24395	84146	15854	08540	91460	14
47	75624	24376	84173	15827	08549	91451	13
48	75642	24358	84200	15800	08558	91442	12
49	75660	24340	84227	15773	08567	91433	11
50	9.75678	10.24322	9.84254	10.15746	10.08575	9.91425	10
51	75696	24304	84280	15720	08584	91416	9
52	75714	24286	84307	15693	08593	91407	8
53	75733	24267	84334	15666	08602	91398	7
54	75751	24249	84361	15639	08611	91389	6
55	9.75769	10.24231	9.84388	10.15612	10.08619	9.91381	5
56	75787	24213	84415	15585	08628	91372	4
57	75805	24195	84442	15558	08637	91363	3
58	75823	24177	84469	15531	08646	91354	2
59	75841	24159	84496	15504	08655	91345	1
60	75859	24141	84523	15477	08664	91336	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

124° 55°

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35° **Logarithms.** 144°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.75859	10.24141	9.84523	10.15477	10.08664	9.91336	60
1	75877	24123	84550	15450	08672	91328	59
2	75895	24105	84576	15424	08681	91319	58
3	75913	24087	84603	15397	08690	91310	57
4	75931	24069	84630	15370	08699	91301	56
5	9.75949	10.24051	9.84657	10.15343	10.08708	9.91292	55
6	75967	24033	84684	15316	08717	91283	54
7	75985	24015	84711	15289	08726	91274	53
8	76003	23997	84738	15262	08734	91266	52
9	76021	23979	84764	15236	08743	91257	51
10	9.76039	10.23961	9.84791	10.15209	10.08752	9.91248	50
11	76057	23943	84818	15182	08761	91239	49
12	76075	23925	84845	15155	08770	91230	48
13	76093	23907	84872	15128	08779	91221	47
14	76111	23889	84899	15101	08788	91212	46
15	9.76129	10.23871	9.84925	10.15075	10.08797	9.91203	45
16	76146	23854	84952	15048	08806	91194	44
17	76164	23836	84979	15021	08815	91185	43
18	76182	23818	85006	14994	08824	91176	42
19	76200	23800	85033	14967	08833	91167	41
20	9.76218	10.23782	9.85059	10.14941	10.08842	9.91158	40
21	76236	23764	85086	14914	08851	91149	39
22	76253	23747	85113	14887	08859	91141	38
23	76271	23729	85140	14860	08868	91132	37
24	76289	23711	85166	14834	08877	91123	36
25	9.76307	10.23693	9.85193	10.14807	10.08886	9.91114	35
26	76324	23676	85220	14780	08895	91105	34
27	76342	23658	85247	14753	08904	91096	33
28	76360	23640	85273	14727	08913	91087	32
29	76378	23622	85300	14700	08922	91078	31
30	9.76395	10.23605	9.85327	10.14673	10.08931	9.91069	30
31	76413	23587	85354	14646	08940	91060	29
32	76431	23569	85380	14620	08949	91051	28
33	76448	23552	85407	14593	08958	91042	27
34	76466	23534	85434	14566	08967	91033	26
35	9.76484	10.23516	9.85460	10.14540	10.08977	9.91023	25
36	76501	23499	85487	14513	08986	91014	24
37	76519	23481	85514	14486	08995	91005	23
38	76537	23463	85540	14460	09004	90996	22
39	76554	23446	85567	14433	09013	90987	21
40	9.76572	10.23428	9.85594	10.14406	10.09022	9.90978	20
41	76590	23410	85620	14380	09031	90969	19
42	76607	23393	85647	14353	09040	90960	18
43	76625	23375	85674	14326	09049	90951	17
44	76642	23358	85700	14300	09058	90942	16
45	9.76660	10.23340	9.85727	10.14273	10.09067	9.90933	15
46	76677	23323	85754	14246	09076	90924	14
47	76695	23305	85780	14220	09085	90915	13
48	76712	23288	85807	14193	09094	90906	12
49	76730	23270	85834	14166	09104	90896	11
50	9.76747	10.23253	9.85860	10.14140	10.09113	9.90887	10
51	76765	23235	85887	14113	09122	90878	9
52	76782	23218	85913	14087	09131	90869	8
53	76800	23200	85940	14060	09140	90860	7
54	76817	23183	85967	14033	09149	90851	6
55	9.76835	10.23165	9.85993	10.14007	10.09158	9.90842	5
56	76852	23148	86020	13980	09168	90832	4
57	76870	23130	86046	13954	09177	90823	3
58	76887	23113	86073	13927	09186	90814	2
59	76904	23096	86100	13900	09195	90805	1
60	76922	23078	86126	13874	09204	90796	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

125° 54°

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36° **Logarithms.** 143°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.76922	10.23078	9.86126	10.13874	10.09204	9.90796	60
1	76939	23061	86153	13847	09213	90787	59
2	76957	23043	86179	13821	09223	90777	58
3	76974	23026	86206	13794	09232	90768	57
4	76991	23009	86232	13768	09241	90759	56
5	9.77009	10.22991	9.86259	10.13741	10.09250	9.90750	55
6	77026	22974	86285	13715	09259	90741	54
7	77043	22957	86312	13688	09269	90731	53
8	77061	22939	86338	13662	09278	90722	52
9	77078	22922	86365	13635	09287	90713	51
10	9.77095	10.22905	9.86392	10.13608	10.09296	9.90704	50
11	77112	22888	86418	13582	09306	90694	49
12	77130	22870	86445	13555	09315	90685	48
13	77147	22853	86471	13529	09324	90676	47
14	77164	22836	86498	13502	09333	90667	46
15	9.77181	10.22819	9.86524	10.13476	10.09343	9.90657	45
16	77199	22801	86551	13449	09352	90648	44
17	77216	22784	86577	13423	09361	90639	43
18	77233	22767	86603	13397	09370	90630	42
19	77250	22750	86630	13370	09380	90620	41
20	9.77268	10.22732	9.86656	10.13344	10.09389	9.90611	40
21	77285	22715	86683	13317	09398	90602	39
22	77302	22698	86709	13291	09408	90592	38
23	77319	22681	86736	13264	09417	90583	37
24	77336	22664	86762	13238	09426	90574	36
25	9.77353	10.22647	9.86789	10.13211	10.09435	9.90565	35
26	77370	22630	86815	13185	09445	90555	34
27	77387	22613	86842	13158	09454	90546	33
28	77405	22595	86868	13132	09463	90537	32
29	77422	22578	86894	13106	09473	90527	31
30	9.77439	10.22561	9.86921	10.13079	10.09482	9.90518	30
31	77456	22544	86947	13053	09491	90509	29
32	77473	22527	86974	13026	09501	90499	28
33	77490	22510	87000	13000	09510	90490	27
34	77507	22493	87027	12973	09520	90480	26
35	9.77524	10.22476	9.87053	10.12947	10.09529	9.90471	25
36	77541	22459	87079	12921	09538	90462	24
37	77558	22442	87106	12894	09548	90452	23
38	77575	22425	87132	12868	09557	90443	22
39	77592	22408	87158	12842	09566	90434	21
40	9.77609	10.22391	9.87185	10.12815	10.09576	9.90424	20
41	77626	22374	87211	12789	09585	90415	19
42	77643	22357	87238	12762	09595	90405	18
43	77660	22340	87264	12736	09604	90396	17
44	77677	22323	87290	12710	09614	90386	16
45	9.77694	10.22306	9.87317	10.12683	10.09623	9.90377	15
46	77711	22289	87343	12657	09632	90368	14
47	77728	22272	87369	12631	09642	90358	13
48	77744	22256	87396	12604	09651	90349	12
49	77761	22239	87422	12578	09661	90339	11
50	9.77778	10.22222	9.87448	10.12552	10.09670	9.90330	10
51	77795	22205	87475	12525	09680	90320	9
52	77812	22188	87501	12499	09689	90311	8
53	77829	22171	87527	12473	09699	90301	7
54	77846	22154	87554	12446	09708	90292	6
55	9.77862	10.22138	9.87580	10.12420	10.09718	9.90282	5
56	77879	22121	87606	12394	09727	90273	4
57	77896	22104	87633	12367	09737	90263	3
58	77913	22087	87659	12341	09746	90254	2
59	77930	22070	87685	12315	09756	90244	1
60	77946	22054	87711	12289	09765	90235	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

126° 53°

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37° **Logarithms.** **142°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.77946	10.22054	9.87711	10.12289	10.09765	9.90235	60
1	77963	22037	87738	12262	09775	90225	59
2	77980	22020	87764	12236	09784	90216	58
3	77997	22003	87790	12210	09794	90206	57
4	78013	21987	87817	12183	09803	90197	56
5	9.78030	10.21970	9.87843	10.12157	10.09813	9.90187	55
6	78047	21953	87869	12131	09822	90178	54
7	78063	21937	87895	12105	09832	90168	53
8	78080	21920	87922	12078	09841	90159	52
9	78097	21903	87948	12052	09851	90149	51
10	9.78113	10.21887	9.87974	10.12026	10.09861	9.90139	50
11	78130	21870	88000	12000	09870	90130	49
12	78147	21853	88027	11973	09880	90120	48
13	78163	21837	88053	11947	09889	90111	47
14	78180	21820	88079	11921	09899	90101	46
15	9.78197	10.21803	9.88105	10.11895	10.09909	9.90091	45
16	78213	21787	88131	11869	09918	90082	44
17	78230	21770	88158	11842	09928	90072	43
18	78246	21754	88184	11816	09937	90063	42
19	78263	21737	88210	11790	09947	90053	41
20	9.78280	10.21720	9.88236	10.11764	10.09957	9.90043	40
21	78296	21704	88262	11738	09966	90034	39
22	78313	21687	88289	11711	09976	90024	38
23	78329	21671	88315	11685	09986	90014	37
24	78346	21654	88341	11659	09995	90005	36
25	9.78362	10.21638	9.88367	10.11633	10.10005	9.89995	35
26	78379	21621	88393	11607	10015	89985	34
27	78395	21605	88420	11580	10024	89976	33
28	78412	21588	88446	11554	10034	89966	32
29	78428	21572	88472	11528	10044	89956	31
30	9.78445	10.21555	9.88498	10.11502	10.10053	9.89947	30
31	78461	21539	88524	11476	10063	89937	29
32	78478	21522	88550	11450	10073	89927	28
33	78494	21506	88577	11423	10082	89918	27
34	78510	21490	88603	11397	10092	89908	26
35	9.78527	10.21473	9.88629	10.11371	10.10102	9.89898	25
36	78543	21457	88655	11345	10112	89888	24
37	78560	21440	88681	11319	10121	89879	23
38	78576	21424	88707	11293	10131	89869	22
39	78592	21408	88733	11267	10141	89859	21
40	9.78609	10.21391	9.88759	10.11241	10.10151	9.89849	20
41	78625	21375	88780	11214	10160	89840	19
42	78642	21358	88812	11188	10170	89830	18
43	78658	21342	88838	11162	10180	89820	17
44	78674	21326	88864	11136	10190	89810	16
45	9.78691	10.21309	9.88890	10.11110	10.10199	9.89801	15
46	78707	21293	88916	11084	10209	89791	14
47	78723	21277	88942	11058	10219	89781	13
48	78739	21261	88968	11032	10229	89771	12
49	78756	21244	88994	11006	10239	89761	11
50	9.78772	10.21228	9.89020	10.10980	10.10248	9.89752	10
51	78788	21212	89046	10954	10258	89742	9
52	78805	21195	89073	10927	10268	89732	8
53	78821	21179	89099	10901	10278	89722	7
54	78837	21163	89125	10875	10288	89712	6
55	9.78853	10.21147	9.89151	10.10849	10.10298	9.89702	5
56	78869	21131	89177	10823	10307	89693	4
57	78886	21114	89203	10797	10317	89683	3
58	78902	21098	89229	10771	10327	89673	2
59	78918	21082	89255	10745	10337	89663	1
60	78934	21066	89281	10719	10347	89653	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

127° **52°**

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38° **Logarithms.** 141°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.78934	10.21066	9.89281	10.10719	10.10347	9.89653	60
1	78950	21050	89307	10693	10357	89643	59
2	78967	21033	89333	10667	10367	89633	58
3	78983	21017	89359	10641	10376	89624	57
4	78999	21001	89385	10615	10386	89614	56
5	9.79015	10.20985	9.89411	10.10589	10.10396	9.89604	55
6	79031	20969	89437	10563	10406	89594	54
7	79047	20953	89463	10537	10416	89584	53
8	79063	20937	89489	10511	10426	89574	52
9	79079	20921	89515	10485	10436	89564	51
10	9.79095	10.20905	9.89541	10.10459	10.10446	9.89554	50
11	79111	20889	89567	10433	10456	89544	49
12	79128	20872	89593	10407	10466	89534	48
13	79144	20856	89619	10381	10476	89524	47
14	79160	20840	89645	10355	10486	89514	46
15	9.79176	10.20824	9.89671	10.10329	10.10496	9.89504	45
16	79192	20808	89697	10303	10505	89495	44
17	79208	20792	89723	10277	10515	89485	43
18	79224	20776	89749	10251	10525	89475	42
19	79240	20760	89775	10225	10535	89465	41
20	9.79256	10.20744	9.89801	10.10199	10.10545	9.89455	40
21	79272	20728	89827	10173	10555	89445	39
22	79288	20712	89853	10147	10565	89435	38
23	79304	20696	89879	10121	10575	89425	37
24	79319	20681	89905	10095	10585	89415	36
25	9.79335	10.20665	9.89931	10.10069	10.10595	9.89405	35
26	79351	20649	89957	10043	10605	89395	34
27	79367	20633	89983	10017	10615	89385	33
28	79383	20617	90009	09991	10625	89375	32
29	79399	20601	90035	09965	10636	89364	31
30	9.79415	10.20585	9.90061	10.09939	10.10646	9.89354	30
31	79431	20569	90086	09914	10656	89344	29
32	79447	20553	90112	09888	10666	89334	28
33	79463	20537	90138	09862	10676	89324	27
34	79478	20522	90164	09836	10686	89314	26
35	9.79494	10.20506	9.90190	10.09810	10.10696	9.89304	25
36	79510	20490	90216	09784	10706	89294	24
37	79526	20474	90242	09758	10716	89284	23
38	79542	20458	90268	09732	10726	89274	22
39	79558	20442	90294	09706	10736	89264	21
40	9.79573	10.20427	9.90320	10.09680	10.10746	9.89254	20
41	79589	20411	90346	09654	10756	89244	19
42	79605	20395	90371	09629	10767	89233	18
43	79621	20379	90397	09603	10777	89223	17
44	79636	20364	90423	09577	10787	89213	16
45	9.79652	10.20348	9.90449	10.09551	10.10797	9.89203	15
46	79668	20332	90475	09525	10807	89193	14
47	79684	20316	90501	09499	10817	89183	13
48	79699	20301	90527	09473	10827	89173	12
49	79715	20285	90553	09447	10838	89162	11
50	9.79731	10.20269	9.90578	10.09422	10.10848	9.89152	10
51	79746	20254	90604	09396	10858	89142	9
52	79762	20238	90630	09370	10868	89132	8
53	79778	20222	90656	09344	10878	89122	7
54	79793	20207	90682	09318	10888	89112	6
55	9.79809	10.20191	9.90708	10.09292	10.10899	9.89101	5
56	79825	20175	90734	09266	10909	89091	4
57	79840	20160	90759	09241	10919	89081	3
58	79856	20144	90785	09215	10929	89071	2
59	79872	20128	90811	09189	10940	89060	1
60	79887	20113	90837	09163	10950	89050	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

128° 51°

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39° **Logarithms.** 140°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.79887	10.20113	9.90837	10.09163	10.10950	9.89050	60
1	79903	20097	90863	09137	10960	89040	59
2	79918	20082	90889	09111	10970	89030	58
3	79934	20066	90914	09086	10980	89020	57
4	79950	20050	90940	09060	10991	89009	56
5	9.79965	10.20035	9.90966	10.09034	10.11001	9.88999	55
6	79981	20019	90992	09008	11011	88989	54
7	79996	20004	91018	08982	11022	88978	53
8	80012	19988	91043	08957	11032	88968	52
9	80027	19973	91069	08931	11042	88958	51
10	9.80043	10.19957	9.91095	10.08905	10.11052	9.88948	50
11	80058	19942	91121	08879	11063	88937	49
12	80074	19926	91147	08853	11073	88927	48
13	80089	19911	91172	08828	11083	88917	47
14	80105	19895	91198	08802	11094	88906	46
15	9.80120	10.19880	9.91224	10.08776	10.11104	9.88896	45
16	80136	19864	91250	08750	11114	88886	44
17	80151	19849	91276	08724	11125	88875	43
18	80166	19834	91301	08699	11135	88865	42
19	80182	19818	91327	08673	11145	88855	41
20	9.80197	10.19803	9.91353	10.08647	10.11156	9.88844	40
21	80213	19787	91379	08621	11166	88834	39
22	80228	19772	91404	08596	11176	88824	38
23	80244	19756	91430	08570	11187	88813	37
24	80259	19741	91456	08544	11197	88803	36
25	9.80274	10.19726	9.91482	10.08518	10.11207	9.88793	35
26	80290	19710	91507	08493	11218	88782	34
27	80305	19695	91533	08467	11228	88772	33
28	80320	19680	91559	08441	11239	88761	32
29	80336	19664	91585	08415	11249	88751	31
30	9.80351	10.19649	9.91610	10.08390	10.11259	9.88741	30
31	80366	19634	91636	08364	11270	88730	29
32	80382	19618	91662	08338	11280	88720	28
33	80397	19603	91688	08312	11291	88709	27
34	80412	19588	91713	08287	11301	88699	26
35	9.80428	10.19572	9.91739	10.08261	10.11312	9.88688	25
36	80443	19557	91765	08235	11322	88678	24
37	80458	19542	91791	08209	11332	88668	23
38	80473	19527	91816	08184	11343	88657	22
39	80489	19511	91842	08158	11353	88647	21
40	9.80504	10.19496	9.91868	10.08132	10.11364	9.88636	20
41	80519	19481	91893	08107	11374	88626	19
42	80534	19466	91919	08081	11385	88615	18
43	80550	19450	91945	08055	11395	88605	17
44	80565	19435	91971	08029	11406	88594	16
45	9.80580	10.19420	9.91996	10.08004	10.11416	9.88584	15
46	80595	19405	92022	07978	11427	88573	14
47	80610	19390	92048	07952	11437	88563	13
48	80625	19375	92073	07927	11448	88552	12
49	80641	19359	92099	07901	11458	88542	11
50	9.80656	10.19344	9.92125	10.07875	10.11469	9.88531	10
51	80671	19329	92150	07850	11479	88521	9
52	80686	19314	92176	07824	11490	88510	8
53	80701	19299	92202	07798	11501	88499	7
54	80716	19284	92227	07773	11511	88489	6
55	9.80731	10.19269	9.92253	10.07747	10.11522	9.88478	5
56	80746	19254	92279	07721	11532	88468	4
57	80762	19238	92304	07696	11543	88457	3
58	80777	19223	92330	07670	11553	88447	2
59	80792	19208	92356	07644	11564	88436	1
60	80807	19193	92381	07619	11575	88425	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

129° 50°

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40° **Logarithms.** 139°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.80807	10.19193	9.92381	10.07619	10.11575	9.88425	60
1	80822	19178	92407	07593	11585	88415	59
2	80837	19163	92433	07567	11596	88404	58
3	80852	19148	92458	07542	11606	88394	57
4	80867	19133	92484	07516	11617	88383	56
5	9.80882	10.19118	9.92510	10.07490	10.11628	9.88372	55
6	80897	19103	92535	07465	11638	88362	54
7	80912	19088	92561	07439	11649	88351	53
8	80927	19073	92587	07413	11660	88340	52
9	80942	19058	92612	07388	11670	88330	51
10	9.80957	10.19043	9.92638	10.07362	10.11681	9.88319	50
11	80972	19028	92663	07337	11692	88308	49
12	80987	19013	92689	07311	11702	88298	48
13	81002	18998	92715	07285	11713	88287	47
14	81017	18983	92740	07260	11724	88276	46
15	9.81032	10.18968	9.92766	10.07234	10.11734	9.88266	45
16	81047	18953	92792	07208	11745	88255	44
17	81061	18939	92817	07183	11756	88244	43
18	81076	18924	92843	07157	11766	88234	42
19	81091	18909	92868	07132	11777	88223	41
20	9.81106	10.18894	9.92894	10.07106	10.11788	9.88212	40
21	81121	18879	92920	07080	11799	88201	39
22	81136	18864	92945	07055	11809	88191	38
23	81151	18849	92971	07029	11820	88180	37
24	81166	18834	92996	07004	11831	88169	36
25	9.81180	10.18820	9.93022	10.06978	10.11842	9.88158	35
26	81195	18805	93048	06952	11852	88148	34
27	81210	18790	93073	06927	11863	88137	33
28	81225	18775	93099	06901	11874	88126	32
29	81240	18760	93124	06876	11885	88115	31
30	9.81254	10.18746	9.93150	10.06850	10.11895	9.88105	30
31	81269	18731	93175	06825	11906	88094	29
32	81284	18716	93201	06799	11917	88083	28
33	81299	18701	93227	06773	11928	88072	27
34	81314	18686	93252	06748	11939	88061	26
35	9.81328	10.18672	9.93278	10.06722	10.11949	9.88051	25
36	81343	18657	93303	06697	11960	88040	24
37	81358	18642	93329	06671	11971	88029	23
38	81372	18628	93354	06646	11982	88018	22
39	81387	18613	93380	06620	11993	88007	21
40	9.81402	10.18598	9.93406	10.06594	10.12004	9.87996	20
41	81417	18583	93431	06569	12015	87985	19
42	81431	18569	93457	06543	12025	87975	18
43	81446	18554	93482	06518	12036	87964	17
44	81461	18539	93508	06492	12047	87953	16
45	9.81475	10.18525	9.93533	10.06467	10.12058	9.87942	15
46	81490	18510	93559	06441	12069	87931	14
47	81505	18495	93584	06416	12080	87920	13
48	81519	18481	93610	06390	12091	87909	12
49	81534	18466	93636	06364	12102	87898	11
50	9.81549	10.18451	9.93661	10.06339	10.12113	9.87887	10
51	81563	18437	93687	06313	12123	87877	9
52	81578	18422	93712	06288	12134	87866	8
53	81592	18408	93738	06262	12145	87855	7
54	81607	18393	93763	06237	12156	87844	6
55	9.81622	10.18378	9.93789	10.06211	10.12167	9.87833	5
56	81636	18364	93814	06186	12178	87822	4
57	81651	18349	93840	06160	12189	87811	3
58	81665	18335	93865	06135	12200	87800	2
59	81680	18320	93891	06109	12211	87789	1
60	81694	18306	93916	06084	12222	87778	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

130° 49°

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41° **Logarithms.** 138°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.81694	10.18306	9.93916	10.06084	10.12222	9.87778	60
1	81709	18291	93942	06058	12233	87767	59
2	81723	18277	93967	06033	12244	87756	58
3	81738	18262	93993	06007	12255	87745	57
4	81752	18248	94018	05982	12266	87734	56
5	9.81767	10.18233	9.94044	10.05956	10.12277	9.87723	55
6	81781	18219	94069	05931	12288	87712	54
7	81796	18204	94095	05905	12299	87701	53
8	81810	18190	94120	05880	12310	87690	52
9	81825	18175	94146	05854	12321	87679	51
10	9.81839	10.18161	9.94171	10.05829	10.12332	9.87668	50
11	81854	18146	94197	05803	12343	87657	49
12	81868	18132	94222	05778	12354	87646	48
13	81882	18118	94248	05752	12365	87635	47
14	81897	18103	94273	05727	12376	87624	46
15	9.81911	10.18089	9.94299	10.05701	10.12387	9.87613	45
16	81926	18074	94324	05676	12399	87601	44
17	81940	18060	94350	05650	12410	87590	43
18	81955	18045	94375	05625	12421	87579	42
19	81969	18031	94401	05599	12432	87568	41
20	9.81983	10.18017	9.94426	10.05574	10.12443	9.87557	40
21	81998	18002	94452	05548	12454	87546	39
22	82012	17988	94477	05523	12465	87535	38
23	82026	17974	94503	05497	12476	87524	37
24	82041	17959	94528	05472	12487	87513	36
25	9.82055	10.17945	9.94554	10.05446	10.12499	9.87501	35
26	82069	17931	94579	05421	12510	87490	34
27	82084	17916	94604	05396	12521	87479	33
28	82098	17902	94630	05370	12532	87468	32
29	82112	17888	94655	05345	12543	87457	31
30	9.82126	10.17874	9.94681	10.05319	10.12554	9.87446	30
31	82141	17859	94706	05294	12566	87434	29
32	82155	17845	94732	05268	12577	87423	28
33	82169	17831	94757	05243	12588	87412	27
34	82184	17816	94783	05217	12599	87401	26
35	9.82198	10.17802	9.94808	10.05192	10.12610	9.87390	25
36	82212	17788	94834	05166	12622	87378	24
37	82226	17774	94859	05141	12633	87367	23
38	82240	17760	94884	05116	12644	87356	22
39	82255	17745	94910	05090	12655	87345	21
40	9.82269	10.17731	9.94935	10.05065	10.12666	9.87334	20
41	82283	17717	94961	05039	12678	87322	19
42	82297	17703	94986	05014	12689	87311	18
43	82311	17689	95012	04988	12700	87300	17
44	82326	17674	95037	04963	12712	87288	16
45	9.82340	10.17660	9.95062	10.04938	10.12723	9.87277	15
46	82354	17646	95088	04912	12734	87266	14
47	82368	17632	95113	04887	12745	87255	13
48	82382	17618	95139	04861	12757	87243	12
49	82396	17604	95164	04836	12768	87232	11
50	9.82410	10.17590	9.95190	10.04810	10.12779	9.87221	10
51	82424	17576	95215	04785	12791	87209	9
52	82439	17561	95240	04760	12802	87198	8
53	82453	17547	95266	04734	12813	87187	7
54	82467	17533	95291	04709	12825	87175	6
55	9.82481	10.17519	9.95317	10.04683	10.12836	9.87164	5
56	82495	17505	95342	04658	12847	87153	4
57	82509	17491	95368	04632	12859	87141	3
58	82523	17477	95393	04607	12870	87130	2
59	82537	17463	95418	04582	12881	87119	1
60	82551	17449	95444	04556	12893	87107	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

131° 48°

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42°			Logarithms.				137°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.82551	10.17449	9.95444	10.04556	10.12893	9.87107	60
1	82565	17435	95469	04531	12904	87096	59
2	82579	17421	95495	04505	12915	87085	58
3	82593	17407	95520	04480	12927	87073	57
4	82607	17393	95545	04455	12938	87062	56
5	9.82621	10.17379	9.95571	10.04429	10.12950	9 87050	55
6	82635	17365	95596	04404	12961	87039	54
7	82649	17351	95622	04378	12972	87028	53
8	82663	17337	95647	04353	12984	87016	52
9	82677	17323	95672	04328	12995	87005	51
10	9.82691	10.17309	9.95698	10.04302	10.13007	9.86993	50
11	82705	17295	95723	04277	13018	86982	49
12	82719	17281	95748	04252	13030	86970	48
13	82733	17267	95774	04226	13041	86959	47
14	82747	17253	95799	04201	13053	86947	46
15	9.82761	10.17239	9.95825	10.04175	10.13064	9.86936	45
16	82775	17225	95850	04150	13076	86924	44
17	82788	17212	95875	04125	13087	86913	43
18	82802	17198	95901	04099	13098	86902	42
19	82816	17184	95926	04074	13110	86890	41
20	9.82830	10.17170	9.95952	10.04048	10.13121	9.86879	40
21	82844	17156	95977	04023	13133	86867	39
22	82858	17142	96002	03998	13145	86855	38
23	82872	17128	96028	03972	13156	86844	37
24	82885	17115	96053	03947	13168	86832	36
25	9.82899	10.17101	9.96078	10.03922	10.13179	9.86821	35
26	82913	17087	96104	03896	13191	86809	34
27	82927	17073	96129	03871	13202	86798	33
28	82941	17059	96155	03845	13214	86786	32
29	82955	17045	96180	03820	13225	86775	31
30	9.82968	10.17032	9.96205	10.03795	10.13237	9.86763	30
31	82982	17018	96231	03769	13248	86752	29
32	82996	17004	96256	03744	13260	86740	28
33	83010	16990	96281	03719	13272	86728	27
34	83023	16977	96307	03693	13283	86717	26
35	9.83037	10.16963	9.96332	10.03668	10.13295	9.86705	25
36	83051	16949	96357	03643	13306	86694	24
37	83065	16935	96383	03617	13318	86682	23
38	83078	16922	96408	03592	13330	86670	22
39	83092	16908	96433	03567	13341	86659	21
40	9.83106	10.16894	9.96459	10.03541	10.13353	9.86647	20
41	83120	16880	96484	03516	13365	86635	19
42	83133	16867	96510	03490	13376	86624	18
43	83147	16853	96535	03465	13388	86612	17
44	83161	16839	96560	03440	13400	86600	16
45	9.83174	10.16826	9.96586	10.03414	10.13411	9.86589	15
46	83188	16812	96611	03389	13423	86577	14
47	83202	16798	96636	03364	13435	86565	13
48	83215	16785	96662	03338	13446	86554	12
49	83229	16771	96687	03313	13458	86542	11
50	9.83242	10.16758	9.96712	10.03288	10.13470	9.86530	10
51	83256	16744	96738	03262	13482	86518	9
52	83270	16730	96763	03237	13493	86507	8
53	83283	16717	96788	03212	13505	86495	7
54	83297	16703	96814	03186	13517	86483	6
55	9.83310	10.16690	9.96839	10.03161	10.13528	9.86472	5
56	83324	16676	96864	03136	13540	86460	4
57	83338	16662	96890	03110	13552	86448	3
58	83351	16649	96915	03085	13564	86436	2
59	83365	16635	96940	03060	13575	86425	1
60	83378	16622	96966	03034	13587	86413	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
132°							47°

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43° **Logarithms.** 136°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.83378	10.16622	9.96966	10.03034	10.13587	9.86413	60
1	83392	16608	96991	03009	13599	86401	59
2	83405	16595	97016	02984	13611	86389	58
3	83419	16581	97042	02958	13623	86377	57
4	83432	16568	97067	02933	13634	86366	56
5	9.83446	10.16554	9.97092	10.02908	10.13646	9.86354	55
6	83459	16541	97118	02882	13658	86342	54
7	83473	16527	97143	02857	13670	86330	53
8	83486	16514	97168	02832	13682	86318	52
9	83500	16500	97193	02807	13694	86306	51
10	9.83513	10.16487	9.97219	10.02781	10.13705	9.86295	50
11	83527	16473	97244	02756	13717	86283	49
12	83540	16460	97269	02731	13729	86271	48
13	83554	16446	97295	02705	13741	86259	47
14	83567	16433	97320	02680	13753	86247	46
15	9.83581	10.16419	9.97345	10.02655	10.13765	9.86235	45
16	83594	16406	97371	02629	13777	86223	44
17	83608	16392	97396	02604	13789	86211	43
18	83621	16379	97421	02579	13800	86200	42
19	83634	16366	97447	02553	13812	86188	41
20	9.83648	10.16352	9.97472	10.02528	10.13824	9.86176	40
21	83661	16339	97497	02503	13836	86164	39
22	83674	16326	97523	02477	13848	86152	38
23	83688	16312	97548	02452	13860	86140	37
24	83701	16299	97573	02427	13872	86128	36
25	9.83715	10.16285	9.97598	10.02402	10.13884	9.86116	35
26	83728	16272	97624	02376	13896	86104	34
27	83741	16259	97649	02351	13908	86092	33
28	83755	16245	97674	02326	13920	86080	32
29	83768	16232	97700	02300	13932	86068	31
30	9.83781	10.16219	9.97725	10.02275	10.13944	9.86056	30
31	83795	16205	97750	02250	13956	86044	29
32	83808	16192	97776	02224	13968	86032	28
33	83821	16179	97801	02199	13980	86020	27
34	83834	16166	97826	02174	13992	86008	26
35	9.83848	10.16152	9.97851	10.02149	10.14004	9.85996	25
36	83861	16139	97877	02123	14016	85984	24
37	83874	16126	97902	02098	14028	85972	23
38	83887	16113	97927	02073	14040	85960	22
39	83901	16099	97953	02047	14052	85948	21
40	9.83914	10.16086	9.97978	10.02022	10.14064	9.85936	20
41	83927	16073	98003	01997	14076	85924	19
42	83940	16060	98029	01971	14088	85912	18
43	83954	16046	98054	01946	14100	85900	17
44	83967	16033	98079	01921	14112	85888	16
45	9.83980	10.16020	9.98104	10.01896	10.14124	9.85876	15
46	83993	16007	98130	01870	14136	85864	14
47	84006	15994	98155	01845	14149	85851	13
48	84020	15980	98180	01820	14161	85839	12
49	84033	15967	98206	01794	14173	85827	11
50	9.84046	10.15954	9.98231	10.01769	10.14185	9.85815	10
51	84059	15941	98256	01744	14197	85803	9
52	84072	15928	98281	01719	14209	85791	8
53	84085	15915	98307	01693	14221	85779	7
54	84098	15902	98332	01668	14234	85766	6
55	9.84112	10.15888	9.98357	10.01643	10.14246	9.85754	5
56	84125	15875	98383	01617	14258	85742	4
57	84138	15862	98408	01592	14270	85730	3
58	84151	15849	98433	01567	14282	85718	2
59	84164	15836	98458	01542	14294	85706	1
60	84177	15823	98484	01516	14307	85693	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

133° 46°

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44° **Logarithms.** 135°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.84177	10.15823	9.98484	10.01516	10.14307	9.85693	60
1	84190	15810	98509	01491	14319	85681	59
2	84203	15797	98534	01466	14331	85669	58
3	84216	15784	98560	01440	14343	85657	57
4	84229	15771	98585	01415	14355	85645	56
5	9.84242	10.15758	9.98610	10.01390	10.14368	9.85632	55
6	84255	15745	98635	01365	14380	85620	54
7	84269	15731	98661	01339	14392	85608	53
8	84282	15718	98686	01314	14404	85596	52
9	84295	15705	98711	01289	14417	85583	51
10	9.84308	10.15692	9.98737	10.01263	10.14429	9.85571	50
11	84321	15679	98762	01238	14441	85559	49
12	84334	15666	98787	01213	14453	85547	48
13	84347	15653	98812	01188	14466	85534	47
14	84360	15640	98838	01162	14478	85522	46
15	9.84373	10.15627	9.98863	10.01137	10.14490	9.85510	45
16	84385	15615	98888	01112	14503	85497	44
17	84398	15602	98913	01087	14515	85485	43
18	84411	15589	98939	01061	14527	85473	42
19	84424	15576	98964	01036	14540	85460	41
20	9.84437	10.15563	9.98989	10.01011	10.14552	9.85448	40
21	84450	15550	99015	00985	14564	85436	39
22	84463	15537	99040	00960	14577	85423	38
23	84476	15524	99065	00935	14589	85411	37
24	84489	15511	99090	00910	14601	85399	36
25	9.84502	10.15498	9.99116	10.00884	10.14614	9.85386	35
26	84515	15485	99141	00859	14626	85374	34
27	84528	15472	99166	00834	14639	85361	33
28	84540	15460	99191	00809	14651	85349	32
29	84553	15447	99217	00783	14663	85337	31
30	9.84566	10.15434	9.99242	10.00758	10.14676	9.85324	30
31	84579	15421	99267	00733	14688	85312	29
32	84592	15408	99293	00707	14701	85299	28
33	84605	15395	99318	00682	14713	85287	27
34	84618	15382	99343	00657	14726	85274	26
35	9.84630	10.15370	9.99368	10.00632	10.14738	9.85262	25
36	84643	15357	99394	00606	14750	85250	24
37	84656	15344	99419	00581	14763	85237	23
38	84669	15331	99444	00556	14775	85225	22
39	84682	15318	99469	00531	14788	85212	21
40	9.84694	10.15306	9.99495	10.00505	10.14800	9.85200	20
41	84707	15293	99520	00480	14813	85187	19
42	84720	15280	99545	00455	14825	85175	18
43	84733	15267	99570	00430	14838	85162	17
44	84745	15255	99596	00404	14850	85150	16
45	9.84758	10.15242	9.99621	10.00379	10.14863	9.85137	15
46	84771	15229	99646	00354	14875	85125	14
47	84784	15216	99672	00328	14888	85112	13
48	84796	15204	99697	00303	14900	85100	12
49	84809	15191	99722	00278	14913	85087	11
50	9.84822	10.15178	9.99747	10.00253	10.14926	9.85074	10
51	84835	15165	99773	00227	14938	85062	9
52	84847	15153	99798	00202	14951	85049	8
53	84860	15140	99823	00177	14963	85037	7
54	84873	15127	99848	00152	14976	85024	6
55	9.84885	10.15115	9.99874	10.00126	10.14988	9.85012	5
56	84898	15102	99899	00101	15001	84999	4
57	84911	15089	99924	00076	15014	84986	3
58	84923	15077	99949	00051	15026	84974	2
59	84936	15064	99975	00025	15039	84961	1
60	84949	15051	10.00000	00000	15051	84949	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

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0°				Natural Trigonometrical Functions.					179°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.00000	1.0000	Infinite.	.00000	Infinite.	1.0000	.00000	1.0000	60
1	. 0029	.99971	3437.7	. 0029	3437.7	.0000	. 0000	.0000	59
2	. 0058	. 9942	1718.9	. 0058	1718.9	.0000	. 0000	.0000	58
3	. 0087	. 9913	1145.9	. 0087	1145.9	.0000	. 0000	.0000	57
4	. 0116	. 9884	859.44	. 0116	859.44	.0000	. 0000	.0000	56
5	.00145	.99854	687.55	.00145	687.55	1.0000	.00000	1.0000	55
6	. 0174	. 9825	572.96	. 0174	572.96	.0000	. 0000	.0000	54
7	. 0204	. 9796	491.11	. 0204	491.11	.0000	. 0000	.0000	53
8	. 0233	. 9767	429.72	. 0233	429.72	.0000	. 0000	.0000	52
9	. 0262	. 9738	381.97	. 0262	381.97	.0000	. 0000	.0000	51
10	.00291	.99709	343.77	.00291	343.77	1.0000	.00000	.99999	50
11	. 0320	. 9680	312.52	. 0320	312.52	.0000	. 0000	. 9999	49
12	. 0349	. 9651	286.48	. 0349	286.48	.0000	. 0001	. 9999	48
13	. 0378	. 9622	64.44	. 0378	64.44	.0000	. 0001	. 9999	47
14	. 0407	. 9593	45.55	. 0407	45.55	.0000	. 0001	. 9999	46
15	.00436	.99564	229.18	.00436	229.18	1.0000	.00001	.99999	45
16	. 0465	. 9534	14.86	. 0465	14.86	.0000	. 0001	. 9999	44
17	. 0494	. 9505	02.22	. 0494	02.22	.0000	. 0001	. 9999	43
18	. 0524	. 9476	190.99	. 0524	190.98	.0000	. 0001	. 9999	42
19	. 0553	. 9447	80.93	. 0553	80.93	.0000	. 0001	. 9998	41
20	.00582	.99418	171.89	.00582	171.88	1.0000	.00002	.99998	40
21	. 0611	. 9389	63.70	. 0611	63.70	.0000	. 0002	. 9998	39
22	. 0640	. 9360	56.26	. 0640	56.26	.0000	. 0002	. 9998	38
23	. 0669	. 9331	49.47	. 0669	49.46	.0000	. 0002	. 9998	37
24	. 0698	. 9302	43.24	. 0698	43.24	.0000	. 0002	. 9997	36
25	.00727	.99273	137.51	.00727	137.51	1.0000	.00003	.99997	35
26	. 0756	. 9244	32.22	. 0756	32.22	.0000	. 0003	. 9997	34
27	. 0785	. 9215	27.32	. 0785	27.32	.0000	. 0003	. 9997	33
28	. 0814	. 9185	22.78	. 0814	22.77	.0000	. 0003	. 9997	32
29	. 0843	. 9156	18.54	. 0844	18.54	.0000	. 0003	. 9996	31
30	.00873	.99127	114.59	.00873	114.59	1.0000	.00004	.99996	30
31	. 0902	. 9098	10.90	. 0902	10.89	.0000	. 0004	. 9996	29
32	. 0931	. 9069	07.43	. 0931	07.43	.0000	. 0004	. 9996	28
33	. 0960	. 9040	04.17	. 0960	04.17	.0000	. 0005	. 9995	27
34	. 0989	. 9011	01.11	. 0989	01.11	.0000	. 0005	. 9995	26
35	.01018	.98982	98.223	.01018	98.218	1.0000	.00005	.99995	25
36	. 1047	. 8953	5.495	. 1047	5.489	.0000	. 0005	. 9994	24
37	. 1076	. 8924	2.914	. 1076	2.908	.0000	. 0006	. 9994	23
38	. 1105	. 8895	0.469	. 1105	0.463	.0001	. 0006	. 9994	22
39	. 1134	. 8865	88.149	. 1134	88.143	.0001	. 0006	. 9993	21
40	.01163	.98836	85.946	.01164	85.940	1.0001	.00007	.99993	20
41	. 1193	. 8807	3.849	. 1193	3.843	.0001	. 0007	. 9993	19
42	. 1222	. 8778	1.853	. 1222	1.847	.0001	. 0007	. 9992	18
43	. 1251	. 8749	79.950	. 1251	79.943	.0001	. 0008	. 9992	17
44	. 1280	. 8720	8.133	. 1280	8.126	.0001	. 0008	. 9992	16
45	.01309	.98691	76.396	.01309	76.390	1.0001	.00008	.99991	15
46	. 1338	. 8662	4.736	. 1338	4.729	.0001	. 0009	. 9991	14
47	. 1367	. 8633	3.146	. 1367	3.139	.0001	. 0009	. 9991	13
48	. 1396	. 8604	1.622	. 1396	1.615	.0001	. 0010	. 9990	12
49	. 1425	. 8575	0.160	. 1425	0.153	.0001	. 0010	. 9990	11
50	.01454	.98546	68.757	.01454	68.750	1.0001	.00010	.99989	10
51	. 1483	. 8516	7.409	. 1484	7.402	.0001	. 0011	. 9989	9
52	. 1512	. 8487	6.113	. 1513	6.105	.0001	. 0011	. 9988	8
53	. 1542	. 8458	4.866	. 1542	4.858	.0001	. 0012	. 9988	7
54	. 1571	. 8429	3.664	. 1571	3.657	.0001	. 0012	. 9988	6
55	.01600	.98400	62.507	.01600	62.499	1.0001	.00013	.99987	5
56	. 1629	. 8371	1.391	. 1629	1.383	.0001	. 0013	. 9987	4
57	. 1658	. 8342	0.314	. 1658	0.306	.0001	. 0014	. 9987	3
58	. 1687	. 8313	59.274	. 1687	59.266	.0001	. 0014	. 9986	2
59	. 1716	. 8284	8.270	. 1716	8.261	.0001	. 0015	. 9985	1
60	. 1745	. 8255	7.299	. 1745	7.290	.0001	. 0015	. 9985	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
90°									89°

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1° **Natural Trigonometrical Functions.** **178°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.01745	.98255	57.299	.01745	57.290	1.0001	.00015	.99985	60
1	. 1774	. 8226	6.359	. 1775	6.350	.0001	. 0016	. 9984	59
2	. 1803	. 8196	5.450	. 1804	5.441	.0001	. 0016	. 9984	58
3	. 1832	. 8167	4.570	. 1833	4.561	.0002	. 0017	. 9983	57
4	. 1861	. 8138	3.718	. 1862	3.708	.0002	. 0017	. 9983	56
5	.01891	.98109	52.891	.01891	52.882	1.0002	.00018	.99982	55
6	. 1920	. 8080	2.090	. 1920	2.081	.0002	. 0018	. 9981	54
7	. 1949	. 8051	1.313	. 1949	1.303	.0002	. 0019	. 9981	53
8	. 1978	. 8022	0.558	. 1978	0.548	.0002	. 0019	. 9980	52
9	. 2007	. 7993	49.826	. 2007	49.816	.0002	. 0020	. 9980	51
10	.02036	.97964	49.114	.02036	49.104	1.0002	.00021	.99979	50
11	. 2065	. 7935	8.422	. 2066	8.412	.0002	. 0021	. 9979	49
12	. 2094	. 7906	7.750	. 2095	7.739	.0002	. 0022	. 9978	48
13	. 2123	. 7877	7.096	. 2124	7.085	.0002	. 0022	. 9977	47
14	. 2152	. 7847	6.460	. 2153	6.449	.0002	. 0023	. 9977	46
15	.02181	.97818	45.840	.02182	45.829	1.0002	.00024	.99976	45
16	. 2210	. 7789	5.237	. 2211	5.226	.0002	. 0024	. 9975	44
17	. 2240	. 7760	4.650	. 2240	4.638	.0002	. 0025	. 9975	43
18	. 2269	. 7731	4.077	. 2269	4.066	.0002	. 0026	. 9974	42
19	. 2298	. 7702	3.520	. 2298	3.508	.0003	. 0026	. 9974	41
20	.02327	.97673	42.976	.02327	42.964	1.0003	.00027	.99973	40
21	. 2356	. 7644	2.445	. 2357	2.433	.0003	. 0028	. 9972	39
22	. 2385	. 7615	1.928	. 2386	1.916	.0003	. 0028	. 9971	38
23	. 2414	. 7586	1.423	. 2415	1.410	.0003	. 0029	. 9971	37
24	. 2443	. 7557	0.930	. 2444	0.917	.0003	. 0030	. 9970	36
25	.02472	.97528	40.448	.02473	40.436	1.0003	.00030	.99969	35
26	. 2501	. 7499	39.978	. 2502	39.965	.0003	. 0031	. 9969	34
27	. 2530	. 7469	9.518	. 2531	9.506	.0003	. 0032	. 9968	33
28	. 2559	. 7440	9.069	. 2560	9.057	.0003	. 0033	. 9967	32
29	. 2589	. 7411	8.631	. 2589	8.618	.0003	. 0033	. 9966	31
30	.02618	.97382	38.201	.02618	38.188	1.0003	.00034	.99966	30
31	. 2647	. 7353	7.782	. 2648	7.769	.0003	. 0035	. 9965	29
32	. 2676	. 7324	7.371	. 2677	7.358	.0003	. 0036	. 9964	28
33	. 2705	. 7295	6.969	. 2706	6.956	.0004	. 0036	. 9963	27
34	. 2734	. 7266	6.576	. 2735	6.563	.0004	. 0037	. 9963	26
35	.02763	.97237	36.191	.02764	36.177	1.0004	.00038	.99962	25
36	. 2792	. 7208	5.814	. 2793	5.800	.0004	. 0039	. 9961	24
37	. 2821	. 7179	5.445	. 2822	5.431	.0004	. 0040	. 9960	23
38	. 2850	. 7150	5.084	. 2851	5.069	.0004	. 0041	. 9959	22
39	. 2879	. 7121	4.729	. 2880	4.715	.0004	. 0041	. 9958	21
40	.02908	.97091	34.382	.02910	34.368	1.0004	.00042	.99958	20
41	. 2937	. 7062	4.042	. 2939	4.027	.0004	. 0043	. 9957	19
42	. 2967	. 7033	3.708	. 2968	3.693	.0004	. 0044	. 9956	18
43	. 2996	. 7004	3.381	. 2997	3.366	.0004	. 0045	. 9955	17
44	. 3025	. 6975	3.060	. 3026	3.045	.0004	. 0046	. 9954	16
45	.03054	.96946	32.745	.03055	32.730	1.0005	.00046	.99953	15
46	. 3083	. 6917	2.437	. 3084	2.421	.0005	. 0047	. 9952	14
47	. 3112	. 6888	2.134	. 3113	2.118	.0005	. 0048	. 9951	13
48	. 3141	. 6859	1.836	. 3143	1.820	.0005	. 0049	. 9951	12
49	. 3170	. 6830	1.544	. 3172	1.528	.0005	. 0050	. 9950	11
50	.03199	.96801	31.257	.03201	31.241	1.0005	.00051	.99949	10
51	. 3228	. 6772	0.976	. 3230	0.960	.0005	. 0052	. 9948	9
52	. 3257	. 6743	0.699	. 3259	0.683	.0005	. 0053	. 9947	8
53	. 3286	. 6713	0.428	. 3288	0.411	.0005	. 0054	. 9946	7
54	. 3315	. 6684	0.161	. 3317	0.145	.0005	. 0055	. 9945	6
55	.03344	.96655	29.899	.03346	29.882	1.0005	.00056	.99944	5
56	. 3374	. 6626	9.641	. 3375	9.624	.0006	. 0057	. 9943	4
57	. 3403	. 6597	9.388	. 3405	9.371	.0006	. 0058	. 9942	3
58	. 3432	. 6568	9.139	. 3434	9.122	.0006	. 0059	. 9941	2
59	. 3461	. 6539	8.894	. 3463	8.877	.0006	. 0060	. 9940	1
60	. 3490	. 6510	8.654	. 3492	8.636	.0006	. 0061	. 9939	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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2° Natural Trigonometrical Functions. 177°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.03490	.96510	28.654	.03492	28.636	1.0006	.00061	.99939	60
1	. 3519	. 6481	8.417	. 3521	8.399	.0006	. 0062	. 9938	59
2	. 3548	. 6452	8.184	. 3550	8.166	.0006	. 0063	. 9937	58
3	. 3577	. 6423	7.955	. 3579	7.937	.0006	. 0064	. 9936	57
4	. 3606	. 6394	7.730	. 3608	7.712	.0006	. 0065	. 9935	56
5	.03635	.96365	27.508	.03638	27.490	1.0007	.00066	.99934	55
6	. 3664	. 6336	7.290	. 3667	7.271	.0007	. 0067	. 9933	54
7	. 3693	. 6306	7.075	. 3696	7.056	.0007	. 0068	. 9932	53
8	. 3722	. 6277	6.864	. 3725	6.845	.0007	. 0069	. 9931	52
9	. 3751	. 6248	6.655	. 3754	6.637	.0007	. 0070	. 9930	51
10	.03781	.96219	26.450	.03783	26.432	1.0007	.00071	.99928	50
11	. 3810	. 6190	6.249	. 3812	6.230	.0007	. 0073	. 9927	49
12	. 3839	. 6161	6.050	. 3842	6.031	.0007	. 0074	. 9926	48
13	. 3868	. 6132	5.854	. 3871	5.835	.0007	. 0075	. 9925	47
14	. 3897	. 6103	5.661	. 3900	5.642	.0008	. 0076	. 9924	46
15	.03926	.96074	25.471	.03929	25.452	1.0008	.00077	.99923	45
16	. 3955	. 6045	5.284	. 3958	5.264	.0008	. 0078	. 9922	44
17	. 3984	. 6016	5.100	. 3987	5.080	.0008	. 0079	. 9921	43
18	. 4013	. 5987	4.918	. 4016	4.898	.0008	. 0080	. 9919	42
19	. 4042	. 5958	4.739	. 4045	4.718	.0008	. 0082	. 9918	41
20	.04071	.95929	24.562	.04075	24.542	1.0008	.00083	.99917	40
21	. 4100	. 5900	4.388	. 4104	4.367	.0008	. 0084	. 9916	39
22	. 4129	. 5870	4.216	. 4133	4.196	.0008	. 0085	. 9915	38
23	. 4158	. 5841	4.047	. 4162	4.026	.0009	. 0086	. 9913	37
24	. 4187	. 5812	3.880	. 4191	3.859	.0009	. 0088	. 9912	36
25	.04217	.95783	23.716	.04220	23.694	1.0009	.00089	.99911	35
26	. 4246	. 5754	3.553	. 4249	3.532	.0009	. 0090	. 9910	34
27	. 4275	. 5725	3.393	. 4279	3.372	.0009	. 0091	. 9908	33
28	. 4304	. 5696	3.235	. 4308	3.214	.0009	. 0093	. 9907	32
29	. 4333	. 5667	3.079	. 4337	3.058	.0009	. 0094	. 9906	31
30	.04362	.95638	22.925	.04366	22.904	1.0009	.00095	.99905	30
31	. 4391	. 5609	2.774	. 4395	2.752	.0010	. 0096	. 9903	29
32	. 4420	. 5580	2.624	. 4424	2.602	.0010	. 0098	. 9902	28
33	. 4449	. 5551	2.476	. 4453	2.454	.0010	. 0099	. 9901	27
34	. 4478	. 5522	2.330	. 4483	2.308	.0010	. 0100	. 9900	26
35	.04507	.95493	22.186	.04512	22.164	1.0010	.00102	.99898	25
36	. 4536	. 5464	2.044	. 4541	2.022	.0010	. 0103	. 9897	24
37	. 4565	. 5435	1.904	. 4570	1.881	.0010	. 0104	. 9896	23
38	. 4594	. 5405	1.765	. 4599	1.742	.0010	. 0106	. 9894	22
39	. 4623	. 5376	1.629	. 4628	1.606	.0011	. 0107	. 9893	21
40	.04652	.95347	21.494	.04657	21.470	1.0011	.00108	.99892	20
41	. 4681	. 5318	1.360	. 4687	1.337	.0011	. 0110	. 9890	19
42	. 4711	. 5289	1.228	. 4716	1.205	.0011	. 0111	. 9889	18
43	. 4740	. 5260	1.098	. 4745	1.075	.0011	. 0112	. 9888	17
44	. 4769	. 5231	0.970	. 4774	0.946	.0011	. 0114	. 9886	16
45	.04798	.95202	20.843	.04803	20.819	1.0011	.00115	.99885	15
46	. 4827	. 5173	0.717	. 4832	0.693	.0012	. 0116	. 9883	14
47	. 4856	. 5144	0.593	. 4862	0.569	.0012	. 0118	. 9882	13
48	. 4885	. 5115	0.471	. 4891	0.446	.0012	. 0119	. 9881	12
49	. 4914	. 5086	0.350	. 4920	0.325	.0012	. 0121	. 9879	11
50	.04943	.95057	20.230	.04949	20.205	1.0012	.00122	.99878	10
51	. 4972	. 5028	0.112	. 4978	0.087	.0012	. 0124	. 9876	9
52	. 5001	. 4999	19.995	. 5007	19.970	.0012	. 0125	. 9875	8
53	. 5030	. 4970	9.880	. 5037	9.854	.0013	. 0127	. 9873	7
54	. 5059	. 4941	9.766	. 5066	9.740	.0013	. 0128	. 9872	6
55	.05088	.94912	19.653	.05095	19.627	1.0013	.00129	.99870	5
56	. 5117	. 4883	9.541	. 5124	9.515	.0013	. 0131	. 9869	4
57	. 5146	. 4853	9.431	. 5153	9.405	.0013	. 0132	. 9867	3
58	. 5175	. 4824	9.322	. 5182	9.296	.0013	. 0134	. 9866	2
59	. 5204	. 4795	9.214	. 5212	9.188	.0013	. 0135	. 9864	1
60	. 5234	. 4766	9.107	. 5241	9.081	.0014	. 0137	. 9863	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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3° Natural Trigonometrical Functions. 176°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.05234	.94766	19.107	.05241	19.081	1.0014	.00137	.99863	60
1	. 5263	. 4737	9.002	. 5270	8.975	.0014	. 0138	. 9861	59
2	. 5292	. 4708	8.897	. 5299	8.871	.0014	. 0140	. 9860	58
3	. 5321	. 4679	8.794	. 5328	8.768	.0014	. 0142	. 9858	57
4	. 5350	. 4650	8.692	. 5357	8.665	.0014	. 0143	. 9857	56
5	.05379	.94621	18.591	.05387	18.564	1.0014	.00145	.99855	55
6	. 5408	. 4592	8.491	. 5416	8.464	.0015	. 0146	. 9854	54
7	. 5437	. 4563	8.393	. 5445	8.365	.0015	. 0148	. 9852	53
8	. 5466	. 4534	8.295	. 5474	8.268	.0015	. 0149	. 9850	52
9	. 5495	. 4505	8.198	. 5503	8.171	.0015	. 0151	. 9849	51
10	.05524	.94476	18.103	.05532	18.075	1.0015	.00153	.99847	50
11	. 5553	. 4447	8.008	. 5562	7.980	.0015	. 0154	. 9846	49
12	. 5582	. 4418	7.914	. 5591	7.886	.0016	. 0156	. 9844	48
13	. 5611	. 4389	7.821	. 5620	7.793	.0016	. 0157	. 9842	47
14	. 5640	. 4360	7.730	. 5649	7.701	.0016	. 0159	. 9841	46
15	.05669	.94331	17.639	.05678	17.610	1.0016	.00161	.99839	45
16	. 5698	. 4302	7.549	. 5707	7.520	.0016	. 0162	. 9837	44
17	. 5727	. 4273	7.460	. 5737	7.431	.0016	. 0164	. 9836	43
18	. 5756	. 4244	7.372	. 5766	7.343	.0017	. 0166	. 9834	42
19	. 5785	. 4214	7.285	. 5795	7.256	.0017	. 0167	. 9832	41
20	.05814	.94185	17.198	.05824	17.169	1.0017	.00169	.99831	40
21	. 5843	. 4156	7.113	. 5853	7.084	.0017	. 0171	. 9829	39
22	. 5872	. 4127	7.028	. 5883	6.999	.0017	. 0172	. 9827	38
23	. 5902	. 4098	6.944	. 5912	6.915	.0017	. 0174	. 9826	37
24	. 5931	. 4069	6.861	. 5941	6.832	.0018	. 0176	. 9824	36
25	.05960	.94040	16.779	.05970	16.750	1.0018	.00178	.99822	35
26	. 5989	. 4011	6.698	. 5999	6.668	.0018	. 0179	. 9820	34
27	. 6018	. 3982	6.617	. 6029	6.587	.0018	. 0181	. 9819	33
28	. 6047	. 3953	6.538	. 6058	6.507	.0018	. 0183	. 9817	32
29	. 6076	. 3924	6.459	. 6087	6.428	.0018	. 0185	. 9815	31
30	.06105	.93895	16.380	.06116	16.350	1.0019	.00186	.99813	30
31	. 6134	. 3866	6.303	. 6145	6.272	.0019	. 0188	. 9812	29
32	. 6163	. 3837	6.226	. 6175	6.195	.0019	. 0190	. 9810	28
33	. 6192	. 3808	6.150	. 6204	6.119	.0019	. 0192	. 9808	27
34	. 6221	. 3777	6.075	. 6233	6.043	.0019	. 0194	. 9806	26
35	.06250	.93750	16.000	.06262	15.969	1.0019	.00195	.99804	25
36	. 6279	. 3721	5.926	. 6291	5.894	.0020	. 0197	. 9803	24
37	. 6308	. 3692	5.853	. 6321	5.821	.0020	. 0199	. 9801	23
38	. 6337	. 3663	5.780	. 6350	5.748	.0020	. 0201	. 9799	22
39	. 6366	. 3634	5.708	. 6379	5.676	.0020	. 0203	. 9797	21
40	.06395	.93605	15.637	.06408	15.605	1.0020	.00205	.99795	20
41	. 6424	. 3576	5.566	. 6437	5.534	.0021	. 0206	. 9793	19
42	. 6453	. 3547	5.496	. 6467	5.464	.0021	. 0208	. 9791	18
43	. 6482	. 3518	5.427	. 6496	5.394	.0021	. 0210	. 9790	17
44	. 6511	. 3489	5.358	. 6525	5.325	.0021	. 0212	. 9788	16
45	.06540	.93460	15.290	.06554	15.257	1.0021	.00214	.99786	15
46	. 6569	. 3431	5.222	. 6583	5.189	.0022	. 0216	. 9784	14
47	. 6598	. 3402	5.155	. 6613	5.122	.0022	. 0218	. 9782	13
48	. 6627	. 3373	5.089	. 6642	5.056	.0022	. 0220	. 9780	12
49	. 6656	. 3343	5.023	. 6671	4.990	.0022	. 0222	. 9778	11
50	.06685	.93314	14.958	.06700	14.924	1.0022	.00224	.99776	10
51	. 6714	. 3285	4.893	. 6730	4.860	.0023	. 0226	. 9774	9
52	. 6743	. 3256	4.829	. 6759	4.795	.0023	. 0228	. 9772	8
53	. 6772	. 3227	4.765	. 6788	4.732	.0023	. 0230	. 9770	7
54	. 6801	. 3198	4.702	. 6817	4.668	.0023	. 0231	. 9768	6
55	.06830	.93169	14.640	.06846	14.606	1.0023	.00233	.99766	5
56	. 6859	. 3140	4.578	. 6876	4.544	.0024	. 0235	. 9764	4
57	. 6888	. 3111	4.517	. 6905	4.482	.0024	. 0237	. 9762	3
58	. 6918	. 3082	4.456	. 6934	4.421	.0024	. 0239	. 9760	2
59	. 6947	. 3053	4.395	. 6963	4.361	.0024	. 0241	. 9758	1
60	. 6976	. 3024	4.335	. 6993	4.301	.0024	. 0243	. 9756	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	.

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4° **Natural Trigonometrical Functions.** 175°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.06976	.93024	14.335	.06993	14.301	1.0024	.00243	.99756	60
1	. 7005	. 2995	4.276	. 7022	4.241	.0025	. 0246	. 9754	59
2	. 7034	. 2966	4.217	. 7051	4.182	.0025	. 0248	. 9752	58
3	. 7063	. 2937	4.159	. 7080	4.123	.0025	. 0250	. 9750	57
4	. 7092	. 2908	4.101	. 7110	4.065	.0025	. 0252	. 9748	56
5	.07121	.92879	14.043	.07139	14.008	1.0025	.00254	.99746	55
6	. 7150	. 2850	3.986	. 7168	3.951	.0026	. 0256	. 9744	54
7	. 7179	. 2821	3.930	. 7197	3.894	.0026	. 0258	. 9742	53
8	. 7208	. 2792	3.874	. 7226	3.838	.0026	. 0260	. 9740	52
9	. 7237	. 2763	3.818	. 7256	3.782	.0026	. 0262	. 9738	51
10	.07266	.92734	13.763	.07285	13.727	1.0026	.00264	.99736	50
11	. 7295	. 2705	3.708	. 7314	3.672	.0027	. 0266	. 9733	49
12	. 7324	. 2676	3.654	. 7343	3.617	.0027	. 0268	. 9731	48
13	. 7353	. 2647	3.600	. 7373	3.563	.0027	. 0271	. 9729	47
14	. 7382	. 2618	3.547	. 7402	3.510	.0027	. 0273	. 9727	46
15	.07411	.92589	13.494	.07431	13.457	1.0027	.00275	.99725	45
16	. 7440	. 2560	3.441	. 7460	3.404	.0028	. 0277	. 9723	44
17	. 7469	. 2531	3.389	. 7490	3.351	.0028	. 0279	. 9721	43
18	. 7498	. 2502	3.337	. 7519	3.299	.0028	. 0281	. 9718	42
19	. 7527	. 2473	3.286	. 7548	3.248	.0028	. 0284	. 9716	41
20	.07556	.92444	13.235	.07577	13.197	1.0029	.00286	.99714	40
21	. 7585	. 2415	3.184	. 7607	3.146	.0029	. 0288	. 9712	39
22	. 7614	. 2386	3.134	. 7636	3.096	.0029	. 0290	. 9710	38
23	. 7643	. 2357	3.084	. 7665	3.046	.0029	. 0292	. 9707	37
24	. 7672	. 2328	3.034	. 7694	2.996	.0029	. 0295	. 9705	36
25	.07701	.92299	12.985	.07724	12.947	1.0030	.00297	.99703	35
26	. 7730	. 2270	2.937	. 7753	2.898	.0030	. 0299	. 9701	34
27	. 7759	. 2241	2.888	. 7782	2.849	.0030	. 0301	. 9698	33
28	. 7788	. 2212	2.840	. 7812	2.801	.0030	. 0304	. 9696	32
29	. 7817	. 2183	2.793	. 7841	2.754	.0031	. 0306	. 9694	31
30	.07846	.92154	12.745	.07870	12.706	1.0031	.00308	.99692	30
31	. 7875	. 2125	2.698	. 7899	2.659	.0031	. 0310	. 9689	29
32	. 7904	. 2096	2.652	. 7929	2.612	.0031	. 0313	. 9687	28
33	. 7933	. 2067	2.606	. 7958	2.566	.0032	. 0315	. 9685	27
34	. 7962	. 2038	2.560	. 7987	2.520	.0032	. 0317	. 9682	26
35	.07991	.92009	12.514	.08016	12.474	1.0032	.00320	.99680	25
36	. 8020	. 1980	2.469	. 8046	2.429	.0032	. 0322	. 9678	24
37	. 8049	. 1951	2.424	. 8075	2.384	.0032	. 0324	. 9675	23
38	. 8078	. 1922	2.379	. 8104	2.339	.0033	. 0327	. 9673	22
39	. 8107	. 1893	2.335	. 8134	2.295	.0033	. 0329	. 9671	21
40	.08136	.91864	12.291	.08163	12.250	1.0033	.00331	.99668	20
41	. 8165	. 1835	2.248	. 8192	2.207	.0033	. 0334	. 9666	19
42	. 8194	. 1806	2.204	. 8221	2.163	.0034	. 0336	. 9664	18
43	. 8223	. 1777	2.161	. 8251	2.120	.0034	. 0339	. 9661	17
44	. 8252	. 1748	2.118	. 8280	2.077	.0034	. 0341	. 9659	16
45	.08281	.91719	12.076	.08309	12.035	1.0034	.00343	.99656	15
46	. 8310	. 1690	2.034	. 8339	1.992	.0035	. 0346	. 9654	14
47	. 8339	. 1661	1.992	. 8368	1.950	.0035	. 0348	. 9652	13
48	. 8368	. 1632	1.950	. 8397	1.909	.0035	. 0351	. 9649	12
49	. 8397	. 1603	1.909	. 8426	1.867	.0035	. 0353	. 9647	11
50	.08426	.91574	11.868	.08456	11.826	1.0036	.00356	.99644	10
51	. 8455	. 1545	1.828	. 8485	1.785	.0036	. 0358	. 9642	9
52	. 8484	. 1516	1.787	. 8514	1.745	.0036	. 0360	. 9639	8
53	. 8513	. 1487	1.747	. 8544	1.704	.0036	. 0363	. 9637	7
54	. 8542	. 1458	1.707	. 8573	1.664	.0037	. 0365	. 9634	6
55	.08571	.91429	11.668	.08602	11.625	1.0037	.00368	.99632	5
56	. 8600	. 1400	1.628	. 8632	1.585	.0037	. 0370	. 9629	4
57	. 8629	. 1371	1.589	. 8661	1.546	.0037	. 0373	. 9627	3
58	. 8658	. 1342	1.550	. 8690	1.507	.0038	. 0375	. 9624	2
59	. 8687	. 1313	1.512	. 8719	1.468	.0038	. 0378	. 9622	1
60	. 8715	. 1284	1.474	. 8749	1.430	.0038	. 0380	. 9619	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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5° **Natural Trigonometrical Functions.** **174°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.08715	.91284	11.474	.08749	11.430	1.0038	.00380	.99619	60
1	. 8744	. 1255	1.436	. 8778	1.392	.0038	. 0383	. 9617	59
2	. 8773	. 1226	1.398	. 8807	1.354	.0039	. 0386	. 9614	58
3	. 8802	. 1197	1.360	. 8837	1.316	.0039	. 0388	. 9612	57
4	. 8831	. 1168	1.323	. 8866	1.279	.0039	. 0391	. 9609	56
5	.08860	.91139	11.286	.08895	11.242	1.0039	.00393	.99607	55
6	. 8889	. 1110	1.249	. 8925	1.205	.0040	. 0396	. 9604	54
7	. 8918	. 1082	1.213	. 8954	1.168	.0040	. 0398	. 9601	53
8	. 8947	. 1053	1.176	. 8983	1.132	.0040	. 0401	. 9599	52
9	. 8976	. 1024	1.140	. 9013	1.095	.0040	. 0404	. 9596	51
10	.09005	.90995	11.104	.09042	11.059	1.0041	.00406	.99594	50
11	. 9034	. 0966	1.069	. 9071	1.024	.0041	. 0409	. 9591	49
12	. 9063	. 0937	1.033	. 9101	0.988	.0041	. 0411	. 9588	48
13	. 9092	. 0908	0.998	. 9130	0.953	.0041	. 0414	. 9586	47
14	. 9121	. 0879	0.963	. 9159	0.918	.0042	. 0417	. 9583	46
15	.09150	.90850	10.929	.09189	10.883	1.0042	.00419	.99580	45
16	. 9179	. 0821	0.894	. 9218	0.848	.0042	. 0422	. 9578	44
17	. 9208	. 0792	0.860	. 9247	0.814	.0043	. 0425	. 9575	43
18	. 9237	. 0763	0.826	. 9277	0.780	.0043	. 0427	. 9572	42
19	. 9266	. 0734	0.792	. 9306	0.746	.0043	. 0430	. 9570	41
20	.09295	.90705	10.758	.09335	10.712	1.0043	.00433	.99567	40
21	. 9324	. 0676	0.725	. 9365	0.678	.0044	. 0436	. 9564	39
22	. 9353	. 0647	0.692	. 9394	0.645	.0044	. 0438	. 9562	38
23	. 9382	. 0618	0.659	. 9423	0.612	.0044	. 0441	. 9559	37
24	. 9411	. 0589	0.626	. 9453	0.579	.0044	. 0444	. 9556	36
25	.09440	.90560	10.593	.09482	10.546	1.0045	.00446	.99553	35
26	. 9469	. 0531	0.561	. 9511	0.514	.0045	. 0449	. 9551	34
27	. 9498	. 0502	0.529	. 9541	0.481	.0045	. 0452	. 9548	33
28	. 9527	. 0473	0.497	. 9570	0.449	.0046	. 0455	. 9545	32
29	. 9556	. 0444	0.465	. 9599	0.417	.0046	. 0458	. 9542	31
30	.09584	.90415	10.433	.09629	10.385	1.0046	.00460	.99540	30
31	. 9613	. 0386	0.402	. 9658	0.354	.0046	. 0463	. 9537	29
32	. 9642	. 0357	0.371	. 9688	0.322	.0047	. 0466	. 9534	28
33	. 9671	. 0328	0.340	. 9717	0.291	.0047	. 0469	. 9531	27
34	. 9700	. 0300	0.309	. 9746	0.260	.0047	. 0472	. 9528	26
35	.09729	.90271	10.278	.09776	10.229	1.0048	.00474	.99525	25
36	. 9758	. 0242	0.248	. 9805	0.199	.0048	. 0477	. 9523	24
37	. 9787	. 0213	0.217	. 9834	0.168	.0048	. 0480	. 9520	23
38	. 9816	. 0184	0.187	. 9864	0.138	.0048	. 0483	. 9517	22
39	. 9845	. 0155	0.157	. 9893	0.108	.0049	. 0486	. 9514	21
40	.09874	.90126	10.127	.09922	10.078	1.0049	.00489	.99511	20
41	. 9903	. 0097	0.098	. 9952	0.048	.0049	. 0491	. 9508	19
42	. 9932	. 0068	0.068	. 9981	0.019	.0050	. 0494	. 9505	18
43	. 9961	. 0039	0.039	.10011	9.9893	.0050	. 0497	. 9503	17
44	. 9990	. 0010	0.010	. 0040	.9601	.0050	. 0500	. 9500	16
45	.10019	.89981	9.9812	.10069	9.9310	1.0050	.00503	.99497	15
46	. 0048	. 9952	.9525	. 0099	.9021	.0051	. 0506	. 9494	14
47	. 0077	. 9923	.9239	. 0128	.8734	.0051	. 0509	. 9491	13
48	. 0106	. 9894	.8955	. 0158	.8448	.0051	. 0512	. 9488	12
49	. 0134	. 9865	.8672	. 0187	.8164	.0052	. 0515	. 9485	11
50	.10163	.89836	9.8391	.10216	9.7882	1.0052	.00518	.99482	10
51	. 0192	. 9807	.8112	. 0246	.7601	.0052	. 0521	. 9479	9
52	. 0221	. 9779	.7834	. 0275	.7322	.0053	. 0524	. 9476	8
53	. 0250	. 9750	.7558	. 0305	.7044	.0053	. 0527	. 9473	7
54	. 0279	. 9721	.7283	. 0334	.6768	.0053	. 0530	. 9470	6
55	.10308	.89692	9.7010	.10363	9.6493	1.0053	.00533	.99467	5
56	. 0337	. 9663	.6739	. 0393	.6220	.0054	. 0536	. 9464	4
57	. 0366	. 9634	.6469	. 0422	.5949	.0054	. 0539	. 9461	3
58	. 0395	. 9605	.6200	. 0452	.5679	.0054	. 0542	. 9458	2
59	. 0424	. 9576	.5933	. 0481	.5411	.0055	. 0545	. 9455	1
60	. 0453	. 9547	.5668	. 0510	.5144	.0055	. 0548	. 9452	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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6° **Natural Trigonometrical Functions.** 173°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.10453	.89547	9.5668	.10510	9.5144	1.0055	.00548	.99452	60
1	. 0482	. 9518	.5404	. 0540	.4878	.0055	. 0551	. 9449	59
2	. 0511	. 9489	.5141	. 0569	.4614	.0056	. 0554	. 9446	58
3	. 0540	. 9460	.4880	. 0599	.4351	.0056	. 0557	. 9443	57
4	. 0568	. 9431	.4620	. 0628	.4090	.0056	. 0560	. 9440	56
5	.10597	.89402	9.4362	.10657	9.3831	1.0057	.00563	.99437	55
6	. 0626	. 9373	.4105	. 0687	.3572	.0057	. 0566	. 9434	54
7	. 0655	. 9345	.3850	. 0716	.3315	.0057	. 0569	. 9431	53
8	. 0684	. 9316	.3596	. 0746	.3060	.0057	. 0572	. 9428	52
9	. 0713	. 9287	.3343	. 0775	.2806	.0058	. 0575	. 9424	51
10	.10742	.89258	9.3092	.10805	9.2553	1.0058	.00579	.99421	50
11	. 0771	. 9229	.2842	. 0834	.2302	.0058	. 0582	. 9418	49
12	. 0800	. 9200	.2593	. 0863	.2051	.0059	. 0585	. 9415	48
13	. 0829	. 9171	.2346	. 0893	.1803	.0059	. 0588	. 9412	47
14	. 0858	. 9142	.2100	. 0922	.1555	.0059	. 0591	. 9409	46
15	.10887	.89113	9.1855	.10952	9.1309	1.0060	.00594	.99406	45
16	. 0916	. 9084	.1612	. 0981	.1064	.0060	. 0597	. 9402	44
17	. 0944	. 9055	.1370	. 1011	.0821	.0060	. 0601	. 9399	43
18	. 0973	. 9026	.1129	. 1040	.0579	.0061	. 0604	. 9396	42
19	. 1002	. 8998	.0890	. 1069	.0338	.0061	. 0607	. 9393	41
20	.11031	.88969	9.0651	.11099	9.0098	1.0061	.00610	.99390	40
21	. 1060	. 8940	.0414	. 1128	8.9860	.0062	. 0613	. 9386	39
22	. 1089	. 8911	.0179	. 1158	.9623	.0062	. 0617	. 9383	38
23	. 1118	. 8882	8.9944	. 1187	.9387	.0062	. 0620	. 9380	37
24	. 1147	. 8853	.9711	. 1217	.9152	.0063	. 0623	. 9377	36
25	.11176	.88824	8.9479	.11246	8.8918	1.0063	.00626	.99373	35
26	. 1205	. 8795	.9248	. 1276	.8686	.0063	. 0630	. 9370	34
27	. 1234	. 8766	.9018	. 1305	.8455	.0064	. 0633	. 9367	33
28	. 1262	. 8737	.8790	. 1335	.8225	.0064	. 0636	. 9364	32
29	. 1291	. 8708	.8563	. 1364	.7996	.0064	. 0639	. 9360	31
30	.11320	.88680	8.8337	.11393	8.7769	1.0065	.00643	.99357	30
31	. 1349	. 8651	.8112	. 1423	.7542	.0065	. 0646	. 9354	29
32	. 1378	. 8622	.7888	. 1452	.7317	.0065	. 0649	. 9350	28
33	. 1407	. 8593	.7665	. 1482	.7093	.0066	. 0653	. 9347	27
34	. 1436	. 8564	.7444	. 1511	.6870	.0066	. 0656	. 9344	26
35	.11465	.88535	8.7223	.11541	8.6648	1.0066	.00659	.99341	25
36	. 1494	. 8506	.7004	. 1570	.6427	.0067	. 0663	. 9337	24
37	. 1523	. 8477	.6786	. 1600	.6208	.0067	. 0666	. 9334	23
38	. 1551	. 8448	.6569	. 1629	.5989	.0067	. 0669	. 9330	22
39	. 1580	. 8420	.6353	. 1659	.5772	.0068	. 0673	. 9327	21
40	.11609	.88391	8.6138	.11688	8.5555	1.0068	.00676	.99324	20
41	. 1638	. 8362	.5924	. 1718	.5340	.0068	. 0679	. 9320	19
42	. 1667	. 8333	.5711	. 1747	.5126	.0069	. 0683	. 9317	18
43	. 1696	. 8304	.5499	. 1777	.4913	.0069	. 0686	. 9314	17
44	. 1725	. 8272	.5289	. 1806	.4701	.0069	. 0690	. 9310	16
45	.11754	.88246	8.5079	.11836	8.4489	1.0070	.00693	.99307	15
46	. 1783	. 8217	.4871	. 1865	.4279	.0070	. 0696	. 9303	14
47	. 1811	. 8188	.4663	. 1895	.4070	.0070	. 0700	. 9300	13
48	. 1840	. 8160	.4457	. 1924	.3862	.0071	. 0703	. 9296	12
49	. 1869	. 8131	.4251	. 1954	.3655	.0071	. 0707	. 9293	11
50	.11898	.88102	8.4046	.11983	8.3449	1.0071	.00710	.99290	10
51	. 1927	. 8073	.3843	. 2013	.3244	.0072	. 0714	. 9286	9
52	. 1956	. 8044	.3640	. 2042	.3040	.0072	. 0717	. 9283	8
53	. 1985	. 8015	.3439	. 2072	.2837	.0073	. 0721	. 9279	7
54	. 2014	. 7986	.3238	. 2101	.2635	.0073	. 0724	. 9276	6
55	.12042	.87957	8.3039	.12131	8.2434	1.0073	.00728	.99272	5
56	. 2071	. 7928	.2840	. 2160	.2234	.0074	. 0731	. 9269	4
57	. 2100	. 7900	.2642	. 2190	.2035	.0074	. 0735	. 9265	3
58	. 2129	. 7871	.2446	. 2219	.1837	.0074	. 0738	. 9262	2
59	. 2158	. 7842	.2250	. 2249	.1640	.0075	. 0742	. 9258	1
60	2187	. 7813	.2055	. 2278	.1443	.0075	. 0745	. 9255	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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7° **Natural Trigonometrical Functions.** **172°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.12187	.87813	8.2055	.12278	8.1443	1.0075	.00745	.99255	60
1	. 2216	. 7787	.1861	. 2308	.1248	.0075	. 0749	. 9251	59
2	. 2245	. 7755	.1668	. 2337	.1053	.0076	. 0752	. 9247	58
3	. 2273	. 7726	.1476	. 2367	.0860	.0076	. 0756	. 9244	57
4	. 2302	. 7697	.1285	. 2396	.0667	.0076	. 0760	. 9240	56
5	.12331	.87669	8.1094	.12426	8.0476	1.0077	.00763	.99237	55
6	. 2360	. 7640	.0905	. 2456	.0285	.0077	. 0767	. 9233	54
7	. 2389	. 7611	.0717	. 2485	.0095	.0078	. 0770	. 9229	53
8	. 2418	. 7582	.0529	. 2515	7.9906	.0078	. 0774	. 9226	52
9	. 2447	. 7553	.0342	. 2544	.9717	.0078	. 0778	. 9222	51
10	.12476	.87524	8.0156	.12574	7.9530	1.0079	.00781	.99219	50
11	. 2504	. 7495	7.9971	. 2603	.9344	.0079	. 0785	. 9215	49
12	. 2533	. 7467	.9787	. 2633	.9158	.0079	. 0788	. 9211	48
13	. 2562	. 7438	.9604	. 2662	.8973	.0080	. 0792	. 9208	47
14	. 2591	. 7409	.9421	. 2692	.8789	.0080	. 0796	. 9204	46
15	.12620	.87380	7.9240	.12722	7.8606	1.0080	.00799	.99200	45
16	. 2649	. 7351	.9059	. 2751	.8424	.0081	. 0803	. 9197	44
17	. 2678	. 7322	.8879	. 2781	.8243	.0081	. 0807	. 9193	43
18	. 2706	. 7293	.8700	. 2810	.8062	.0082	. 0810	. 9189	42
19	. 2735	. 7265	.8522	. 2840	.7882	.0082	. 0814	. 9186	41
20	.12764	.87236	7.8344	.12869	7.7703	1.0082	.00818	.99182	40
21	. 2793	. 7207	.8168	. 2899	.7525	.0083	. 0822	. 9178	39
22	. 2822	. 7178	.7992	. 2928	.7348	.0083	. 0825	. 9174	38
23	. 2851	. 7149	.7817	. 2958	.7171	.0084	. 0829	. 9171	37
24	. 2879	. 7120	.7642	. 2988	.6996	.0084	. 0833	. 9167	36
25	.12908	.87091	7.7469	.13017	7.6821	1.0084	.00837	.99163	35
26	. 2937	. 7063	.7296	. 3047	.6646	.0085	. 0840	. 9160	34
27	. 2966	. 7034	.7124	. 3076	.6473	.0085	. 0844	. 9156	33
28	. 2995	. 7005	.6953	. 3106	.6300	.0085	. 0848	. 9152	32
29	. 3024	. 6976	.6783	. 3136	.6129	.0086	. 0852	. 9148	31
30	.13053	.86947	7.6613	.13165	7.5957	1.0086	.00855	.99144	30
31	. 3081	. 6918	.6444	. 3195	.5787	.0087	. 0859	. 9141	29
32	. 3110	. 6890	.6276	. 3224	.5617	.0087	. 0863	. 9137	28
33	. 3139	. 6861	.6108	. 3254	.5449	.0087	. 0867	. 9133	27
34	. 3168	. 6832	.5942	. 3284	.5280	.0088	. 0871	. 9129	26
35	.13197	.86803	7.5776	.13313	7.5113	1.0088	.00875	.99125	25
36	. 3226	. 6774	.5611	. 3343	.4946	.0089	. 0878	. 9121	24
37	. 3254	. 6745	.5446	. 3372	.4780	.0089	. 0882	. 9118	23
38	. 3283	. 6717	.5282	. 3402	.4615	.0089	. 0886	. 9114	22
39	. 3312	. 6688	.5119	. 3432	.4451	.0090	. 0890	. 9110	21
40	.13341	.86659	7.4957	.13461	7.4287	1.0090	.00894	.99106	20
41	. 3370	. 6630	.4795	. 3491	.4124	.0090	. 0898	. 9102	19
42	. 3399	. 6601	.4634	. 3520	.3961	.0091	. 0902	. 9098	18
43	. 3427	. 6572	.4474	. 3550	.3800	.0091	. 0905	. 9094	17
44	. 3456	. 6544	.4315	. 3580	.3639	.0092	. 0909	. 9090	16
45	.13485	.86515	7.4156	.13609	7.3479	1.0092	.00913	.99086	15
46	. 3514	. 6486	.3998	. 3639	.3319	.0092	. 0917	. 9083	14
47	. 3543	. 6457	.3840	. 3669	.3160	.0093	. 0921	. 9079	13
48	. 3571	. 6428	.3683	. 3698	.3002	.0093	. 0925	. 9075	12
49	. 3600	. 6400	.3527	. 3728	.2844	.0094	. 0929	. 9070	11
50	.13629	.86371	7.3372	.13757	7.2687	1.0094	.00933	.99067	10
51	. 3658	. 6342	.3217	. 3787	.2531	.0094	. 0937	. 9063	9
52	. 3687	. 6313	.3063	. 3817	.2375	.0095	. 0941	. 9059	8
53	. 3716	. 6284	.2909	. 3846	.2220	.0095	. 0945	. 9055	7
54	. 3744	. 6255	.2757	. 3876	.2066	.0096	. 0949	. 9051	6
55	.13773	.86227	7.2604	.13906	7.1912	1.0096	.00953	.99047	5
56	. 3802	. 6198	.2453	. 3935	.1759	.0097	. 0957	. 9043	4
57	. 3831	. 6169	.2302	. 3965	.1607	.0097	. 0961	. 9039	3
58	. 3860	. 6140	.2152	. 3995	.1455	.0097	. 0965	. 9035	2
59	. 3888	. 6111	.2002	. 4024	.1304	.0098	. 0969	. 9031	1
60	. 3917	. 6083	.1853	. 4054	.1154	.0098	. 0973	. 9027	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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8° Natural Trigonometrical Functions. 171°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.13917	.86083	7.1853	.14054	7.1154	1.0098	.00973	.99027	60
1	. 3946	. 6054	.1704	. 4084	.1004	.0099	. 0977	. 9023	59
2	. 3975	. 6025	.1557	. 4113	.0854	.0099	. 0981	. 9019	58
3	. 4004	. 5996	.1409	. 4143	.0706	.0099	. 0985	. 9015	57
4	. 4032	. 5967	.1263	. 4173	.0558	.0100	. 0989	. 9010	56
5	.14061	.85939	7.1117	.14202	7.0410	1.0100	.00993	.99006	55
6	. 4090	. 5910	.0972	. 4232	.0264	.0101	. 0998	. 9002	54
7	. 4119	. 5881	.0827	. 4262	.0117	.0101	. 1002	. 8998	53
8	. 4148	. 5852	.0683	. 4291	6.9972	.0102	. 1006	. 8994	52
9	. 4176	. 5823	.0539	. 4321	.9827	.0102	. 1010	. 8990	51
10	.14205	.85795	7.0396	.14351	6.9682	1.0102	.01014	.98986	50
11	. 4234	. 5766	.0254	. 4380	.9538	.0103	. 1018	. 8982	49
12	. 4263	. 5737	.0112	. 4410	.9395	.0103	. 1022	. 8978	48
13	. 4292	. 5708	6.9971	. 4440	.9252	.0104	. 1026	. 8973	47
14	. 4320	. 5679	.9830	. 4470	.9110	.0104	. 1031	. 8969	46
15	.14349	.85651	6.9690	.14499	6.8969	1.0104	.01035	.98965	45
16	. 4378	. 5622	.9550	. 4529	.8828	.0105	. 1039	. 8961	44
17	. 4407	. 5593	.9411	. 4559	.8687	.0105	. 1043	. 8957	43
18	. 4436	. 5564	.9273	. 4588	.8547	.0106	. 1047	. 8952	42
19	. 4464	. 5536	.9135	. 4618	.8408	.0106	. 1052	. 8948	41
20	.14493	.85507	6.8998	.14648	6.8269	1.0107	.01056	.98944	40
21	. 4522	. 5478	.8861	. 4677	.8131	.0107	. 1060	. 8940	39
22	. 4551	. 5449	.8725	. 4707	.7993	.0107	. 1064	. 8936	38
23	. 4579	. 5420	.8589	. 4737	.7856	.0108	. 1068	. 8931	37
24	. 4608	. 5392	.8454	. 4767	.7720	.0108	. 1073	. 8927	36
25	.14637	.85363	6.8320	.14796	6.7584	1.0109	.01077	.98923	35
26	. 4666	. 5334	.8185	. 4826	.7448	.0109	. 1081	. 8919	34
27	. 4695	. 5305	.8052	. 4856	.7313	.0110	. 1085	. 8914	33
28	. 4723	. 5277	.7919	. 4886	.7179	.0110	. 1090	. 8910	32
29	. 4752	. 5248	.7787	. 4915	.7045	.0111	. 1094	. 8906	31
30	.14781	.85219	6.7655	.14945	6.6911	1.0111	.01098	.98901	30
31	. 4810	. 5190	.7523	. 4975	.6779	.0111	. 1103	. 8897	29
32	. 4838	. 5161	.7392	. 5004	.6646	.0112	. 1107	. 8893	28
33	. 4867	. 5133	.7262	. 5034	.6514	.0112	. 1111	. 8889	27
34	. 4896	. 5104	.7132	. 5064	.6383	.0113	. 1116	. 8884	26
35	.14925	.85075	6.7003	.15094	6.6252	1.0113	.01120	.98880	25
36	. 4953	. 5046	.6874	. 5123	.6122	.0114	. 1124	. 8876	24
37	. 4982	. 5018	.6745	. 5153	.5992	.0114	. 1129	. 8871	23
38	. 5011	. 4989	.6617	. 5183	.5863	.0115	. 1133	. 8867	22
39	. 5040	. 4960	.6490	. 5213	.5734	.0115	. 1137	. 8862	21
40	.15068	.84931	6.6363	.15243	6.5605	1.0115	.01142	.98858	20
41	. 5097	. 4903	.6237	. 5272	.5478	.0116	. 1146	. 8854	19
42	. 5126	. 4874	.6111	. 5302	.5350	.0116	. 1151	. 8849	18
43	. 5155	. 4845	.5985	. 5332	.5223	.0117	. 1155	. 8845	17
44	. 5183	. 4816	.5860	. 5362	.5097	.0117	. 1159	. 8840	16
45	.15212	.84788	6.5736	.15391	6.4971	1.0118	.01164	.98836	15
46	. 5241	. 4759	.5612	. 5421	.4845	.0118	. 1168	. 8832	14
47	. 5270	. 4730	.5488	. 5451	.4720	.0119	. 1173	. 8827	13
48	. 5298	. 4701	.5365	. 5481	.4596	.0119	. 1177	. 8823	12
49	. 5328	. 4672	.5243	. 5511	.4472	.0119	. 1182	. 8818	11
50	.15356	.84644	6.5121	.15540	6.4348	1.0120	.01186	.98814	10
51	. 5385	. 4615	.4999	. 5570	.4225	.0120	. 1190	. 8809	9
52	. 5413	. 4586	.4878	. 5600	.4103	.0121	. 1195	. 8805	8
53	. 5442	. 4558	.4757	. 5630	.3980	.0121	. 1199	. 8800	7
54	. 5471	. 4529	.4637	. 5659	.3859	.0122	. 1204	. 8796	6
55	.15500	.84500	6.4517	.15689	6.3737	1.0122	.01208	.98791	5
56	. 5528	. 4471	.4398	. 5719	.3616	.0123	. 1213	. 8787	4
57	. 5557	. 4443	.4279	. 5749	.3496	.0123	. 1217	. 8782	3
58	. 5586	. 4414	.4160	. 5779	.3376	.0124	. 1222	. 8778	2
59	. 5615	. 4385	.4042	. 5809	.3257	.0124	. 1227	. 8773	1
60	. 5643	. 4356	.3924	. 5838	.3137	.0125	. 1231	. 8769	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Sine.	Vrs. cos.	M.

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9° Natural Trigonometrical Functions. 170°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.15643	.84356	6.3924	.15838	6.3137	1.0125	.01231	.98769	60
1	.5672	.4328	.3807	.5868	.3019	.0125	.1236	.8764	59
2	.5701	.4299	.3690	.5898	.2901	.0125	.1240	.8760	58
3	.5730	.4270	.3574	.5928	.2783	.0126	.1245	.8755	57
4	.5758	.4242	.3458	.5958	.2665	.0126	.1249	.8750	56
5	.15787	.84213	6.3343	.15987	6.2548	1.0127	.01254	.98746	55
6	.5816	.4184	.3228	.6017	.2432	.0127	.1259	.8741	54
7	.5844	.4156	.3113	.6047	.2316	.0128	.1263	.8737	53
8	.5873	.4127	.2999	.6077	.2200	.0128	.1268	.8732	52
9	.5902	.4098	.2885	.6107	.2085	.0129	.1272	.8727	51
10	.15931	.84069	6.2772	.16137	6.1970	1.0129	.01277	.98723	50
11	.5959	.4041	.2659	.6167	.1856	.0130	.1282	.8718	49
12	.5988	.4012	.2546	.6196	.1742	.0130	.1286	.8714	48
13	.6017	.3983	.2434	.6226	.1628	.0131	.1291	.8709	47
14	.6045	.3954	.2322	.6256	.1515	.0131	.1296	.8704	46
15	.16074	.83926	6.2211	.16286	6.1402	1.0132	.01300	.98700	45
16	.6103	.3897	.2100	.6316	.1290	.0132	.1305	.8695	44
17	.6132	.3868	.1990	.6346	.1178	.0133	.1310	.8690	43
18	.6160	.3840	.1880	.6376	.1066	.0133	.1314	.8685	42
19	.6189	.3811	.1770	.6405	.0955	.0134	.1319	.8681	41
20	.16218	.83782	6.1661	.16435	6.0844	1.0134	.01324	.98676	40
21	.6246	.3753	.1552	.6465	.0734	.0135	.1328	.8671	39
22	.6275	.3725	.1443	.6495	.0624	.0135	.1333	.8667	38
23	.6304	.3696	.1335	.6525	.0514	.0136	.1338	.8662	37
24	.6333	.3667	.1227	.6555	.0405	.0136	.1343	.8657	36
25	.16361	.83639	6.1120	.16585	6.0296	1.0136	.01347	.98652	35
26	.6390	.3610	.1013	.6615	.0188	.0137	.1352	.8648	34
27	.6419	.3581	.0906	.6644	.0080	.0137	.1357	.8643	33
28	.6447	.3553	.0800	.6674	5.9972	.0138	.1362	.8638	32
29	.6476	.3524	.0694	.6704	.9865	.0138	.1367	.8633	31
30	.16505	.83495	6.0588	.16734	5.9758	1.0139	.01371	.98628	30
31	.6533	.3466	.0483	.6764	.9651	.0139	.1376	.8624	29
32	.6562	.3438	.0379	.6794	.9545	.0140	.1381	.8619	28
33	.6591	.3409	.0274	.6824	.9439	.0140	.1386	.8614	27
34	.6619	.3380	.0170	.6854	.9333	.0141	.1391	.8609	26
35	.16648	.83352	6.0066	.16884	5.9228	1.0141	.01395	.98604	25
36	.6677	.3323	5.9963	.6914	.9123	.0142	.1400	.8600	24
37	.6705	.3294	.9860	.6944	.9019	.0142	.1405	.8595	23
38	.6734	.3266	.9758	.6973	.8915	.0143	.1410	.8590	22
39	.6763	.3237	.9655	.7003	.8811	.0143	.1415	.8585	21
40	.16791	.83208	5.9554	.17033	5.8708	1.0144	.01420	.98580	20
41	.6820	.3180	.9452	.7063	.8605	.0144	.1425	.8575	19
42	.6849	.3151	.9351	.7093	.8502	.0145	.1430	.8570	18
43	.6878	.3122	.9250	.7123	.8400	.0145	.1434	.8565	17
44	.6906	.3094	.9150	.7153	.8298	.0146	.1439	.8560	16
45	.16935	.83065	5.9049	.17183	5.8196	1.0146	.01444	.98556	15
46	.6964	.3036	.8950	.7213	.8095	.0147	.1449	.8551	14
47	.6992	.3008	.8850	.7243	.7994	.0147	.1454	.8546	13
48	.7021	.2979	.8751	.7273	.7894	.0148	.1459	.8541	12
49	.7050	.2950	.8652	.7303	.7793	.0148	.1464	.8536	11
50	.17078	.82922	5.8554	.17333	5.7694	1.0149	.01469	.98531	10
51	.7107	.2893	.8456	.7363	.7594	.0150	.1474	.8526	9
52	.7136	.2864	.8358	.7393	.7495	.0150	.1479	.8521	8
53	.7164	.2836	.8261	.7423	.7396	.0151	.1484	.8516	7
54	.7193	.2807	.8163	.7453	.7297	.0151	.1489	.8511	6
55	.17221	.82778	5.8067	.17483	5.7199	1.0152	.01494	.98506	5
56	.7250	.2750	.7970	.7513	.7101	.0152	.1499	.8501	4
57	.7279	.2721	.7874	.7543	.7004	.0153	.1504	.8496	3
58	.7307	.2692	.7778	.7573	.6906	.0153	.1509	.8491	2
59	.7336	.2664	.7683	.7603	.6809	.0154	.1514	.8486	1
60	.7365	.2635	.7588	.7633	.6713	.0154	.1519	.8481	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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10° **Natural Trigonometrical Functions.** **169°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.17365	.82635	5.7588	.17633	5.6713	1.0154	.01519	.98481	60
1	. 7393	. 2606	.7493	. 7663	.6616	.0155	. 1524	. 8476	59
2	. 7422	. 2578	.7398	. 7693	.6520	.0155	. 1529	. 8471	58
3	. 7451	. 2549	.7304	. 7723	.6425	.0156	. 1534	. 8465	57
4	. 7479	. 2521	.7210	. 7753	.6329	.0156	. 1539	. 8460	56
5	.17508	.82492	5.7117	.17783	5.6234	1.0157	.01544	.98455	55
6	. 7537	. 2463	.7023	. 7813	.6140	.0157	. 1550	. 8450	54
7	. 7565	. 2435	.6930	. 7843	.6045	.0158	. 1555	. 8445	53
8	. 7594	. 2406	.6838	. 7873	.5951	.0158	. 1560	. 8440	52
9	. 7622	. 2377	.6745	. 7903	.5857	.0159	. 1565	. 8435	51
10	.17651	.82349	5.6653	.17933	5.5764	1.0159	.01570	.98430	50
11	. 7680	. 2320	.6561	. 7963	.5670	.0160	. 1575	. 8425	49
12	. 7708	. 2291	.6470	. 7993	.5578	.0160	. 1580	. 8419	48
13	. 7737	. 2263	.6379	. 8023	.5485	.0161	. 1585	. 8414	47
14	. 7766	. 2234	.6288	. 8053	.5393	.0162	. 1591	. 8409	46
15	.17794	.82206	5.6197	.18083	5.5301	1.0162	.01596	.98404	45
16	. 7823	. 2177	.6107	. 8113	.5209	.0163	. 1601	. 8399	44
17	. 7852	. 2148	.6017	. 8143	.5117	.0163	. 1606	. 8394	43
18	. 7880	. 2120	.5928	. 8173	.5026	.0164	. 1611	. 8388	42
19	. 7909	. 2091	.5838	. 8203	.4936	.0164	. 1617	. 8383	41
20	.17937	.82062	5.5749	.18233	5.4845	1.0165	.01622	.98378	40
21	. 7966	. 2034	.5660	. 8263	.4755	.0165	. 1627	. 8373	39
22	. 7995	. 2005	.5572	. 8293	.4665	.0166	. 1632	. 8368	38
23	. 8023	. 1977	.5484	. 8323	.4575	.0166	. 1638	. 8362	37
24	. 8052	. 1948	.5396	. 8353	.4486	.0167	. 1643	. 8357	36
25	.18080	.81919	5.5308	.18383	5.4396	1.0167	.01648	.98352	35
26	. 8109	. 1891	.5221	. 8413	.4308	.0168	. 1653	. 8347	34
27	. 8138	. 1862	.5134	. 8444	.4219	.0169	. 1659	. 8341	33
28	. 8166	. 1834	.5047	. 8474	.4131	.0169	. 1664	. 8336	32
29	. 8195	. 1805	.4960	. 8504	.4043	.0170	. 1669	. 8331	31
30	.18223	.81776	5.4874	.18534	5.3955	1.0170	.01674	.98325	30
31	. 8252	. 1748	.4788	. 8564	.3868	.0171	. 1680	. 8320	29
32	. 8281	. 1719	.4702	. 8594	.3780	.0171	. 1685	. 8315	28
33	. 8309	. 1691	.4617	. 8624	.3694	.0172	. 1690	. 8309	27
34	. 8338	. 1662	.4532	. 8654	.3607	.0172	. 1696	. 8304	26
35	.18366	.81633	5.4447	.18684	5.3521	1.0173	.01701	.98299	25
36	. 8395	. 1605	.4362	. 8714	.3434	.0174	. 1706	. 8293	24
37	. 8424	. 1576	.4278	. 8745	.3349	.0174	. 1712	. 8288	23
38	. 8452	. 1548	.4194	. 8775	.3263	.0175	. 1717	. 8283	22
39	. 8481	. 1519	.4110	. 8805	.3178	.0175	. 1722	. 8277	21
40	.18509	.81490	5.4026	.18835	5.3093	1.0176	.01728	.98272	20
41	. 8538	. 1462	.3943	. 8865	.3008	.0176	. 1733	. 8267	19
42	. 8567	. 1433	.3860	. 8895	.2923	.0177	. 1739	. 8261	18
43	. 8595	. 1405	.3777	. 8925	.2839	.0177	. 1744	. 8256	17
44	. 8624	. 1376	.3695	. 8955	.2755	.0178	. 1749	. 8250	16
45	.18652	.81348	5.3612	.18985	5.2671	1.0179	.01755	.98245	15
46	. 8681	. 1319	.3530	. 9016	.2588	.0179	. 1760	. 8240	14
47	. 8709	. 1290	.3449	. 9046	.2505	.0180	. 1766	. 8234	13
48	. 8738	. 1262	.3367	. 9076	.2422	.0180	. 1771	. 8229	12
49	. 8767	. 1233	.3286	. 9106	.2339	.0181	. 1777	. 8223	11
50	.18795	.81205	5.3205	.19136	5.2257	1.0181	.01782	.98218	10
51	. 8824	. 1176	.3124	. 9166	.2174	.0182	. 1788	. 8212	9
52	. 8852	. 1147	.3044	. 9197	.2092	.0182	. 1793	. 8207	8
53	. 8881	. 1119	.2963	. 9227	.2011	.0183	. 1799	. 8201	7
54	. 8909	. 1090	.2883	. 9257	.1929	.0184	. 1804	. 8196	6
55	.18938	.81062	5.2803	.19287	5.1848	1.0184	.01810	.98190	5
56	. 8967	. 1033	.2724	. 9317	.1767	.0185	. 1815	. 8185	4
57	. 8995	. 1005	.2645	. 9347	.1686	.0185	. 1821	. 8179	3
58	. 9024	. 0976	.2566	. 9378	.1606	.0186	. 1826	. 8174	2
59	. 9052	. 0948	.2487	. 9408	.1525	.0186	. 1832	. 8168	1
60	. 9081	. 0919	.2408	. 9438	.1445	.0187	. 1837	. 8163	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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11° Natural Trigonometrical Functions. 168°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.19081	.80919	5.2408	.19438	5.1445	1.0187	.01837	.98163	60
1	. 9109	. 0890	.2330	. 9468	.1366	.0188	. 1843	. 8157	59
2	. 9138	. 0862	.2252	. 9498	.1286	.0188	. 1848	. 8152	58
3	. 9166	. 0833	.2174	. 9529	.1207	.0189	. 1854	. 8146	57
4	. 9195	. 0805	.2097	. 9559	.1128	.0189	. 1859	. 8140	56
5	.19224	.80776	5.2019	.19589	5.1049	1.0190	.01865	.98135	55
6	. 9252	. 0748	.1942	. 9619	.0970	.0191	. 1871	. 8129	54
7	. 9281	. 0719	.1865	. 9649	.0892	.0191	. 1876	. 8124	53
8	. 9309	. 0691	.1788	. 9680	.0814	.0192	. 1882	. 8118	52
9	. 9338	. 0662	.1712	. 9710	.0736	.0192	. 1887	. 8112	51
10	.19366	.80634	5.1636	.19740	5.0658	1.0193	.01893	.98107	50
11	. 9395	. 0605	.1560	. 9770	.0581	.0193	. 1899	. 8101	49
12	. 9423	. 0576	.1484	. 9800	.0504	.0194	. 1904	. 8095	48
13	. 9452	. 0548	.1409	. 9831	.0427	.0195	. 1910	. 8090	47
14	. 9480	. 0519	.1333	. 9861	.0350	.0195	. 1916	. 8084	46
15	.19509	.80491	5.1258	.19891	5.0273	1.0196	.01921	.98078	45
16	. 9537	. 0462	.1183	. 9921	.0197	.0196	. 1927	. 8073	44
17	. 9566	. 0434	.1109	. 9952	.0121	.0197	. 1933	. 8067	43
18	. 9595	. 0405	.1034	. 9982	.0045	.0198	. 1938	. 8061	42
19	. 9623	. 0377	.0960	.20012	4.9969	.0198	. 1944	. 8056	41
20	.19652	.80348	5.0886	.20042	4.9894	1.0199	.01950	.98050	40
21	. 9680	. 0320	.0812	. 0073	.9819	.0199	. 1956	. 8044	39
22	. 9709	. 0291	.0739	. 0103	.9744	.0200	. 1961	. 8039	38
23	. 9737	. 0263	.0666	. 0133	.9669	.0201	. 1967	. 8033	37
24	. 9766	. 0234	.0593	. 0163	.9594	.0201	. 1973	. 8027	36
25	.19794	.80206	5.0520	.20194	4.9520	1.0202	.01979	.98021	35
26	. 9823	. 0177	.0447	. 0224	.9446	.0202	. 1984	. 8016	34
27	. 9851	. 0149	.0375	. 0254	.9372	.0203	. 1990	. 8010	33
28	. 9880	. 0120	.0302	. 0285	.9298	.0204	. 1996	. 8004	32
29	. 9908	. 0092	.0230	. 0315	.9225	.0204	. 2002	. 7998	31
30	.19937	.80063	5.0158	.20345	4.9151	1.0205	.02007	.97992	30
31	. 9965	. 0035	.0087	. 0375	.9078	.0205	. 2013	. 7987	29
32	. 9994	. 0006	.0015	. 0406	.9006	.0206	. 2019	. 7981	28
33	.20022	.79978	4.9944	. 0436	.8933	.0207	. 2025	. 7975	27
34	. 0051	. 9949	.9873	. 0466	.8860	.0207	. 2031	. 7969	26
35	.20079	.79921	4.9802	.20497	4.8788	1.0208	.02037	.97963	25
36	. 0108	. 9892	.9732	. 0527	.8716	.0208	. 2042	. 7957	24
37	. 0136	. 9863	.9661	. 0557	.8644	.0209	. 2048	. 7952	23
38	. 0165	. 9835	.9591	. 0588	.8573	.0210	. 2054	. 7946	22
39	. 0193	. 9807	.9521	. 0618	.8501	.0210	. 2060	. 7940	21
40	.20222	.79778	4.9452	.20648	4.8430	1.0211	.02066	.97934	20
41	. 0250	. 9750	.9382	. 0679	.8359	.0211	. 2072	. 7928	19
42	. 0279	. 9721	.9313	. 0709	.8288	.0212	. 2078	. 7922	18
43	. 0307	. 9693	.9243	. 0739	.8217	.0213	. 2084	. 7916	17
44	. 0336	. 9664	.9175	. 0770	.8147	.0213	. 2089	. 7910	16
45	.20364	.79636	4.9106	.20800	4.8077	1.0214	.02095	.97904	15
46	. 0393	. 9607	.9037	. 0830	.8007	.0215	. 2101	. 7899	14
47	. 0421	. 9579	.8969	. 0861	.7937	.0215	. 2107	. 7893	13
48	. 0450	. 9550	.8901	. 0891	.7867	.0216	. 2113	. 7887	12
49	. 0478	. 9522	.8833	. 0921	.7798	.0216	. 2119	. 7881	11
50	.20506	.79493	4.8765	.20952	4.7728	1.0217	.02125	.97875	10
51	. 0535	. 9465	.8697	. 0982	.7659	.0218	. 2131	. 7869	9
52	. 0563	. 9436	.8630	. 1012	.7591	.0218	. 2137	. 7863	8
53	. 0592	. 9408	.8563	. 1043	.7522	.0219	. 2143	. 7857	7
54	. 0620	. 9379	.8496	. 1073	.7453	.0220	. 2149	. 7851	6
55	.20649	.79351	4.8429	.21104	4.7385	1.0220	.02155	.97845	5
56	. 0677	. 9323	.8362	. 1134	.7317	.0221	. 2161	. 7839	4
57	. 0706	. 9294	.8296	. 1164	.7249	.0221	. 2167	. 7833	3
58	. 0734	. 9266	.8229	. 1195	.7181	.0222	. 2173	. 7827	2
59	. 0763	. 9237	.8163	. 1225	.7114	.0223	. 2179	. 7821	1
60	. 0791	. 9209	.8097	. 1256	.7046	.0223	. 2185	. 7815	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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12° **Natural Trigonometrical Functions.** **167°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.20791	.79209	4.8097	.21256	4.7046	1.0223	.02185	.97815	60
1	. 0820	. 9180	.8032	. 1286	.6979	.0224	. 2191	. 7809	59
2	. 0848	. 9152	.7966	. 1316	.6912	.0225	. 2197	. 7803	58
3	. 0876	. 9123	.7901	. 1347	.6845	.0225	. 2203	. 7806	57
4	. 0905	. 9105	.7835	. 1377	.6778	.0226	. 2209	. 7790	56
5	.20933	.79066	4.7770	.21408	4.6712	1.0226	.02215	.97784	55
6	. 0962	. 9038	.7706	. 1438	.6646	.0227	. 2222	. 7778	54
7	. 0990	. 9010	.7641	. 1468	.6580	.0228	. 2228	. 7772	53
8	. 1019	. 8981	.7576	. 1499	.6514	.0228	. 2234	. 7766	52
9	. 1047	. 8953	.7512	. 1529	.6448	.0229	. 2240	. 7760	51
10	.21076	.78924	4.7448	.21560	4.6382	1.0230	.02246	.97754	50
11	. 1104	. 8896	.7384	. 1590	.6317	.0230	. 2252	. 7748	49
12	. 1132	. 8867	.7320	. 1621	.6252	.0231	. 2258	. 7741	48
13	. 1161	. 8839	.7257	. 1651	.6187	.0232	. 2264	. 7735	47
14	. 1189	. 8811	.7193	. 1682	.6122	.0232	. 2271	. 7729	46
15	.21218	.78782	4.7130	.21712	4.6057	1.0233	.02277	.97723	45
16	. 1246	. 8754	.7067	. 1742	.5993	.0234	. 2283	. 7717	44
17	. 1275	. 8725	.7004	. 1773	.5928	.0234	. 2289	. 7711	43
18	. 1303	. 8697	.6942	. 1803	.5864	.0235	. 2295	. 7704	42
19	. 1331	. 8668	.6879	. 1834	.5800	.0235	. 2302	. 7698	41
20	.21360	.78640	4.6817	.21864	4.5736	1.0236	.02308	.97692	40
21	. 1388	. 8612	.6754	. 1895	.5673	.0237	. 2314	. 7686	39
22	. 1417	. 8583	.6692	. 1925	.5609	.0237	. 2320	. 7680	38
23	. 1445	. 8555	.6631	. 1956	.5546	.0238	. 2326	. 7673	37
24	. 1473	. 8526	.6569	. 1986	.5483	.0239	. 2333	. 7667	36
25	.21502	.78508	4.6507	.22017	4.5420	1.0239	.02339	.97661	35
26	. 1530	. 8470	.6446	. 2047	.5357	.0240	. 2345	. 7655	34
27	. 1559	. 8441	.6385	. 2078	.5294	.0241	. 2351	. 7648	33
28	. 1587	. 8413	.6324	. 2108	.5232	.0241	. 2358	. 7642	32
29	. 1615	. 8384	.6263	. 2139	.5169	.0242	. 2364	. 7636	31
30	.21644	.78356	4.6202	.22169	4.5107	1.0243	.02370	.97630	30
31	. 1672	. 8328	.6142	. 2200	.5045	.0243	. 2377	. 7623	29
32	. 1701	. 8299	.6081	. 2230	.4983	.0244	. 2383	. 7617	28
33	. 1729	. 8271	.6021	. 2261	.4921	.0245	. 2389	. 7611	27
34	. 1757	. 8242	.5961	. 2291	.4860	.0245	. 2396	. 7604	26
35	.21786	.78214	4.5901	.22322	4.4799	1.0246	.02402	.97598	25
36	. 1814	. 8186	.5841	. 2353	.4737	.0247	. 2408	. 7592	24
37	. 1843	. 8154	.5782	. 2383	.4676	.0247	. 2415	. 7585	23
38	. 1871	. 8129	.5722	. 2414	.4615	.0248	. 2421	. 7579	22
39	. 1899	. 8100	.5663	. 2444	.4555	.0249	. 2427	. 7573	21
40	.21928	.78072	4.5604	.22475	4.4494	1.0249	.02434	.97566	20
41	. 1956	. 8043	.5545	. 2505	.4434	.0250	. 2440	. 7560	19
42	. 1985	. 8015	.5486	. 2536	.4373	.0251	. 2446	. 7553	18
43	. 2013	. 7987	.5428	. 2566	.4313	.0251	. 2453	. 7547	17
44	. 2041	. 7959	.5369	. 2597	.4253	.0252	. 2459	. 7541	16
45	.22070	.77930	4.5311	.22628	4.4194	1.0253	.02466	.97534	15
46	. 2098	. 7902	.5253	. 2658	.4134	.0253	. 2472	. 7528	14
47	. 2126	. 7873	.5195	. 2689	.4074	.0254	. 2479	. 7521	13
48	. 2155	. 7845	.5137	. 2719	.4015	.0255	. 2485	. 7515	12
49	. 2183	. 7817	.5079	. 2750	.3956	.0255	. 2491	. 7508	11
50	.22211	.77788	4.5021	.22781	4.3897	1.0256	.02498	.97502	10
51	. 2240	. 7760	.4964	. 2811	.3838	.0257	. 2504	. 7495	9
52	. 2268	. 7732	.4907	. 2842	.3779	.0257	. 2511	. 7489	8
53	. 2297	. 7703	.4850	. 2872	.3721	.0258	. 2517	. 7483	7
54	. 2325	. 7675	.4793	. 2903	.3662	.0259	. 2524	. 7476	6
55	.22353	.77647	4.4736	.22934	4.3604	1.0260	.02530	.97470	5
56	. 2382	. 7618	.4679	. 2964	.3546	.0260	. 2537	. 7463	4
57	. 2410	. 7590	.4623	. 2995	.3488	.0261	. 2543	. 7457	3
58	. 2438	. 7561	.4566	. 3025	.3430	.0262	. 2550	. 7450	2
59	. 2467	. 7533	.4510	. 3056	.3372	.0262	. 2556	. 7443	1
60	. 2495	. 7505	.4454	. 3087	.3315	.0263	. 2563	. 7437	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

102° **77°**

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13° Natural Trigonometrical Functions. 166°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.22495	.77505	4.4454	.23087	4.3315	1.0263	.02563	.97437	60
1	. 2523	. 7476	.4398	. 3117	.3257	.0264	. 2569	. 7430	59
2	. 2552	. 7448	.4342	. 3148	.3200	.0264	. 2576	. 7424	58
3	. 2580	. 7420	.4287	. 3179	.3143	.0265	. 2583	. 7417	57
4	. 2608	. 7391	.4231	. 3209	.3086	.0266	. 2589	. 7411	56
5	.22637	.77363	4.4176	.23240	4.3029	1.0266	.02596	.97404	55
6	. 2665	. 7335	.4121	. 3270	.2972	.0267	. 2602	. 7398	54
7	. 2693	. 7306	.4065	. 3301	.2916	.0268	. 2609	. 7391	53
8	. 2722	. 7278	.4011	. 3332	.2859	.0268	. 2616	. 7384	52
9	. 2750	. 7250	.3956	. 3363	.2803	.0269	. 2622	. 7378	51
10	.22778	.77221	4.3901	.23393	4.2747	1.0270	.02629	.97371	50
11	. 2807	. 7193	.3847	. 3424	.2691	.0271	. 2635	. 7364	49
12	. 2835	. 7165	.3792	. 3455	.2635	.0271	. 2642	. 7358	48
13	. 2863	. 7136	.3738	. 3485	.2579	.0272	. 2649	. 7351	47
14	. 2892	. 7108	.3684	. 3516	.2524	.0273	. 2655	. 7344	46
15	.22920	.77080	4.3630	.23547	4.2468	1.0273	.02662	.97338	45
16	. 2948	. 7052	.3576	. 3577	.2413	.0274	. 2669	. 7331	44
17	. 2977	. 7023	.3522	. 3608	.2358	.0275	. 2675	. 7324	43
18	. 3005	. 6995	.3469	. 3639	.2303	.0276	. 2682	. 7318	42
19	. 3033	. 6967	.3415	. 3670	.2248	.0276	. 2689	. 7311	41
20	.23061	.76938	4.3362	.23700	4.2193	1.0277	.02695	.97304	40
21	. 3090	. 6910	.3309	. 3731	.2139	.0278	. 2702	. 7298	39
22	. 3118	. 6882	.3256	. 3762	.2084	.0278	. 2709	. 7291	38
23	. 3146	. 6853	.3203	. 3793	.2030	.0279	. 2716	. 7284	37
24	. 3175	. 6825	.3150	. 3823	.1976	.0280	. 2722	. 7277	36
25	.23203	.76797	4.3098	.23854	4.1921	1.0280	.02729	.97271	35
26	. 3231	. 6769	.3045	. 3885	.1867	.0281	. 2736	. 7264	34
27	. 3260	. 6740	.2993	. 3916	.1814	.0282	. 2743	. 7257	33
28	. 3288	. 6712	.2941	. 3946	.1760	.0283	. 2749	. 7250	32
29	. 3316	. 6684	.2888	. 3977	.1706	.0283	. 2756	. 7244	31
30	.23344	.76655	4.2836	.24008	4.1653	1.0284	.02763	.97237	30
31	. 3373	. 6627	.2785	. 4039	.1600	.0285	. 2770	. 7230	29
32	. 3401	. 6599	.2733	. 4069	.1546	.0285	. 2777	. 7223	28
33	. 3429	. 6571	.2681	. 4100	.1493	.0286	. 2783	. 7216	27
34	. 3458	. 6542	.2630	. 4131	.1440	.0287	. 2790	. 7210	26
35	.23486	.76514	4.2579	.24162	4.1388	1.0288	.02797	.97203	25
36	. 3514	. 6486	.2527	. 4192	.1335	.0288	. 2804	. 7196	24
37	. 3542	. 6457	.2476	. 4223	.1282	.0289	. 2811	. 7189	23
38	. 3571	. 6429	.2425	. 4254	.1230	.0290	. 2818	. 7182	22
39	. 3599	. 6401	.2375	. 4285	.1178	.0291	. 2824	. 7175	21
40	.23627	.76373	4.2324	.24316	4.1126	1.0291	.02831	.97169	20
41	. 3655	. 6344	.2273	. 4346	.1073	.0292	. 2838	. 7162	19
42	. 3684	. 6316	.2223	. 4377	.1022	.0293	. 2845	. 7155	18
43	. 3712	. 6288	.2173	. 4408	.0970	.0293	. 2852	. 7148	17
44	. 3740	. 6260	.2122	. 4439	.0918	.0294	. 2859	. 7141	16
45	.23768	.76231	4.2072	.24470	4.0867	1.0295	.02866	.97134	15
46	. 3797	. 6203	.2022	. 4501	.0815	.0296	. 2873	. 7127	14
47	. 3825	. 6175	.1972	. 4531	.0764	.0296	. 2880	. 7120	13
48	. 3853	. 6147	.1923	. 4562	.0713	.0297	. 2886	. 7113	12
49	. 3881	. 6118	.1873	. 4593	.0662	.0298	. 2893	. 7106	11
50	.23910	.76090	4.1824	.24624	4.0611	1.0299	.02900	.97099	10
51	. 3938	. 6062	.1774	. 4655	.0560	.0299	. 2907	. 7092	9
52	. 3966	. 6034	.1725	. 4686	.0509	.0300	. 2914	. 7086	8
53	. 3994	. 6005	.1676	. 4717	.0458	.0301	. 2921	. 7079	7
54	. 4023	. 5977	.1627	. 4747	.0408	.0302	. 2928	. 7072	6
55	.24051	.75949	4.1578	.24778	4.0358	1.0302	.02935	.97065	5
56	. 4079	. 5921	.1529	. 4809	.0307	.0303	. 2942	. 7058	4
57	. 4107	. 5892	.1481	. 4840	.0257	.0304	. 2949	. 7051	3
58	. 4136	. 5864	.1432	. 4871	.0207	.0305	. 2956	. 7044	2
59	. 4164	. 5836	.1384	. 4902	.0157	.0305	. 2963	. 7037	1
60	. 4192	. 5808	.1336	. 4933	.0108	.0306	. 2970	. 7029	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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14° **Natural Trigonometrical Functions.** **165°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.24192	.75808	4.1336	.24933	4.0108	1.0306	.02970	.97029	60
1	. 4220	. 5779	.1287	. 4964	.0058	.0307	. 2977	. 7022	59
2	. 4249	. 5751	.1239	. 4995	.0009	.0308	. 2984	. 7015	58
3	. 4277	. 5723	.1191	. 5025	3.9959	.0308	. 2991	. 7008	57
4	. 4305	. 5695	.1144	. 5056	.9910	.0309	. 2999	. 7001	56
5	.24333	.75667	4.1096	.25087	3.9861	1.0310	.03006	.96994	55
6	. 4361	. 5638	.1048	. 5118	.9812	.0311	. 3013	. 6987	54
7	. 4390	. 5610	.1001	. 5149	.9763	.0311	. 3020	. 6980	53
8	. 4418	. 5582	.0953	. 5180	.9714	.0312	. 3027	. 6973	52
9	. 4446	. 5554	.0906	. 5211	.9665	.0313	. 3034	. 6966	51
10	.24474	.75526	4.0859	.25242	3.9616	1.0314	.03041	.96959	50
11	. 4502	. 5497	.0812	. 5273	.9568	.0314	. 3048	. 6952	49
12	. 4531	. 5469	.0765	. 5304	.9520	.0315	. 3055	. 6944	48
13	. 4559	. 5441	.0718	. 5335	.9471	.0316	. 3063	. 6937	47
14	. 4587	. 5413	.0672	. 5366	.9423	.0317	. 3070	. 6930	46
15	.24615	.75385	4.0625	.25397	3.9375	1.0317	.03077	.96923	45
16	. 4643	. 5356	.0579	. 5428	.9327	.0318	. 3084	. 6916	44
17	. 4672	. 5328	.0532	. 5459	.9279	.0319	. 3091	. 6909	43
18	. 4700	. 5300	.0486	. 5490	.9231	.0320	. 3098	. 6901	42
19	. 4728	. 5272	.0440	. 5521	.9184	.0320	. 3106	. 6894	41
20	.24756	.75244	4.0394	.25552	3.9136	1.0321	.03113	.96887	40
21	. 4784	. 5215	.0348	. 5583	.9089	.0322	. 3120	. 6880	39
22	. 4813	. 5187	.0302	. 5614	.9042	.0323	. 3127	. 6873	38
23	. 4841	. 5159	.0256	. 5645	.8994	.0323	. 3134	. 6865	37
24	. 4869	. 5131	.0211	. 5676	.8947	.0324	. 3142	. 6858	36
25	.24897	.75103	4.0165	.25707	3.8900	1.0325	.03149	.96851	35
26	. 4925	. 5075	.0120	. 5738	.8853	.0326	. 3156	. 6844	34
27	. 4953	. 5046	.0074	. 5769	.8807	.0327	. 3163	. 6836	33
28	. 4982	. 5018	.0029	. 5800	.8760	.0327	. 3171	. 6829	32
29	. 5010	. 4990	3.9984	. 5831	.8713	.0328	. 3178	. 6822	31
30	.25038	.74962	3.9939	.25862	3.8667	1.0329	.03185	.96815	30
31	. 5066	. 4934	.9894	. 5893	.8621	.0330	. 3192	. 6807	29
32	. 5094	. 4906	.9850	. 5924	.8574	.0330	. 3200	. 6800	28
33	. 5122	. 4877	.9805	. 5955	.8528	.0331	. 3207	. 6793	27
34	. 5151	. 4849	.9760	. 5986	.8482	.0332	. 3214	. 6785	26
35	.25179	.74821	3.9716	.26017	3.8436	1.0333	.03222	.96778	25
36	. 5207	. 4793	.9672	. 6048	.8390	.0334	. 3229	. 6771	24
37	. 5235	. 4765	.9627	. 6079	.8345	.0334	. 3236	. 6763	23
38	. 5263	. 4737	.9583	. 6110	.8299	.0335	. 3244	. 6756	22
39	. 5291	. 4709	.9539	. 6141	.8254	.0336	. 3251	. 6749	21
40	.25319	.74680	3.9495	.26172	3.8208	1.0337	.03258	.96741	20
41	. 5348	. 4652	.9451	. 6203	.8163	.0338	. 3266	. 6734	19
42	. 5376	. 4624	.9408	. 6234	.8118	.0338	. 3273	. 6727	18
43	. 5404	. 4596	.9364	. 6266	.8073	.0339	. 3281	. 6719	17
44	. 5432	. 4568	.9320	. 6297	.8027	.0340	. 3288	. 6712	16
45	.25460	.74540	3.9277	.26328	3.7983	1.0341	.03295	.96704	15
46	. 5488	. 4512	.9234	. 6359	.7938	.0341	. 3303	. 6697	14
47	. 5516	. 4483	.9190	. 6390	.7893	.0342	. 3310	. 6690	13
48	. 5544	. 4455	.9147	. 6421	.7848	.0343	. 3318	. 6682	12
49	. 5573	. 4427	.9104	. 6452	.7804	.0344	. 3325	. 6675	11
50	.25601	.74399	3.9061	.26483	3.7759	1.0345	.03332	.96667	10
51	. 5629	. 4371	.9018	. 6514	.7715	.0345	. 3340	. 6660	9
52	. 5657	. 4344	.8976	. 6546	.7671	.0346	. 3347	. 6652	8
53	. 5685	. 4315	.8933	. 6577	.7627	.0347	. 3355	. 6645	7
54	. 5713	. 4287	.8890	. 6608	.7583	.0348	. 3362	. 6638	6
55	.25741	.74259	3.8848	.26639	3.7539	1.0349	.03370	.96630	5
56	. 5769	. 4230	.8805	. 6670	.7495	.0349	. 3377	. 6623	4
57	. 5798	. 4202	.8763	. 6701	.7451	.0350	. 3385	. 6615	3
58	. 5826	. 4174	.8721	. 6732	.7407	.0351	. 3392	. 6608	2
59	. 5854	. 4146	.8679	. 6764	.7364	.0352	. 3400	. 6600	1
60	. 5882	. 4118	.8637	. 6795	.7320	.0353	. 3407	. 6592	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

104° **75°**

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15° Natural Trigonometrical Functions. 164°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.25882	.74118	3.8637	.26795	3.7320	1.0353	.03407	.96592	60
1	. 5910	. 4090	.8595	. 6826	.7277	.0353	. 3415	. 6585	59
2	. 5938	. 4062	.8553	. 6857	.7234	.0354	. 3422	. 6577	58
3	. 5966	. 4034	.8512	. 6888	.7191	.0355	. 3430	. 6570	57
4	. 5994	. 4006	.8470	. 6920	.7147	.0356	. 3438	. 6562	56
5	.26022	.73978	3.8428	.26951	3.7104	1.0357	.03445	.96555	55
6	. 6050	. 3949	.8387	. 6982	.7062	.0358	. 3453	. 6547	54
7	. 6078	. 3921	.8346	. 7013	.7019	.0358	. 3460	. 6540	53
8	. 6107	. 3893	.8304	. 7044	.6976	.0359	. 3468	. 6532	52
9	. 6135	. 3865	.8263	. 7076	.6933	.0360	. 3475	. 6524	51
10	.26163	.73837	3.8222	.27107	3.6891	1.0361	.03483	.96517	50
11	. 6191	. 3809	.8181	. 7138	.6848	.0362	. 3491	. 6509	49
12	. 6219	. 3781	.8140	. 7169	.6806	.0362	. 3498	. 6502	48
13	. 6247	. 3753	.8100	. 7201	.6764	.0363	. 3506	. 6494	47
14	. 6275	. 3725	.8059	. 7232	.6722	.0364	. 3514	. 6486	46
15	.26303	.73697	3.8018	.27263	3.6679	1.0365	.03521	.96479	45
16	. 6331	. 3669	.7978	. 7294	.6637	.0366	. 3529	. 6471	44
17	. 6359	. 3641	.7937	. 7326	.6596	.0367	. 3536	. 6463	43
18	. 6387	. 3613	.7897	. 7357	.6554	.0367	. 3544	. 6456	42
19	. 6415	. 3585	.7857	. 7388	.6512	.0368	. 3552	. 6448	41
20	.26443	.73556	3.7816	.27419	3.6470	1.0369	.03560	.96440	40
21	. 6471	. 3528	.7776	. 7451	.6429	.0370	. 3567	. 6433	39
22	. 6499	. 3500	.7736	. 7482	.6387	.0371	. 3575	. 6425	38
23	. 6527	. 3472	.7697	. 7513	.6346	.0371	. 3583	. 6417	37
24	. 6556	. 3444	.7657	. 7544	.6305	.0372	. 3590	. 6409	36
25	.26584	.73416	3.7617	.27576	3.6263	1.0373	.03598	.96402	35
26	. 6612	. 3388	.7577	. 7607	.6222	.0374	. 3606	. 6394	34
27	. 6640	. 3360	.7538	. 7638	.6181	.0375	. 3614	. 6386	33
28	. 6668	. 3332	.7498	. 7670	.6140	.0376	. 3621	. 6378	32
29	. 6696	. 3304	.7459	. 7701	.6100	.0376	. 3629	. 6371	31
30	.26724	.73276	3.7420	.27732	3.6059	1.0377	.03637	.96363	30
31	. 6752	. 3248	.7380	. 7764	.6018	.0378	. 3645	. 6355	29
32	. 6780	. 3220	.7341	. 7795	.5977	.0379	. 3652	. 6347	28
33	. 6808	. 3192	.7302	. 7826	.5937	.0380	. 3660	. 6340	27
34	. 6836	. 3164	.7263	. 7858	.5896	.0381	. 3668	. 6332	26
35	.26864	.73136	3.7224	.27889	3.5856	1.0382	.03676	.96324	25
36	. 6892	. 3108	.7186	. 7920	.5816	.0382	. 3684	. 6316	24
37	. 6920	. 3080	.7147	. 7952	.5776	.0383	. 3691	. 6308	23
38	. 6948	. 3052	.7108	. 7983	.5736	.0384	. 3699	. 6301	22
39	. 6976	. 3024	.7070	. 8014	.5696	.0385	. 3707	. 6293	21
40	.27004	.72996	3.7031	.28046	3.5656	1.0386	.03715	.96285	20
41	. 7032	. 2968	.6993	. 8077	.5616	.0387	. 3723	. 6277	19
42	. 7060	. 2940	.6955	. 8109	.5576	.0387	. 3731	. 6269	18
43	. 7088	. 2912	.6917	. 8140	.5536	.0388	. 3739	. 6261	17
44	. 7116	. 2884	.6878	. 8171	.5497	.0389	. 3746	. 6253	16
45	.27144	.72856	3.6840	.28203	3.5457	1.0390	.03754	.96245	15
46	. 7172	. 2828	.6802	. 8234	.5418	.0391	. 3762	. 6238	14
47	. 7200	. 2800	.6765	. 8266	.5378	.0392	. 3770	. 6230	13
48	. 7228	. 2772	.6727	. 8297	.5339	.0393	. 3778	. 6222	12
49	. 7256	. 2744	.6689	. 8328	.5300	.0393	. 3786	. 6214	11
50	.27284	.72716	3.6651	.28360	3.5261	1.0394	.03794	.96206	10
51	. 7312	. 2688	.6614	. 8391	.5222	.0395	. 3802	. 6198	9
52	. 7340	. 2660	.6576	. 8423	.5183	.0396	. 3810	. 6190	8
53	. 7368	. 2632	.6539	. 8454	.5144	.0397	. 3818	. 6182	7
54	. 7396	. 2604	.6502	. 8486	.5105	.0398	. 3826	. 6174	6
55	.27424	.72576	3.6464	.28517	3.5066	1.0399	.03834	.96166	5
56	. 7452	. 2548	.6427	. 8549	.5028	.0399	. 3842	. 6158	4
57	. 7480	. 2520	.6390	. 8580	.4989	.0400	. 3850	. 6150	3
58	. 7508	. 2492	.6353	. 8611	.4951	.0401	. 3858	. 6142	2
59	. 7536	. 2464	.6316	. 8643	.4912	.0402	. 3866	. 6134	1
60	. 7564	. 2436	.6279	. 8674	.4874	.0403	. 3874	. 6126	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

105° 74°

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16° **Natural Trigonometrical Functions.** **163°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.27564	.72436	3.6279	.28674	3.4874	1.0403	.03874	.96126	60
1	. 7592	. 2408	.6243	. 8706	.4836	.0404	. 3882	. 6118	59
2	. 7620	. 2380	.6206	. 8737	.4798	.0405	. 3890	. 6110	58
3	. 7648	. 2352	.6169	. 8769	.4760	.0406	. 3898	. 6102	57
4	. 7675	. 2324	.6133	. 8800	.4722	.0406	. 3906	. 6094	56
5	.27703	.72296	3.6096	.28832	3.4684	1.0407	.03914	.96086	55
6	. 7731	. 2268	.6060	. 8863	.4646	.0408	. 3922	. 6078	54
7	. 7759	. 2240	.6024	. 8895	.4608	.0409	. 3930	. 6070	53
8	. 7787	. 2213	.5987	. 8926	.4570	.0410	. 3938	. 6062	52
9	. 7815	. 2185	.5951	. 8958	.4533	.0411	. 3946	. 6054	51
10	.27843	.72157	3.5915	.28990	3.4495	1.0412	.03954	.96045	50
11	. 7871	. 2129	.5879	. 9021	.4458	.0413	. 3962	. 6037	49
12	. 7899	. 2101	.5843	. 9053	.4420	.0413	. 3971	. 6029	48
13	. 7927	. 2073	.5807	. 9084	.4383	.0414	. 3979	. 6021	47
14	. 7955	. 2045	.5772	. 9116	.4346	.0415	. 3987	. 6013	46
15	.27983	.72017	3.5736	.29147	3.4308	1.0416	.03995	.96005	45
16	. 8011	. 1989	.5700	. 9179	.4271	.0417	. 4003	. 5997	44
17	. 8039	. 1961	.5665	. 9210	.4234	.0418	. 4011	. 5989	43
18	. 8067	. 1933	.5629	. 9242	.4197	.0419	. 4019	. 5980	42
19	. 8094	. 1905	.5594	. 9274	.4160	.0420	. 4028	. 5972	41
20	.28122	.71877	3.5559	.29305	3.4124	1.0420	.04036	.95964	40
21	. 8150	. 1849	.5523	. 9337	.4087	.0421	. 4044	. 5956	39
22	. 8178	. 1822	.5488	. 9368	.4050	.0422	. 4052	. 5948	38
23	. 8206	. 1794	.5453	. 9400	.4014	.0423	. 4060	. 5940	37
24	. 8234	. 1766	.5418	. 9432	.3977	.0424	. 4069	. 5931	36
25	.28262	.71738	3.5383	.29463	3.3941	1.0425	.04077	.95923	35
26	. 8290	. 1710	.5348	. 9495	.3904	.0426	. 4085	. 5915	34
27	. 8318	. 1682	.5313	. 9526	.3868	.0427	. 4093	. 5907	33
28	. 8346	. 1654	.5279	. 9558	.3832	.0428	. 4101	. 5898	32
29	. 8374	. 1626	.5244	. 9590	.3795	.0428	. 4110	. 5890	31
30	.28401	.71608	3.5209	.29621	3.3759	1.0429	.04118	.95882	30
31	. 8429	. 1570	.5175	. 9653	.3723	.0430	. 4126	. 5874	29
32	. 8457	. 1543	.5140	. 9685	.3687	.0431	. 4134	. 5865	28
33	. 8485	. 1515	.5106	. 9716	.3651	.0432	. 4143	. 5857	27
34	. 8513	. 1487	.5072	. 9748	.3616	.0433	. 4151	. 5849	26
35	.28541	.71459	3.5037	.29780	3.3580	1.0434	.04159	.95840	25
36	. 8569	. 1431	.5003	. 9811	.3544	.0435	. 4168	. 5832	24
37	. 8597	. 1403	.4969	. 9843	.3509	.0436	. 4176	. 5824	23
38	. 8624	. 1375	.4935	. 9875	.3473	.0437	. 4184	. 5816	22
39	. 8652	. 1347	.4901	. 9906	.3438	.0438	. 4193	. 5807	21
40	.28680	.71320	3.4867	.29938	3.3402	1.0438	.04201	.95799	20
41	. 8708	. 1292	.4833	. 9970	.3367	.0439	. 4209	. 5791	19
42	. 8736	. 1264	.4799	.30001	.3332	.0440	. 4218	. 5782	18
43	. 8764	. 1236	.4766	. 0033	.3296	.0441	. 4226	. 5774	17
44	. 8792	. 1208	.4732	. 0065	.3261	.0442	. 4234	. 5765	16
45	.28820	.71180	3.4698	.30096	3.3226	1.0443	.04243	.95757	15
46	. 8847	. 1152	.4665	. 0128	.3191	.0444	. 4251	. 5749	14
47	. 8875	. 1125	.4632	. 0160	.3156	.0445	. 4260	. 5740	13
48	. 8903	. 1097	.4598	. 0192	.3121	.0446	. 4268	. 5732	12
49	. 8931	. 1069	.4565	. 0223	.3087	.0447	. 4276	. 5723	11
50	.28959	.71041	3.4532	.30255	3.3052	1.0448	.04285	.95715	10
51	. 8987	. 1013	.4498	. 0287	.3017	.0448	. 4293	. 5707	9
52	. 9014	. 0985	.4465	. 0319	.2983	.0449	. 4302	. 5698	8
53	. 9042	. 0958	.4432	. 0350	.2948	.0450	. 4310	. 5690	7
54	. 9070	. 0930	.4399	. 0382	.2914	.0451	. 4319	. 5681	6
55	.29098	.70902	3.4366	.30414	3.2879	1.0452	.04327	.95673	5
56	. 9126	. 0874	.4334	. 0446	.2845	.0453	. 4335	. 5664	4
57	. 9154	. 0846	.4301	. 0478	.2811	.0454	. 4344	. 5656	3
58	. 9181	. 0818	.4268	. 0509	.2777	.0455	. 4352	. 5647	2
59	. 9209	. 0791	.4236	. 0541	.2742	.0456	. 4361	. 5639	1
60	. 9237	. 0763	.4203	. 0573	.2708	.0457	. 4369	. 5630	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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17° **Natural Trigonometrical Functions.** **162°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.29237	.70763	3.4203	.30573	3.2708	1.0457	.04369	.95630	60
1	. 9265	. 0735	.4170	. 0605	.2674	.0458	. 4378	. 5622	59
2	. 9293	. 0707	.4138	. 0637	.2640	.0459	. 4386	. 5613	58
3	. 9321	. 0679	.4106	. 0668	.2607	.0460	. 4395	. 5605	57
4	. 9348	. 0651	.4073	. 0700	.2573	.0461	. 4404	. 5596	56
5	.29376	.70624	3.4041	.30732	3.2539	1.0461	.04412	.95588	55
6	. 9404	. 0596	.4009	. 0764	.2505	.0462	. 4421	. 5579	54
7	. 9432	. 0568	.3977	. 0796	.2472	.0463	. 4426	. 5571	53
8	. 9460	. 0540	.3945	. 0828	.2438	.0464	. 4438	. 5562	52
9	. 9487	. 0512	.3913	. 0859	.2405	.0465	. 4446	. 5554	51
10	.29515	.70485	3.3881	.30891	3.2371	1.0466	.04455	.95545	50
11	. 9543	. 0457	.3849	. 0923	.2338	.0467	. 4463	. 5536	49
12	. 9571	. 0429	.3817	. 0955	.2305	.0468	. 4472	. 5528	48
13	. 9598	. 0401	.3785	. 0987	.2271	.0469	. 4481	. 5519	47
14	. 9626	. 0374	.3754	. 1019	.2238	.0470	. 4489	. 5511	46
15	.29654	.70346	3.3722	.31051	3.2205	1.0471	.04498	.95502	45
16	. 9682	. 0318	.3690	. 1083	.2172	.0472	. 4507	. 5493	44
17	. 9710	. 0290	.3659	. 1115	.2139	.0473	. 4515	. 5485	43
18	. 9737	. 0262	.3627	. 1146	.2106	.0474	. 4524	. 5476	42
19	. 9765	. 0235	.3596	. 1178	.2073	.0475	. 4532	. 5467	41
20	.29793	.70207	3.3565	.31210	3.2041	1.0476	.04541	.95459	40
21	. 9821	. 0179	.3534	. 1242	.2008	.0477	. 4550	. 5450	39
22	. 9848	. 0151	.3502	. 1274	.1975	.0478	. 4558	. 5441	38
23	. 9876	. 0124	.3471	. 1306	.1942	.0478	. 4567	. 5433	37
24	. 9904	. 0096	.3440	. 1338	.1910	.0479	. 4576	. 5424	36
25	.29932	.70068	3.3409	.31370	3.1877	1.0480	.04585	.95415	35
26	. 9959	. 0040	.3378	. 1402	.1845	.0481	. 4593	. 5407	34
27	. 9987	. 0013	.3347	. 1434	.1813	.0482	. 4602	. 5398	33
28	.30015	.69982	.3316	. 1466	.1780	.0483	. 4611	. 5389	32
29	. 0043	. 9957	.3286	. 1498	.1748	.0484	. 4619	. 5380	31
30	.30070	.69929	3.3255	.31530	3.1716	1.0485	.04628	.95372	30
31	. 0098	. 9902	.3224	. 1562	.1684	.0486	. 4637	. 5363	29
32	. 0126	. 9874	.3194	. 1594	.1652	.0487	. 4646	. 5354	28
33	. 0154	. 9846	.3163	. 1626	.1620	.0488	. 4654	. 5345	27
34	. 0181	. 9818	.3133	. 1658	.1588	.0489	. 4663	. 5337	26
35	.30209	.69791	3.3102	.31690	3.1556	1.0490	.04672	.95328	25
36	. 0237	. 9763	.3072	. 1722	.1524	.0491	. 4681	. 5319	24
37	. 0265	. 9735	.3042	. 1754	.1492	.0492	. 4690	. 5310	23
38	. 0292	. 9707	.3011	. 1786	.1460	.0493	. 4698	. 5301	22
39	. 0320	. 9680	.2981	. 1818	.1429	.0494	. 4707	. 5293	21
40	.30348	.69652	3.2951	.31850	3.1397	1.0495	.04716	.95284	20
41	. 0375	. 9624	.2921	. 1882	.1366	.0496	. 4725	. 5275	19
42	. 0403	. 9597	.2891	. 1914	.1334	.0497	. 4734	. 5266	18
43	. 0431	. 9569	.2861	. 1946	.1303	.0498	. 4743	. 5257	17
44	. 0459	. 9541	.2831	. 1978	.1271	.0499	. 4751	. 5248	16
45	.30486	.69513	3.2801	.32010	3.1240	1.0500	.04760	.95239	15
46	. 0514	. 9486	.2772	. 2042	.1209	.0501	. 4769	. 5231	14
47	. 0542	. 9458	.2742	. 2074	.1177	.0502	. 4778	. 5222	13
48	. 0569	. 9430	.2712	. 2106	.1146	.0503	. 4787	. 5213	12
49	. 0597	. 9403	.2683	. 2138	.1115	.0504	. 4796	. 5204	11
50	.30625	.69375	3.2653	.32171	3.1084	1.0505	.04805	.95195	10
51	. 0653	. 9347	.2624	. 2203	.1053	.0506	. 4814	. 5186	9
52	. 0680	. 9320	.2594	. 2235	.1022	.0507	. 4823	. 5177	8
53	. 0708	. 9292	.2565	. 2267	.0991	.0508	. 4832	. 5168	7
54	. 0736	. 9264	.2535	. 2299	.0960	.0509	. 4840	. 5159	6
55	.30763	.69237	3.2506	.32331	3.0930	1.0510	.04849	.95150	5
56	. 0791	. 9209	.2477	. 2363	.0899	.0511	. 4858	. 5141	4
57	. 0819	. 9181	.2448	. 2395	.0868	.0512	. 4867	. 5132	3
58	. 0846	. 9154	.2419	. 2428	.0838	.0513	. 4876	. 5124	2
59	. 0874	. 9126	.2390	. 2460	.0807	.0514	. 4885	. 5115	1
60	. 0902	. 9098	.2361	. 2492	.0777	.0515	. 4894	. 5106	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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18° **Natural Trigonometrical Functions.** **161°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.30902	.69098	3.2361	.32492	3.0777	1.0515	.04894	.95106	60
1	. 0929	. 9071	.2332	. 2524	.0746	.0516	. 4903	. 5097	59
2	. 0957	. 9043	.2303	. 2556	.0716	.0517	. 4912	. 5088	58
3	. 0985	. 9015	.2274	. 2588	.0686	.0518	. 4921	. 5079	57
4	. 1012	. 8988	.2245	. 2621	.0655	.0519	. 4930	. 5070	56
5	.31040	.68960	3.2216	.32653	3.0625	1.0520	.04939	.95061	55
6	. 1068	. 8932	.2188	. 2685	.0595	.0521	. 4948	. 5051	54
7	. 1095	. 8905	.2159	. 2717	.0565	.0522	. 4957	. 5042	53
8	. 1123	. 8877	.2131	. 2749	.0535	.0523	. 4966	. 5033	52
9	. 1150	. 8849	.2102	. 2782	.0505	.0524	. 4975	. 5024	51
10	.31178	.68822	3.2074	.32814	3.0475	1.0525	.04985	.95015	50
11	. 1206	. 8794	.2045	. 2846	.0445	.0526	. 4994	. 5006	49
12	. 1233	. 8766	.2017	. 2878	.0415	.0527	. 5003	. 4997	48
13	. 1261	. 8739	.1989	. 2910	.0385	.0528	. 5012	. 4988	47
14	. 1289	. 8711	.1960	. 2943	.0356	.0529	. 5021	. 4979	46
15	.31316	.68684	3.1932	.32975	3.0326	1.0530	.05030	.94970	45
16	. 1344	. 8656	.1904	. 3007	.0296	.0531	. 5039	. 4961	44
17	. 1372	. 8628	.1876	. 3039	.0267	.0532	. 5048	. 4952	43
18	. 1399	. 8601	.1848	. 3072	.0237	.0533	. 5057	. 4942	42
19	. 1427	. 8573	.1820	. 3104	.0208	.0534	. 5066	. 4933	41
20	.31454	.68545	3.1792	.33136	3.0178	1.0535	.05076	.94924	40
21	. 1482	. 8518	.1764	. 3169	.0149	.0536	. 5085	. 4915	39
22	. 1510	. 8490	.1736	. 3201	.0120	.0537	. 5094	. 4906	38
23	. 1537	. 8463	.1708	. 3233	.0090	.0538	. 5103	. 4897	37
24	. 1565	. 8435	.1681	. 3265	.0061	.0539	. 5112	. 4888	36
25	.31592	.68407	3.1653	.33298	3.0032	1.0540	.05121	.94878	35
26	. 1620	. 8380	.1625	. 3330	.0003	.0541	. 5131	. 4869	34
27	. 1648	. 8352	.1598	. 3362	2.9974	.0542	. 5140	. 4860	33
28	. 1675	. 8325	.1570	. 3395	.9945	.0543	. 5149	. 4851	32
29	. 1703	. 8297	.1543	. 3427	.9916	.0544	. 5158	. 4841	31
30	.31730	.68269	3.1515	.33459	2.9887	1.0545	.05168	.94832	30
31	. 1758	. 8242	.1488	. 3492	.9858	.0546	. 5177	. 4823	29
32	. 1786	. 8214	.1461	. 3524	.9829	.0547	. 5186	. 4814	28
33	. 1813	. 8187	.1433	. 3557	.9800	.0548	. 5195	. 4805	27
34	. 1841	. 8159	.1406	. 3589	.9772	.0549	. 5205	. 4795	26
35	.31868	.68132	3.1379	.33621	2.9743	1.0550	.05214	.94786	25
36	. 1896	. 8104	.1352	. 3654	.9714	.0551	. 5223	. 4777	24
37	. 1923	. 8076	.1325	. 3686	.9686	.0552	. 5232	. 4767	23
38	. 1951	. 8049	.1298	. 3718	.9657	.0553	. 5242	. 4758	22
39	. 1978	. 8021	.1271	. 3751	.9629	.0554	. 5251	. 4749	21
40	.32006	.67994	3.1244	.33783	2.9600	1.0555	.05260	.94740	20
41	. 2034	. 7966	.1217	. 3816	.9572	.0556	. 5270	. 4730	19
42	. 2061	. 7939	.1190	. 3848	.9544	.0557	. 5279	. 4721	18
43	. 2089	. 7911	.1163	. 3880	.9515	.0558	. 5288	. 4712	17
44	. 2116	. 7884	.1137	. 3913	.9487	.0559	. 5297	. 4702	16
45	.32144	.67856	3.1110	.33945	2.9459	1.0560	.05307	.94693	15
46	. 2171	. 7828	.1083	. 3978	.9431	.0561	. 5316	. 4684	14
47	. 2199	. 7801	.1057	. 4010	.9403	.0562	. 5326	. 4674	13
48	. 2226	. 7773	.1030	. 4043	.9375	.0563	. 5335	. 4665	12
49	. 2254	. 7746	.1004	. 4075	.9347	.0565	. 5344	. 4655	11
50	.32282	.67718	3.0977	.34108	2.9319	1.0566	.05354	.94646	10
51	. 2309	. 7691	.0951	. 4140	.9291	.0567	. 5363	. 4637	9
52	. 2337	. 7663	.0925	. 4173	.9263	.0568	. 5373	. 4627	8
53	. 2364	. 7636	.0898	. 4205	.9235	.0569	. 5382	. 4618	7
54	. 2392	. 7608	.0872	. 4238	.9208	.0570	. 5391	. 4608	6
55	.32419	.67581	3.0846	.34270	2.9180	1.0571	.05401	.94599	5
56	. 2447	. 7553	.0820	. 4303	.9152	.0572	. 5410	. 4590	4
57	. 2474	. 7526	.0793	. 4335	.9125	.0573	. 5420	. 4580	3
58	. 2502	. 7498	.0767	. 4368	.9097	.0574	. 5429	. 4571	2
59	. 2529	. 7471	.0741	. 4400	.9069	.0575	. 5439	. 4561	1
60	. 2557	. 7443	.0715	. 4433	.9042	.0576	. 5448	. 4552	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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19° Natural Trigonometrical Functions. 160°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.32557	.67443	3.0715	.34433	2.9042	1.0576	.05448	.94552	60
1	. 2584	. 7416	.0690	. 4465	.9015	.0577	. 5458	. 4542	59
2	. 2612	. 7388	.0664	. 4498	.8987	.0578	. 5467	. 4533	58
3	. 2639	. 7361	.0638	. 4530	.8960	.0579	. 5476	. 4523	57
4	. 2667	. 7333	.0612	. 4563	.8933	.0580	. 5486	. 4514	56
5	.32694	.67306	3.0586	.34595	2.8905	1.0581	.05495	.94504	55
6	. 2722	. 7278	.0561	. 4628	.8878	.0582	. 5505	. 4495	54
7	. 2749	. 7251	.0535	. 4661	.8851	.0584	. 5515	. 4485	53
8	. 2777	. 7223	.0509	. 4693	.8824	.0585	. 5524	. 4476	52
9	. 2804	. 7196	.0484	. 4726	.8797	.0586	. 5534	. 4466	51
10	.32832	.67168	3.0458	.34758	2.8770	1.0587	.05543	.94457	50
11	. 2859	. 7141	.0433	. 4791	.8743	.0588	. 5553	. 4447	49
12	. 2887	. 7113	.0407	. 4824	.8716	.0589	. 5562	. 4438	48
13	. 2914	. 7086	.0382	. 4856	.8689	.0590	. 5572	. 4428	47
14	. 2942	. 7058	.0357	. 4889	.8662	.0591	. 5581	. 4418	46
15	.32969	.67031	3.0331	.34921	2.8636	1.0592	.05591	.94409	45
16	. 2996	. 7003	.0306	. 4954	.8609	.0593	. 5601	. 4399	44
17	. 3024	. 6976	.0281	. 4987	.8582	.0594	. 5610	. 4390	43
18	. 3051	. 6948	.0256	. 5019	.8555	.0595	. 5620	. 4380	42
19	. 3079	. 6921	.0231	. 5052	.8529	.0596	. 5629	. 4370	41
20	.33106	.66894	3.0206	.35085	2.8502	1.0598	.05639	.94361	40
21	. 3134	. 6866	.0181	. 5117	.8476	.0599	. 5649	. 4351	39
22	. 3161	. 6839	.0156	. 5150	.8449	.0600	. 5658	. 4341	38
23	. 3189	. 6811	.0131	. 5183	.8423	.0601	. 5668	. 4332	37
24	. 3216	. 6784	.0106	. 5215	.8396	.0602	. 5678	. 4322	36
25	.33243	.66756	3.0081	.35248	2.8370	1.0603	.05687	.94313	35
26	. 3271	. 6729	.0056	. 5281	.8344	.0604	. 5697	. 4303	34
27	. 3298	. 6701	.0031	. 5314	.8318	.0605	. 5707	. 4293	33
28	. 3326	. 6674	.0007	. 5346	.8291	.0606	. 5716	. 4283	32
29	. 3353	. 6647	2.9982	. 5379	.8265	.0607	. 5726	. 4274	31
30	.33381	.66619	2.9957	.35412	2.8239	1.0608	.05736	.94264	30
31	. 3408	. 6592	.9933	. 5445	.8213	.0609	. 5745	. 4254	29
32	. 3435	. 6564	.9908	. 5477	.8187	.0611	. 5755	. 4245	28
33	. 3463	. 6537	.9884	. 5510	.8161	.0612	. 5765	. 4235	27
34	. 3490	. 6510	.9859	. 5543	.8135	.0613	. 5775	. 4225	26
35	.33518	.66482	2.9835	.35576	2.8109	1.0614	.05784	.94215	25
36	. 3545	. 6455	.9810	. 5608	.8083	.0615	. 5794	. 4206	24
37	. 3572	. 6427	.9786	. 5641	.8057	.0616	. 5804	. 4196	23
38	. 3600	. 6400	.9762	. 5674	.8032	.0617	. 5814	. 4186	22
39	. 3627	. 6373	.9738	. 5707	.8006	.0618	. 5823	. 4176	21
40	.33655	.66345	2.9713	.35739	2.7980	1.0619	.05833	.94167	20
41	. 3682	. 6318	.9689	. 5772	.7954	.0620	. 5843	. 4157	19
42	. 3709	. 6290	.9665	. 5805	.7929	.0622	. 5853	. 4147	18
43	. 3737	. 6263	.9641	. 5838	.7903	.0623	. 5863	. 4137	17
44	. 3764	. 6236	.9617	. 5871	.7878	.0624	. 5872	. 4127	16
45	.33792	.66208	2.9593	.35904	2.7852	1.0625	.05882	.94118	15
46	. 3819	. 6181	.9569	. 5936	.7827	.0626	. 5892	. 4108	14
47	. 3846	. 6153	.9545	. 5969	.7801	.0627	. 5902	. 4098	13
48	. 3874	. 6126	.9521	. 6002	.7776	.0628	. 5912	. 4088	12
49	. 3901	. 6099	.9497	. 6035	.7751	.0629	. 5922	. 4078	11
50	.33928	.66071	2.9474	.36068	2.7725	1.0630	.05932	.94068	10
51	. 3956	. 6044	.9450	. 6101	.7700	.0632	. 5941	. 4058	9
52	. 3983	. 6017	.9426	. 6134	.7675	.0633	. 5951	. 4049	8
53	. 4011	. 5989	.9402	. 6167	.7650	.0634	. 5961	. 4039	7
54	. 4038	. 5962	.9379	. 6199	.7625	.0635	. 5971	. 4029	6
55	.34065	.65935	2.9355	.36232	2.7600	1.0636	.05981	.94019	5
56	. 4093	. 5907	.9332	. 6265	.7574	.0637	. 5991	. 4009	4
57	. 4120	. 5880	.9308	. 6298	.7549	.0638	. 6001	. 3999	3
58	. 4147	. 5853	.9285	. 6331	.7524	.0639	. 6011	. 3989	2
59	. 4175	. 5825	.9261	. 6364	.7500	.0641	. 6021	. 3979	1
60	. 4202	. 5798	.9238	. 6397	.7475	.0642	. 6031	. 3969	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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20° Natural Trigonometrical Functions. 159°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.34202	.65798	2.9238	.36397	2.7475	1.0642	.06031	.93969	60
1	. 4229	. 5771	.9215	. 6430	.7450	.0643	. 6041	. 3959	59
2	. 4257	. 5743	.9191	. 6463	.7425	.0644	. 6051	. 3949	58
3	. 4284	. 5716	.9168	. 6496	.7400	.0645	. 6061	. 3939	57
4	. 4311	. 5689	.9145	. 6529	.7376	.0646	. 6071	. 3929	56
5	.34339	.65661	2.9122	.36562	2.7351	1.0647	.06080	.93919	55
6	. 4366	. 5634	.9098	. 6595	.7326	.0648	. 6090	. 3909	54
7	. 4393	. 5607	.9075	. 6628	.7302	.0650	. 6100	. 3899	53
8	. 4421	. 5579	.9052	. 6661	.7277	.0651	. 6110	. 3889	52
9	. 4448	. 5552	.9029	. 6694	.7252	.0652	. 6121	. 3879	51
10	.34475	.65525	2.9006	.36727	2.7228	1.0653	.06131	.93869	50
11	. 4502	. 5497	.8983	. 6760	.7204	.0654	. 6141	. 3859	49
12	. 4530	. 5470	.8960	. 6793	.7179	.0655	. 6151	. 3849	48
13	. 4557	. 5443	.8937	. 6826	.7155	.0656	. 6161	. 3839	47
14	. 4584	. 5415	.8915	. 6859	.7130	.0658	. 6171	. 3829	46
15	.34612	.65388	2.8892	.36892	2.7106	1.0659	.06181	.93819	45
16	. 4639	. 5361	.8869	. 6925	.7082	.0660	. 6191	. 3809	44
17	. 4666	. 5334	.8846	. 6958	.7058	.0661	. 6201	. 3799	43
18	. 4693	. 5306	.8824	. 6991	.7033	.0662	. 6211	. 3789	42
19	. 4721	. 5279	.8801	. 7024	.7009	.0663	. 6221	. 3779	41
20	.34748	.65252	2.8778	.37057	2.6985	1.0664	.06231	.93769	40
21	. 4775	. 5225	.8756	. 7090	.6961	.0666	. 6241	. 3758	39
22	. 4803	. 5197	.8733	. 7123	.6937	.0667	. 6251	. 3748	38
23	. 4830	. 5170	.8711	. 7156	.6913	.0668	. 6262	. 3738	37
24	. 4857	. 5143	.8688	. 7190	.6889	.0669	. 6272	. 3728	36
25	.34884	.65115	2.8666	.37223	2.6865	1.0670	.06282	.93718	35
26	. 4912	. 5088	.8644	. 7256	.6841	.0671	. 6292	. 3708	34
27	. 4939	. 5061	.8621	. 7289	.6817	.0673	. 6302	. 3698	33
28	. 4966	. 5034	.8599	. 7322	.6794	.0674	. 6312	. 3687	32
29	. 4993	. 5006	.8577	. 7355	.6770	.0675	. 6323	. 3677	31
30	.35021	.64979	2.8554	.37388	2.6746	1.0676	.06333	.93667	30
31	. 5048	. 4952	.8532	. 7422	.6722	.0677	. 6343	. 3657	29
32	. 5075	. 4925	.8510	. 7455	.6699	.0678	. 6353	. 3647	28
33	. 5102	. 4897	.8488	. 7488	.6675	.0679	. 6363	. 3637	27
34	. 5130	. 4870	.8466	. 7521	.6652	.0681	. 6373	. 3626	26
35	.35157	.64843	2.8444	.37554	2.6628	1.0682	.06384	.93616	25
36	. 5184	. 4816	.8422	. 7587	.6604	.0683	. 6394	. 3606	24
37	. 5211	. 4789	.8400	. 7621	.6581	.0684	. 6404	. 3596	23
38	. 5239	. 4761	.8378	. 7654	.6558	.0685	. 6414	. 3585	22
39	. 5266	. 4734	.8356	. 7687	.6534	.0686	. 6425	. 3575	21
40	.35293	.64707	2.8334	.37720	2.6511	1.0688	.06435	.93565	20
41	. 5320	. 4680	.8312	. 7754	.6487	.0689	. 6445	. 3555	19
42	. 5347	. 4652	.8290	. 7787	.6464	.0690	. 6456	. 3544	18
43	. 5375	. 4625	.8269	. 7820	.6441	.0691	. 6466	. 3534	17
44	. 5402	. 4598	.8247	. 7853	.6418	.0692	. 6476	. 3524	16
45	.35429	.64571	2.8225	.37887	2.6394	1.0694	.06486	.93513	15
46	. 5456	. 4544	.8204	. 7920	.6371	.0695	. 6497	. 3503	14
47	. 5483	. 4516	.8182	. 7953	.6348	.0696	. 6507	. 3493	13
48	. 5511	. 4489	.8160	. 7986	.6325	.0697	. 6517	. 3482	12
49	. 5538	. 4462	.8139	. 8020	.6302	.0698	. 6528	. 3472	11
50	.35565	.64435	2.8117	.38053	2.6279	1.0699	.06538	.93462	10
51	. 5592	. 4408	.8096	. 8086	.6256	.0701	. 6548	. 3451	9
52	. 5619	. 4380	.8074	. 8120	.6233	.0702	. 6559	. 3441	8
53	. 5647	. 4353	.8053	. 8153	.6210	.0703	. 6569	. 3431	7
54	. 5674	. 4326	.8032	. 8186	.6187	.0704	. 6579	. 3420	6
55	.35701	.64299	2.8010	.38220	2.6164	1.0705	.06590	.93410	5
56	. 5728	. 4272	.7989	. 8253	.6142	.0707	. 6600	. 3400	4
57	. 5755	. 4245	.7968	. 8286	.6119	.0708	. 6611	. 3389	3
58	. 5782	. 4217	.7947	. 8320	.6096	.0709	. 6621	. 3379	2
59	. 5810	. 4190	.7925	. 8353	.6073	.0710	. 6631	. 3368	1
60	. 5837	. 4163	.7904	. 8386	.6051	.0711	. 6642	. 3358	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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21° **Natural Trigonometrical Functions.** **158°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.35837	.64163	2.7904	.38386	2.6051	1.0711	.06642	.93358	60
1	. 5864	. 4136	.7883	. 8420	.6028	.0713	. 6652	. 3348	59
2	. 5891	. 4109	.7862	. 8453	.6006	.0714	. 6663	. 3337	58
3	. 5918	. 4082	.7841	. 8486	.5983	.0715	. 6673	. 3327	57
4	. 5945	. 4055	.7820	. 8520	.5960	.0716	. 6684	. 3316	56
5	.35972	.64027	2.7799	.38553	2.5938	1.0717	.06694	.93306	55
6	. 6000	. 4000	.7778	. 8587	.5916	.0719	. 6705	. 3295	54
7	. 6027	. 3973	.7757	. 8620	.5893	.0720	. 6715	. 3285	53
8	. 6054	. 3946	.7736	. 8654	.5871	.0721	. 6726	. 3274	52
9	. 6081	. 3919	.7715	. 8687	.5848	.0722	. 6736	. 3264	51
10	.36108	.63892	2.7694	.38720	2.5826	1.0723	.06747	.93253	50
11	. 6135	. 3865	.7674	. 8754	.5804	.0725	. 6757	. 3243	49
12	. 6162	. 3837	.7653	. 8787	.5781	.0726	. 6768	. 3232	48
13	. 6189	. 3810	.7632	. 8821	.5759	.0727	. 6778	. 3222	47
14	. 6217	. 3783	.7611	. 8854	.5737	.0728	. 6789	. 3211	46
15	.36244	.63756	2.7591	.38888	2.5715	1.0729	.06799	.93201	45
16	. 6271	. 3729	.7570	. 8921	.5693	.0731	. 6810	. 3190	44
17	. 6298	. 3702	.7550	. 8955	.5671	.0732	. 6820	. 3180	43
18	. 6325	. 3675	.7529	. 8988	.5640	.0733	. 6831	. 3169	42
19	. 6352	. 3648	.7509	. 9022	.5627	.0734	. 6841	. 3158	41
20	.36379	.63621	2.7488	.39055	2.5605	1.0736	.06852	.93148	40
21	. 6406	. 3593	.7468	. 9089	.5583	.0737	. 6863	. 3137	39
22	. 6433	. 3566	.7447	. 9122	.5561	.0738	. 6873	. 3127	38
23	. 6460	. 3539	.7427	. 9156	.5539	.0739	. 6884	. 3116	37
24	. 6488	. 3512	.7406	. 9189	.5517	.0740	. 6894	. 3105	36
25	.36515	.63485	2.7386	.39223	2.5495	1.0742	.06905	.93095	35
26	. 6542	. 3458	.7366	. 9257	.5473	.0743	. 6916	. 3084	34
27	. 6569	. 3431	.7346	. 9290	.5451	.0744	. 6926	. 3074	33
28	. 6596	. 3404	.7325	. 9324	.5430	.0745	. 6937	. 3063	32
29	. 6623	. 3377	.7305	. 9357	.5408	.0747	. 6947	. 3052	31
30	.36650	.63350	2.7285	.39391	2.5386	1.0748	.06958	.93042	30
31	. 6677	. 3323	.7265	. 9425	.5365	.0749	. 6969	. 3031	29
32	. 6704	. 3296	.7245	. 9458	.5343	.0750	. 6979	. 3020	28
33	. 6731	. 3269	.7225	. 9492	.5322	.0751	. 6990	. 3010	27
34	. 6758	. 3242	.7205	. 9525	.5300	.0753	. 7001	. 2999	26
35	.36785	.63214	2.7185	.39559	2.5278	1.0754	.07012	.92988	25
36	. 6812	. 3187	.7165	. 9593	.5257	.0755	. 7022	. 2978	24
37	. 6839	. 3160	.7145	. 9626	.5236	.0756	. 7033	. 2967	23
38	. 6866	. 3133	.7125	. 9660	.5214	.0758	. 7044	. 2956	22
39	. 6893	. 3106	.7105	. 9694	.5193	.0759	. 7054	. 2945	21
40	.36921	.63079	2.7085	.39727	2.5171	1.0760	.07065	.92935	20
41	. 6948	. 3052	.7065	. 9761	.5150	.0761	. 7076	. 2924	19
42	. 6975	. 3025	.7045	. 9795	.5129	.0763	. 7087	. 2913	18
43	. 7002	. 2998	.7026	. 9828	.5108	.0764	. 7097	. 2902	17
44	. 7029	. 2971	.7006	. 9862	.5086	.0765	. 7108	. 2892	16
45	.37056	.62944	2.6986	.39896	2.5065	1.0766	.07119	.92881	15
46	. 7083	. 2917	.6967	. 9930	.5044	.0768	. 7130	. 2870	14
47	. 7110	. 2890	.6947	. 9963	.5023	.0769	. 7141	. 2859	13
48	. 7137	. 2863	.6927	. 9997	.5002	.0770	. 7151	. 2848	12
49	. 7164	. 2836	.6908	.40031	.4981	.0771	. 7162	. 2838	11
50	.37191	.62809	2.6888	.40065	2.4960	1.0773	.07173	.92827	10
51	. 7218	. 2782	.6869	. 0098	.4939	.0774	. 7184	. 2816	9
52	. 7245	. 2755	.6849	. 0132	.4918	.0775	. 7195	. 2805	8
53	. 7272	. 2728	.6830	. 0166	.4897	.0776	. 7205	. 2794	7
54	. 7299	. 2701	.6810	. 0200	.4876	.0778	. 7216	. 2784	6
55	.37326	.62674	2.6791	.40233	2.4855	1.0779	.07227	.92773	5
56	. 7353	. 2647	.6772	. 0267	.4834	.0780	. 7238	. 2762	4
57	. 7380	. 2620	.6752	. 0301	.4813	.0781	. 7249	. 2751	3
58	. 7407	. 2593	.6733	. 0335	.4792	.0783	. 7260	. 2740	2
59	. 7434	. 2566	.6714	. 0369	.4772	.0784	. 7271	. 2729	1
60	. 7461	. 2539	.6695	. 0403	.4751	.0785	. 7282	. 2718	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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22° **Natural Trigonometrical Functions.** **157°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.37461	.62539	2.6695	.40403	2.4751	1.0785	.07282	.92718	60
1	.7488	.2512	.6675	.0436	.4730	.0787	.7292	.2707	59
2	.7514	.2485	.6656	.0470	.4709	.0788	.7303	.2696	58
3	.7541	.2458	.6637	.0504	.4689	.0789	.7314	.2686	57
4	.7568	.2431	.6618	.0538	.4668	.0790	.7325	.2675	56
5	.37595	.62404	2.6599	.40572	2.4647	1.0792	.07336	.92664	55
6	.7622	.2377	.6580	.0606	.4627	.0793	.7347	.2653	54
7	.7649	.2351	.6561	.0640	.4606	.0794	.7358	.2642	53
8	.7676	.2324	.6542	.0673	.4586	.0795	.7369	.2631	52
9	.7703	.2297	.6523	.0707	.4565	.0797	.7380	.2620	51
10	.37730	.62270	2.6504	.40741	2.4545	1.0798	.07391	.92609	50
11	.7757	.2243	.6485	.0775	.4525	.0799	.7402	.2598	49
12	.7784	.2216	.6466	.0809	.4504	.0801	.7413	.2587	48
13	.7811	.2189	.6447	.0843	.4484	.0802	.7424	.2576	47
14	.7838	.2162	.6428	.0877	.4463	.0803	.7435	.2565	46
15	.37865	.62135	2.6410	.40911	2.4443	1.0804	.07446	.92554	45
16	.7892	.2108	.6391	.0945	.4423	.0806	.7457	.2543	44
17	.7919	.2081	.6372	.0979	.4403	.0807	.7468	.2532	43
18	.7946	.2054	.6353	.1013	.4382	.0808	.7479	.2521	42
19	.7972	.2027	.6335	.1047	.4362	.0810	.7490	.2510	41
20	.37999	.62000	2.6316	.41081	2.4342	1.0811	.07501	.92499	40
21	.8026	.1974	.6297	.1115	.4322	.0812	.7512	.2488	39
22	.8053	.1947	.6279	.1149	.4302	.0813	.7523	.2477	38
23	.8080	.1920	.6260	.1183	.4282	.0815	.7534	.2466	37
24	.8107	.1893	.6242	.1217	.4262	.0816	.7545	.2455	36
25	.38134	.61866	2.6223	.41251	2.4242	1.0817	.07556	.92443	35
26	.8161	.1839	.6205	.1285	.4222	.0819	.7567	.2432	34
27	.8188	.1812	.6186	.1319	.4202	.0820	.7579	.2421	33
28	.8214	.1785	.6168	.1353	.4182	.0821	.7590	.2410	32
29	.8241	.1758	.6150	.1387	.4162	.0823	.7601	.2399	31
30	.38268	.61732	2.6131	.41421	2.4142	1.0824	.07612	.92388	30
31	.8295	.1705	.6113	.1455	.4122	.0825	.7623	.2377	29
32	.8322	.1678	.6095	.1489	.4102	.0826	.7634	.2366	28
33	.8349	.1651	.6076	.1524	.4083	.0828	.7645	.2354	27
34	.8376	.1624	.6058	.1558	.4063	.0829	.7657	.2343	26
35	.38403	.61597	2.6040	.41592	2.4043	1.0830	.07668	.92332	25
36	.8429	.1570	.6022	.1626	.4023	.0832	.7679	.2321	24
37	.8456	.1544	.6003	.1660	.4004	.0833	.7690	.2310	23
38	.8483	.1517	.5985	.1694	.3984	.0834	.7701	.2299	22
39	.8510	.1490	.5967	.1728	.3964	.0836	.7712	.2287	21
40	.38537	.61463	2.5949	.41762	2.3945	1.0837	.07724	.92276	20
41	.8564	.1436	.5931	.1797	.3925	.0838	.7735	.2265	19
42	.8591	.1409	.5913	.1831	.3906	.0840	.7746	.2254	18
43	.8617	.1382	.5895	.1865	.3886	.0841	.7757	.2242	17
44	.8644	.1356	.5877	.1899	.3867	.0842	.7769	.2231	16
45	.38671	.61329	2.5859	.41933	2.3847	1.0844	.07780	.92220	15
46	.8698	.1302	.5841	.1968	.3828	.0845	.7791	.2209	14
47	.8725	.1275	.5823	.2002	.3808	.0846	.7802	.2197	13
48	.8751	.1248	.5805	.2036	.3789	.0847	.7814	.2186	12
49	.8778	.1222	.5787	.2070	.3770	.0849	.7825	.2175	11
50	.38805	.61195	2.5770	.42105	2.3750	1.0850	.07836	.92164	10
51	.8832	.1168	.5752	.2139	.3731	.0851	.7847	.2152	9
52	.8859	.1141	.5734	.2173	.3712	.0853	.7859	.2141	8
53	.8886	.1114	.5716	.2207	.3692	.0854	.7870	.2130	7
54	.8912	.1088	.5699	.2242	.3673	.0855	.7881	.2118	6
55	.38939	.61061	2.5681	.42276	2.3654	1.0857	.07893	.92107	5
56	.8966	.1034	.5663	.2310	.3635	.0858	.7904	.2096	4
57	.8993	.1007	.5646	.2344	.3616	.0859	.7915	.2084	3
58	.9019	.0980	.5628	.2379	.3597	.0861	.7927	.2073	2
59	.9046	.0954	.5610	.2413	.3577	.0862	.7938	.2062	1
60	.9073	.0927	.5593	.2447	.3558	.0864	.7949	.2050	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

112° **67°**

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23° Natural Trigonometrical Functions. 156°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.39073	.60927	2.5593	.42447	2.3558	1.0864	.07949	.92050	60
1	.9100	.0900	.5575	.2482	.3539	.0865	.7961	.2039	59
2	.9126	.0873	.5558	.2516	.3520	.0866	.7972	.2028	58
3	.9153	.0846	.5540	.2550	.3501	.0868	.7984	.2016	57
4	.9180	.0820	.5523	.2585	.3482	.0869	.7995	.2005	56
5	.39207	.60793	2.5506	.42619	2.3463	1.0870	.08006	.91993	55
6	.9234	.0766	.5488	.2654	.3445	.0872	.8018	.1982	54
7	.9260	.0739	.5471	.2688	.3426	.0873	.8029	.1971	53
8	.9287	.0713	.5453	.2722	.3407	.0874	.8041	.1959	52
9	.9314	.0686	.5436	.2757	.3388	.0876	.8052	.1948	51
10	.39341	.60659	2.5419	.42791	2.3369	1.0877	.08063	.91936	50
11	.9367	.0632	.5402	.2826	.3350	.0878	.8075	.1925	49
12	.9394	.0606	.5384	.2860	.3332	.0880	.8086	.1913	48
13	.9421	.0579	.5367	.2894	.3313	.0881	.8098	.1902	47
14	.9448	.0552	.5350	.2929	.3294	.0882	.8109	.1891	46
15	.39474	.60526	2.5333	.42963	2.3276	1.0884	.08121	.91879	45
16	.9501	.0499	.5316	.2998	.3257	.0885	.8132	.1868	44
17	.9528	.0472	.5299	.3032	.3238	.0886	.8144	.1856	43
18	.9554	.0445	.5281	.3067	.3220	.0888	.8155	.1845	42
19	.9581	.0419	.5264	.3101	.3201	.0889	.8167	.1833	41
20	.39608	.60392	2.5247	.43136	2.3183	1.0891	.08178	.91822	40
21	.9635	.0365	.5230	.3170	.3164	.0892	.8190	.1810	39
22	.9661	.0339	.5213	.3205	.3145	.0893	.8201	.1798	38
23	.9688	.0312	.5196	.3239	.3127	.0895	.8213	.1787	37
24	.9715	.0285	.5179	.3274	.3109	.0896	.8224	.1775	36
25	.39741	.60258	2.5163	.43308	2.3090	1.0897	.08236	.91764	35
26	.9768	.0232	.5146	.3343	.3072	.0899	.8248	.1752	34
27	.9795	.0205	.5129	.3377	.3053	.0900	.8259	.1741	33
28	.9821	.0178	.5112	.3412	.3035	.0902	.8271	.1729	32
29	.9848	.0152	.5095	.3447	.3017	.0903	.8282	.1718	31
30	.39875	.60125	2.5078	.43481	2.2998	1.0904	.08294	.91706	30
31	.9901	.0098	.5062	.3516	.2980	.0906	.8306	.1694	29
32	.9928	.0072	.5045	.3550	.2962	.0907	.8317	.1683	28
33	.9955	.0045	.5028	.3585	.2944	.0908	.8329	.1671	27
34	.9981	.0018	.5011	.3620	.2925	.0910	.8340	.1659	26
35	.40008	.59992	2.4995	.43654	2.2907	1.0911	.08352	.91648	25
36	.0035	.9965	.4978	.3689	.2889	.0913	.8364	.1636	24
37	.0061	.9938	.4961	.3723	.2871	.0914	.8375	.1625	23
38	.0088	.9912	.4945	.3758	.2853	.0915	.8387	.1613	22
39	.0115	.9885	.4928	.3793	.2835	.0917	.8399	.1601	21
40	.40141	.59858	2.4912	.43827	2.2817	1.0918	.08410	.91590	20
41	.0168	.9832	.4895	.3862	.2799	.0920	.8422	.1578	19
42	.0195	.9805	.4879	.3897	.2781	.0921	.8434	.1566	18
43	.0221	.9778	.4862	.3932	.2763	.0922	.8445	.1554	17
44	.0248	.9752	.4846	.3966	.2745	.0924	.8457	.1543	16
45	.40275	.59725	2.4829	.44001	2.2727	1.0925	.08469	.91531	15
46	.0301	.9699	.4813	.4036	.2709	.0927	.8480	.1519	14
47	.0328	.9672	.4797	.4070	.2691	.0928	.8492	.1508	13
48	.0354	.9645	.4780	.4105	.2673	.0929	.8504	.1496	12
49	.0381	.9619	.4764	.4140	.2655	.0931	.8516	.1484	11
50	.40408	.59592	2.4748	.44175	2.2637	1.0932	.08527	.91472	10
51	.0434	.9566	.4731	.4209	.2619	.0934	.8539	.1461	9
52	.0461	.9539	.4715	.4244	.2602	.0935	.8551	.1449	8
53	.0487	.9512	.4699	.4279	.2584	.0936	.8563	.1437	7
54	.0514	.9486	.4683	.4314	.2566	.0938	.8575	.1425	6
55	.40541	.59459	2.4666	.44349	2.2548	1.0939	.08586	.91414	5
56	.0567	.9433	.4650	.4383	.2531	.0941	.8598	.1402	4
57	.0594	.9406	.4634	.4418	.2513	.0942	.8610	.1390	3
58	.0620	.9379	.4618	.4453	.2495	.0943	.8622	.1378	2
59	.0647	.9353	.4602	.4488	.2478	.0945	.8634	.1366	1
60	.0674	.9326	.4586	.4523	.2460	.0946	.8645	.1354	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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24° **Natural Trigonometrical Functions.** **155°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.40674	.59326	2.4586	.44523	2.2460	1.0946	.08645	.91354	60
1	. 0700	. 9300	.4570	. 4558	.2443	.0948	. 8657	. 1343	59
2	. 0727	. 9273	.4554	. 4593	.2425	.0949	. 8669	. 1331	58
3	. 0753	. 9247	.4538	. 4627	.2408	.0951	. 8681	. 1319	57
4	. 0780	. 9220	.4522	. 4662	.2390	.0952	. 8693	. 1307	56
5	.40806	.59193	2.4506	.44697	2.2373	1.0953	.08705	.91295	55
6	. 0833	. 9167	.4490	. 4732	.2355	.0955	. 8716	. 1283	54
7	. 0860	. 9140	.4474	. 4767	.2338	.0956	. 8728	. 1271	53
8	. 0886	. 9114	.4458	. 4802	.2320	.0958	. 8740	. 1260	52
9	. 0913	. 9087	.4442	. 4837	.2303	.0959	. 8752	. 1248	51
10	.40939	.59061	2.4426	.44872	2.2286	1.0961	.08764	.91236	50
11	. 0966	. 9034	.4411	. 4907	.2268	.0962	. 8776	. 1224	49
12	. 0992	. 9008	.4395	. 4942	.2251	.0963	. 8788	. 1212	48
13	. 1019	. 8981	.4379	. 4977	.2234	.0965	. 8800	. 1200	47
14	. 1045	. 8955	.4363	. 5012	.2216	.0966	. 8812	. 1188	46
15	.41072	.58928	2.4347	.45047	2.2199	1.0968	.08824	.91176	45
16	. 1098	. 8901	.4332	. 5082	.2182	.0969	. 8836	. 1164	44
17	. 1125	. 8875	.4316	. 5117	.2165	.0971	. 8848	. 1152	43
18	. 1151	. 8848	.4300	. 5152	.2147	.0972	. 8860	. 1140	42
19	. 1178	. 8822	.4285	. 5187	.2130	.0973	. 8872	. 1128	41
20	.41204	.58795	2.4269	.45222	2.2113	1.0975	.08884	.91116	40
21	. 1231	. 8769	.4254	. 5257	.2096	.0976	. 8896	. 1104	39
22	. 1257	. 8742	.4238	. 5292	.2079	.0978	. 8908	. 1092	38
23	. 1284	. 8716	.4222	. 5327	.2062	.0979	. 8920	. 1080	37
24	. 1310	. 8689	.4207	. 5362	.2045	.0981	. 8932	. 1068	36
25	.41337	.58663	2.4191	.45397	2.2028	1.0982	.08944	.91056	35
26	. 1363	. 8636	.4176	. 5432	.2011	.0984	. 8956	. 1044	34
27	. 1390	. 8610	.4160	. 5467	.1994	.0985	. 8968	. 1032	33
28	. 1416	. 8584	.4145	. 5502	.1977	.0986	. 8980	. 1020	32
29	. 1443	. 8557	.4130	. 5537	.1960	.0988	. 8992	. 1008	31
30	.41469	.58531	2.4114	.45573	2.1943	1.0989	.09004	.90996	30
31	. 1496	. 8504	.4099	. 5608	.1926	.0991	. 9016	. 0984	29
32	. 1522	. 8478	.4083	. 5643	.1909	.0992	. 9028	. 0972	28
33	. 1549	. 8451	.4068	. 5678	.1892	.0994	. 9040	. 0960	27
34	. 1575	. 8425	.4053	. 5713	.1875	.0995	. 9052	. 0948	26
35	41602	.58398	2.4037	.45748	2.1859	1.0997	.09064	.90936	25
36	. 1628	. 8372	.4022	. 5783	.1842	.0998	. 9076	. 0924	24
37	. 1654	. 8345	.4007	. 5819	.1825	.1000	. 9088	. 0911	23
38	. 1681	. 8319	.3992	. 5854	.1808	.1001	. 9101	. 0899	22
39	. 1707	. 8292	.3976	. 5889	.1792	.1003	. 9113	. 0887	21
40	.41734	.58266	2.3961	.45924	2.1775	1.1004	.09125	.90875	20
41	. 1760	. 8240	.3946	. 5960	.1758	.1005	. 9137	. 0863	19
42	. 1787	. 8213	.3931	. 5995	.1741	.1007	. 9149	. 0851	18
43	. 1813	. 8187	.3916	. 6030	.1725	.1008	. 9161	. 0839	17
44	. 1839	. 8160	.3901	. 6065	.1708	.1010	. 9173	. 0826	16
45	.41866	.58134	2.3886	.46101	2.1692	1.1011	.09186	.90814	15
46	. 1892	. 8108	.3871	. 6136	.1675	.1013	. 9198	. 0802	14
47	. 1919	. 8081	.3856	. 6171	.1658	.1014	. 9210	. 0790	13
48	. 1945	. 8055	.3841	. 6206	.1642	.1016	. 9222	. 0778	12
49	. 1972	. 8028	.3826	. 6242	.1625	.1017	. 9234	. 0765	11
50	.41998	.58002	2.3811	.46277	2.1609	1.1019	.09247	.90753	10
51	. 2024	. 7975	.3796	. 6312	.1592	.1020	. 9259	. 0741	9
52	. 2051	. 7949	.3781	. 6348	.1576	.1022	. 9271	. 0729	8
53	. 2077	. 7923	.3766	. 6383	.1559	.1023	. 9283	. 0717	7
54	. 2103	. 7896	.3751	. 6418	.1543	.1025	. 9296	. 0704	6
55	.42130	.57870	2.3736	.46454	2.1527	1.1026	.09308	.90692	5
56	. 2156	. 7844	.3721	. 6489	.1510	.1028	. 9320	. 0680	4
57	. 2183	. 7817	.3706	. 6524	.1494	.1029	. 9332	. 0668	3
58	. 2209	. 7791	.3691	. 6560	.1478	.1031	. 9345	. 0655	2
59	. 2235	. 7764	.3677	. 6595	.1461	.1032	. 9357	. 0643	1
60	. 2262	. 7738	.3662	. 6631	.1445	.1034	. 9369	. 0631	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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25°				Natural Trigonometrical Functions.					154°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.42262	.57738	2.3662	.46631	2.1445	1.1034	.09369	.90631	60
1	. 2288	. 7712	.3647	. 6666	.1429	.1035	. 9381	. 0618	59
2	. 2314	. 7685	.3632	. 6702	.1412	.1037	. 9394	. 0606	58
3	. 2341	. 7659	.3618	. 6737	.1396	.1038	. 9406	. 0594	57
4	. 2367	. 7633	.3603	. 6772	.1380	.1040	. 9418	. 0581	56
5	.42394	.57606	2.3588	.46808	2.1364	1.1041	.09431	.90569	55
6	. 2420	. 7580	.3574	. 6843	.1348	.1043	. 9443	. 0557	54
7	. 2446	. 7554	.3559	. 6879	.1331	.1044	. 9455	. 0544	53
8	. 2473	. 7527	.3544	. 6914	.1315	.1046	. 9468	. 0532	52
9	. 2499	. 7501	.3530	. 6950	.1299	.1047	. 9480	. 0520	51
10	.42525	.57475	2.3515	.46985	2.1283	1.1049	.09492	.90507	50
11	. 2552	. 7448	.3501	. 7021	.1267	.1050	. 9505	. 0495	49
12	. 2578	. 7422	.3486	. 7056	.1251	.1052	. 9517	. 0483	48
13	. 2604	. 7396	.3472	. 7092	.1235	.1053	. 9530	. 0470	47
14	. 2630	. 7369	.3457	. 7127	.1219	.1055	. 9542	. 0458	46
15	.42657	.57343	2.3443	.47163	2.1203	1.1056	.09554	.90445	45
16	. 2683	. 7317	.3428	. 7199	.1187	.1058	. 9567	. 0433	44
17	. 2709	. 7290	.3414	. 7234	.1171	.1059	. 9579	. 0421	43
18	. 2736	. 7264	.3399	. 7270	.1155	.1061	. 9592	. 0408	42
19	. 2762	. 7238	.3385	. 7305	.1139	.1062	. 9604	. 0396	41
20	.42788	.57212	2.3371	.47341	2.1123	1.1064	.09617	.90383	40
21	. 2815	. 7185	.3356	. 7376	.1107	.1065	. 9629	. 0371	39
22	. 2841	. 7159	.3342	. 7412	.1092	.1067	. 9641	. 0358	38
23	. 2867	. 7133	.3328	. 7448	.1076	.1068	. 9654	. 0346	37
24	. 2893	. 7106	.3313	. 7483	.1060	.1070	. 9666	. 0333	36
25	.42920	.57080	2.3299	.47519	2.1044	1.1072	.09679	.90321	35
26	. 2946	. 7054	.3285	. 7555	.1028	.1073	. 9691	. 0308	34
27	. 2972	. 7028	.3271	. 7590	.1013	.1075	. 9704	. 0296	33
28	. 2998	. 7001	.3256	. 7626	.0997	.1076	. 9716	. 0283	32
29	. 3025	. 6975	.3242	. 7662	.0981	.1078	. 9729	. 0271	31
30	.43051	.56949	2.3228	.47697	2.0965	1.1079	.09741	.90258	30
31	. 3077	. 6923	.3214	. 7733	.0950	.1081	. 9754	. 0246	29
32	. 3104	. 6896	.3200	. 7769	.0934	.1082	. 9766	. 0233	28
33	. 3130	. 6870	.3186	. 7805	.0918	.1084	. 9779	. 0221	27
34	. 3156	. 6844	.3172	. 7840	.0903	.1085	. 9792	. 0208	26
35	.43182	.56818	2.3158	.47876	2.0887	1.1087	.09804	.90196	25
36	. 3208	. 6791	.3143	. 7912	.0872	.1088	. 9817	. 0183	24
37	. 3235	. 6765	.3129	. 7948	.0856	.1090	. 9829	. 0171	23
38	. 3261	. 6739	.3115	. 7983	.0840	.1092	. 9842	. 0158	22
39	. 3287	. 6713	.3101	. 8019	.0825	.1093	. 9854	. 0145	21
40	.43313	.56686	2.3087	.48055	2.0809	1.1095	.09867	.90133	20
41	. 3340	. 6660	.3073	. 8091	.0794	.1096	. 9880	. 0120	19
42	. 3366	. 6634	.3059	. 8127	.0778	.1098	. 9892	. 0108	18
43	. 3392	. 6608	.3046	. 8162	.0763	.1099	. 9905	. 0095	17
44	. 3418	. 6582	.3032	. 8198	.0747	.1101	. 9917	. 0082	16
45	.43444	.56555	2.3018	.48234	2.0732	1.1102	.09930	.90070	15
46	. 3471	. 6529	.3004	. 8270	.0717	.1104	. 9943	. 0057	14
47	. 3497	. 6503	.2990	. 8306	.0701	.1106	. 9955	. 0044	13
48	. 3523	. 6477	.2976	. 8342	.0686	.1107	. 9968	. 0032	12
49	. 3549	. 6451	.2962	. 8378	.0671	.1109	. 9981	. 0019	11
50	.43575	.56424	2.2949	.48414	2.0655	1.1110	.09993	.90006	10
51	. 3602	. 6398	.2935	. 8449	.0640	.1112	.10006	.89994	9
52	. 3628	. 6372	.2921	. 8485	.0625	.1113	. 0019	. 9981	8
53	. 3654	. 6346	.2907	. 8521	.0609	.1115	. 0031	. 9968	7
54	. 3680	. 6320	.2894	. 8557	.0594	.1116	. 0044	. 9956	6
55	.43706	.56294	2.2880	.48593	2.0579	1.1118	.10057	.89943	5
56	. 3732	. 6267	.2866	. 8629	.0564	.1120	. 0070	. 9930	4
57	. 3759	. 6241	.2853	. 8665	.0548	.1121	. 0082	. 9918	3
58	. 3785	. 6215	.2839	. 8701	.0533	.1123	. 0095	. 9905	2
59	. 3811	. 6189	.2825	. 8737	.0518	.1124	. 0108	. 9892	1
60	. 3837	. 6163	.2812	. 8773	.0503	.1126	. 0121	. 9879	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
115°									64°

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26° **Natural Trigonometrical Functions.** **153°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.43837	.56163	2.2812	.48773	2.0503	1.1126	.10121	.89879	60
1	. 3863	. 6137	.2798	. 8809	.0488	.1127	. 0133	. 9867	59
2	. 3889	. 6111	.2784	. 8845	.0473	.1129	. 0146	. 9854	58
3	. 3915	. 6084	.2771	. 8881	.0458	.1131	. 0159	. 9841	57
4	. 3942	. 6058	.2757	. 8917	.0443	.1132	. 0172	. 9828	56
5	.43968	.56032	2.2744	.48953	2.0427	1.1134	.10184	.89815	55
6	. 3994	. 6006	.2730	. 8989	.0412	.1135	. 0197	. 9803	54
7	. 4020	. 5980	.2717	. 9025	.0397	.1137	. 0210	. 9790	53
8	. 4046	. 5954	.2703	. 9062	.0382	.1139	. 0223	. 9777	52
9	. 4072	. 5928	.2690	. 9098	.0367	.1140	. 0236	. 9764	51
10	.44098	.55902	2.2676	.49134	2.0352	1.1142	.10248	.89751	50
11	. 4124	. 5875	.2663	. 9170	.0338	.1143	. 0261	. 9739	49
12	. 4150	. 5849	.2650	. 9206	.0323	.1145	. 0274	. 9726	48
13	. 4177	. 5823	.2636	. 9242	.0308	.1147	. 0287	. 9713	47
14	. 4203	. 5797	.2623	. 9278	.0293	.1148	. 0300	. 9700	46
15	.44229	.55771	2.2610	.49314	2.0278	1.1150	.10313	.89687	45
16	. 4255	. 5745	.2596	. 9351	.0263	.1151	. 0326	. 9674	44
17	. 4281	. 5719	.2583	. 9387	.0248	.1153	. 0338	. 9661	43
18	. 4307	. 5693	.2570	. 9423	.0233	.1155	. 0351	. 9649	42
19	. 4333	. 5667	.2556	. 9459	.0219	.1156	. 0364	. 9636	41
20	.44359	.55641	2.2543	.49495	2.0204	1.1158	.10377	.89623	40
21	. 4385	. 5615	.2530	. 9532	.0189	.1159	. 0390	. 9610	39
22	. 4411	. 5589	.2517	. 9568	.0174	.1161	. 0403	. 9597	38
23	. 4437	. 5562	.2503	. 9604	.0159	.1163	. 0416	. 9584	37
24	. 4463	. 5536	.2490	. 9640	.0145	.1164	. 0429	. 9571	36
25	.44489	.55510	2.2477	.49677	2.0130	1.1166	.10442	.89558	35
26	. 4516	. 5484	.2464	. 9713	.0115	.1167	. 0455	. 9545	34
27	. 4542	. 5458	.2451	. 9749	.0101	.1169	. 0468	. 9532	33
28	. 4568	. 5432	.2438	. 9785	.0086	.1171	. 0481	. 9519	32
29	. 4594	. 5406	.2425	. 9822	.0071	.1172	. 0493	. 9506	31
30	.44620	.55380	2.2411	.49858	2.0058	1.1174	.10506	.89493	30
31	. 4646	. 5354	.2398	. 9894	.0042	.1176	. 0519	. 9480	29
32	. 4672	. 5328	.2385	. 9931	.0028	.1177	. 0532	. 9467	28
33	. 4698	. 5302	.2372	. 9967	.0013	.1179	. 0545	. 9454	27
34	. 4724	. 5276	.2359	.50003	1.9998	.1180	. 0558	. 9441	26
35	.44750	.55250	2.2346	.50040	1.9984	1.1182	.10571	.89428	25
36	. 4776	. 5224	.2333	. 0076	.9969	.1184	. 0584	. 9415	24
37	. 4802	. 5198	.2320	. 0113	.9955	.1185	. 0598	. 9402	23
38	. 4828	. 5172	.2307	. 0149	.9940	.1187	. 0611	. 9389	22
39	. 4854	. 5146	.2294	. 0185	.9926	.1189	. 0624	. 9376	21
40	.44880	.55120	2.2282	.50222	1.9912	1.1190	.10637	.89363	20
41	. 4906	. 5094	.2269	. 0258	.9897	.1192	. 0650	. 9350	19
42	. 4932	. 5068	.2256	. 0295	.9883	.1193	. 0663	. 9337	18
43	. 4958	. 5042	.2243	. 0331	.9868	.1195	. 0676	. 9324	17
44	. 4984	. 5016	.2230	. 0368	.9854	.1197	. 0689	. 9311	16
45	.45010	.54990	2.2217	.50404	1.9840	1.1198	.10702	.89298	15
46	. 5036	. 4964	.2204	. 0441	.9825	.1200	. 0715	. 9285	14
47	. 5062	. 4938	.2192	. 0477	.9811	.1202	. 0728	. 9272	13
48	. 5088	. 4912	.2179	. 0514	.9797	.1203	. 0741	. 9258	12
49	. 5114	. 4886	.2166	. 0550	.9782	.1205	. 0754	. 9245	11
50	.45140	.54860	2.2153	.50587	1.9768	1.1207	.10768	.89232	10
51	. 5166	. 4834	.2141	. 0623	.9754	.1208	. 0781	. 9219	9
52	. 5191	. 4808	.2128	. 0660	.9739	.1210	. 0794	. 9206	8
53	. 5217	. 4782	.2115	. 0696	.9725	.1212	. 0807	. 9193	7
54	. 5243	. 4756	.2103	. 0733	.9711	.1213	. 0820	. 9180	6
55	.45269	.54730	2.2090	.50769	1.9697	1.1215	.10833	.89166	5
56	. 5295	. 4705	.2077	. 0806	.9683	.1217	. 0846	. 9153	4
57	. 5321	. 4679	.2065	. 0843	.9668	.1218	. 0860	. 9140	3
58	5347	. 4653	.2052	. 0879	.9654	.1220	. 0873	. 9127	2
59	. 5373	. 4627	.2039	. 0916	.9640	.1222	. 0886	. 9114	1
60	. 5399	. 4601	.2027	. 0952	.9626	.1223	. 0899	. 9101	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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27° **Natural Trigonometrical Functions.** **152°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.45399	.54601	2.2027	.50952	1.9626	1.1223	.10899	.89101	60
1	. 5425	. 4575	.2014	. 0989	.9612	.1225	. 0912	. 9087	59
2	. 5451	. 4549	.2002	. 1026	.9598	.1226	. 0926	. 9074	58
3	. 5477	. 4523	.1989	. 1062	.9584	.1228	. 0939	. 9061	57
4	. 5503	. 4497	.1977	. 1099	.9570	.1230	. 0952	. 9048	56
5	.45528	.54471	2.1964	.51136	1.9556	1.1231	.10965	.89034	55
6	. 5554	. 4445	.1952	. 1172	.9542	.1233	. 0979	. 9021	54
7	. 5580	. 4420	.1939	. 1209	.9528	.1235	. 0992	. 9008	53
8	. 5606	. 4394	.1927	. 1246	.9514	.1237	. 1005	. 8995	52
9	. 5632	. 4368	.1914	. 1283	.9500	.1238	. 1018	. 8981	51
10	.45658	.54342	2.1902	.51319	1.9486	1.1240	.11032	.88968	50
11	. 5684	. 4316	.1889	. 1356	.9472	.1242	. 1045	. 8955	49
12	. 5710	. 4290	.1877	. 1393	.9458	.1243	. 1058	. 8942	48
13	. 5736	. 4264	.1865	. 1430	.9444	.1245	. 1072	. 8928	47
14	. 5761	. 4238	.1852	. 1466	.9430	.1247	. 1085	. 8915	46
15	.45787	.54213	2.1840	.51503	1.9416	1.1248	.11098	.88902	45
16	. 5813	. 4187	.1828	. 1540	.9402	.1250	. 1112	. 8888	44
17	. 5839	. 4161	.1815	. 1577	.9388	.1252	. 1125	. 8875	43
18	. 5865	. 4135	.1803	. 1614	.9375	.1253	. 1138	. 8862	42
19	. 5891	. 4109	.1791	. 1651	.9361	.1255	. 1152	. 8848	41
20	.45917	.54083	2.1778	.51687	1.9347	1.1257	.11165	.88835	40
21	. 5942	. 4057	.1766	. 1724	.9333	.1258	. 1178	. 8822	39
22	. 5968	. 4032	.1754	. 1761	.9319	.1260	. 1192	. 8808	38
23	. 5994	. 4006	.1742	. 1798	.9306	.1262	. 1205	. 8795	37
24	. 6020	. 3980	.1730	. 1835	.9292	.1264	. 1218	. 8781	36
25	.46046	.53954	2.1717	.51872	1.9278	1.1265	.11232	.88768	35
26	. 6072	. 3928	.1705	. 1909	.9264	.1267	. 1245	. 8755	34
27	. 6097	. 3902	.1693	. 1946	.9251	.1269	. 1259	. 8741	33
28	. 6123	. 3877	.1681	. 1983	.9237	.1270	. 1272	. 8728	32
29	. 6149	. 3851	.1669	. 2020	.9223	.1272	. 1285	. 8714	31
30	.46175	.53825	2.1657	.52057	1.9210	1.1274	.11299	.88701	30
31	. 6201	. 3799	.1645	. 2094	.9196	.1275	. 1312	. 8688	29
32	. 6226	. 3773	.1633	. 2131	.9182	.1277	. 1326	. 8674	28
33	. 6252	. 3748	.1620	. 2168	.9169	.1279	. 1339	. 8661	27
34	. 6278	. 3722	.1608	. 2205	.9155	.1281	. 1353	. 8647	26
35	.46304	.53696	2.1596	.52242	1.9142	1.1282	.11366	.88634	25
36	. 6330	. 3670	.1584	. 2279	.9128	.1284	. 1380	. 8620	24
37	. 6355	. 3645	.1572	. 2316	.9115	.1286	. 1393	. 8607	23
38	. 6381	. 3619	.1560	. 2353	.9101	.1287	. 1407	. 8593	22
39	. 6407	. 3593	.1548	. 2390	.9088	.1289	. 1420	. 8580	21
40	.46433	.53567	2.1536	.52427	1.9074	1.1291	.11434	.88566	20
41	. 6458	. 3541	.1525	. 2464	.9061	.1293	. 1447	. 8553	19
42	. 6484	. 3516	.1513	. 2501	.9047	.1294	. 1461	. 8539	18
43	. 6510	. 3490	.1501	. 2538	.9034	.1296	. 1474	. 8526	17
44	. 6536	. 3464	.1489	. 2575	.9020	.1298	. 1488	. 8512	16
45	.46561	.53438	2.1477	.52612	1.9007	1.1299	.11501	.88499	15
46	. 6587	. 3413	.1465	. 2650	.8993	.1301	. 1515	. 8485	14
47	. 6613	. 3387	.1453	. 2687	.8980	.1303	. 1528	. 8472	13
48	. 6639	. 3361	.1441	. 2724	.8967	.1305	. 1542	. 8458	12
49	. 6664	. 3336	.1430	. 2761	.8953	.1306	. 1555	. 8444	11
50	.46690	.53310	2.1418	.52798	1.8940	1.1308	.11569	.88431	10
51	. 6716	. 3284	.1406	. 2836	.8927	.1310	. 1583	. 8417	9
52	. 6741	. 3258	.1394	. 2873	.8913	.1312	. 1596	. 8404	8
53	. 6767	. 3233	.1382	. 2910	.8900	.1313	. 1610	. 8390	7
54	. 6793	. 3207	.1371	. 2947	.8887	.1315	. 1623	. 8376	6
55	.46819	.53181	2.1359	.52984	1.8873	1.1317	.11637	.88363	5
56	. 6844	. 3156	.1347	. 3022	.8860	.1319	. 1651	. 8349	4
57	. 6870	. 3130	.1335	. 3059	.8847	.1320	. 1664	. 8336	3
58	. 6896	. 3104	.1324	. 3096	.8834	.1322	. 1678	. 8322	2
59	. 6921	. 3078	.1312	. 3134	.8820	.1324	. 1691	. 8308	1
60	. 6947	. 3053	.1300	. 3171	.8807	.1326	. 1705	. 8295	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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28° **Natural Trigonometrical Functions.** **151°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.46947	.53053	2.1300	.53171	1.8807	1.1326	.11705	.88295	60
1	. 6973	. 3027	.1289	. 3208	.8794	.1327	. 1719	. 8281	59
2	. 6998	. 3001	.1277	. 3245	.8781	.1329	. 1732	. 8267	58
3	. 7024	. 2976	.1266	. 3283	.8768	.1331	. 1746	. 8254	57
4	. 7050	. 2950	.1254	. 3320	.8754	.1333	. 1760	. 8240	56
5	.47075	.52924	2.1242	.53358	1.8741	1.1334	.11774	.88226	55
6	. 7101	. 2899	.1231	. 3395	.8728	.1336	. 1787	. 8213	54
7	. 7127	. 2873	.1219	. 3432	.8715	.1338	. 1801	. 8199	53
8	. 7152	. 2847	.1208	. 3470	.8702	.1340	. 1815	. 8185	52
9	. 7178	. 2822	.1196	. 3507	.8689	.1341	. 1828	. 8171	51
10	.47204	.52796	2.1185	.53545	1.8676	1.1343	.11842	.88158	50
11	. 7229	. 2770	.1173	. 3582	.8663	.1345	. 1856	. 8144	49
12	. 7255	. 2745	.1162	. 3619	.8650	.1347	. 1870	. 8130	48
13	. 7281	. 2719	.1150	. 3657	.8637	.1349	. 1883	. 8117	47
14	. 7306	. 2694	.1139	. 3694	.8624	.1350	. 1897	. 8103	46
15	.47332	.52668	2.1127	.53732	1.8611	1.1352	.11911	.88089	45
16	. 7357	. 2642	.1116	. 3769	.8598	.1354	. 1925	. 8075	44
17	. 7383	. 2617	.1104	. 3807	.8585	.1356	. 1938	. 8061	43
18	. 7409	. 2591	.1093	. 3844	.8572	.1357	. 1952	. 8048	42
19	. 7434	. 2565	.1082	. 3882	.8559	.1359	. 1966	. 8034	41
20	.47460	.52540	2.1070	.53919	1.8546	1.1361	.11980	.88020	40
21	. 7486	. 2514	.1059	. 3957	.8533	.1363	. 1994	. 8006	39
22	. 7511	. 2489	.1048	. 3995	.8520	.1365	. 2007	. 7992	38
23	. 7537	. 2463	.1036	. 4032	.8507	.1366	. 2021	. 7979	37
24	. 7562	. 2437	.1025	. 4070	.8495	.1368	. 2035	. 7965	36
25	.47588	.52412	2.1014	.54107	1.8482	1.1370	.12049	.87951	35
26	. 7613	. 2386	.1002	. 4145	.8469	.1372	. 2063	. 7937	34
27	. 7639	. 2361	.0991	. 4183	.8456	.1373	. 2077	. 7923	33
28	. 7665	. 2335	.0980	. 4220	.8443	.1375	. 2090	. 7909	32
29	. 7690	. 2310	.0969	. 4258	.8430	.1377	. 2104	. 7895	31
30	.47716	.52284	2.0957	.54295	1.8418	1.1379	.12118	.87882	30
31	. 7741	. 2258	.0946	. 4333	.8405	.1381	. 2132	. 7868	29
32	. 7767	. 2233	.0935	. 4371	.8392	.1382	. 2146	. 7854	28
33	. 7792	. 2207	.0924	. 4409	.8379	.1384	. 2160	. 7840	27
34	. 7818	. 2182	.0912	. 4446	.8367	.1386	. 2174	. 7826	26
35	.47844	.52156	2.0901	.54484	1.8354	1.1388	.12188	.87812	25
36	. 7869	. 2131	.0890	. 4522	.8341	.1390	. 2202	. 7798	24
37	. 7895	. 2105	.0879	. 4559	.8329	.1391	. 2216	. 7784	23
38	. 7920	. 2080	.0868	. 4597	.8316	.1393	. 2229	. 7770	22
39	. 7946	. 2054	.0857	. 4635	.8303	.1395	. 2243	. 7756	21
40	.47971	.52029	2.0846	.54673	1.8291	1.1397	.12257	.87742	20
41	. 7997	. 2003	.0835	. 4711	.8278	.1399	. 2271	. 7728	19
42	. 8022	. 1978	.0824	. 4748	.8265	.1401	. 2285	. 7715	18
43	. 8048	. 1952	.0812	. 4786	.8253	.1402	. 2299	. 7701	17
44	. 8073	. 1927	.0801	. 4824	.8240	.1404	. 2313	. 7687	16
45	.48099	.51901	2.0790	.54862	1.8227	1.1406	.12327	.87673	15
46	. 8124	. 1876	.0779	. 4900	.8215	.1408	. 2341	. 7659	14
47	. 8150	. 1850	.0768	. 4937	.8202	.1410	. 2355	. 7645	13
48	. 8175	. 1825	.0757	. 4975	.8190	.1411	. 2369	. 7631	12
49	. 8201	. 1799	.0746	. 5013	.8177	.1413	. 2383	. 7617	11
50	.48226	.51774	2.0735	.55051	1.8165	1.1415	.12397	.87603	10
51	. 8252	. 1748	.0725	. 5089	.8152	.1417	. 2411	. 7588	9
52	. 8277	. 1723	.0714	. 5127	.8140	.1419	. 2425	. 7574	8
53	. 8303	. 1697	.0703	. 5165	.8127	.1421	. 2439	. 7560	7
54	. 8328	. 1672	.0692	. 5203	.8115	.1422	. 2453	. 7546	6
55	.48354	.51646	2.0681	.55241	1.8102	1.1424	.12468	.87532	5
56	. 8379	. 1621	.0670	. 5279	.8090	.1426	. 2482	. 7518	4
57	. 8405	. 1595	.0659	. 5317	.8078	.1428	. 2496	. 7504	3
58	. 8430	. 1570	.0648	. 5355	.8065	.1430	. 2510	. 7490	2
59	. 8455	. 1544	.0637	. 5393	.8053	.1432	. 2524	. 7476	1
60	. 8481	. 1519	.0627	. 5431	.8040	.1433	. 2538	. 7462	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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29° **Natural Trigonometrical Functions.** **150°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.48481	.51519	2.0627	.55431	1.8040	1.1433	.12538	.87462	60
1	. 8506	. 1493	.0616	. 5469	.8028	.1435	. 2552	. 7448	59
2	. 8532	. 1468	.0605	. 5507	.8016	.1437	. 2566	. 7434	58
3	. 8557	. 1443	.0594	. 5545	.8003	.1439	. 2580	. 7420	57
4	. 8583	. 1417	.0583	. 5583	.7991	.1441	. 2594	. 7406	56
5	.48608	.51392	2.0573	.55621	1.7979	1.1443	.12609	.87391	55
6	. 8633	. 1366	.0562	. 5659	.7966	.1445	. 2623	. 7377	54
7	. 8659	. 1341	.0551	. 5697	.7954	.1446	. 2637	. 7363	53
8	. 8684	. 1316	.0540	. 5735	.7942	.1448	. 2651	. 7349	52
9	. 8710	. 1290	.0530	. 5774	.7930	.1450	. 2665	. 7335	51
10	.48735	.51265	2.0519	.55812	1.7917	1.1452	.12679	.87320	50
11	. 8760	. 1239	.0508	. 5850	.7905	.1454	. 2694	. 7306	49
12	. 8786	. 1214	.0498	. 5888	.7893	.1456	. 2708	. 7292	48
13	. 8811	. 1189	.0487	. 5926	.7881	.1458	. 2722	. 7278	47
14	. 8837	. 1163	.0476	. 5964	.7868	.1459	. 2736	. 7264	46
15	.48862	.51138	2.0466	.56003	1.7856	1.1461	.12750	.87250	45
16	. 8887	. 1112	.0455	. 6041	.7844	.1463	. 2765	. 7235	44
17	. 8913	. 1087	.0444	. 6079	.7832	.1465	. 2779	. 7221	43
18	. 8938	. 1062	.0434	. 6117	.7820	.1467	. 2793	. 7207	42
19	. 8964	. 1036	.0423	. 6156	.7808	.1469	. 2807	. 7193	41
20	.48989	.51011	2.0413	.56194	1.7795	1.1471	.12821	.87178	40
21	. 9014	. 0986	.0402	. 6232	.7783	.1473	. 2836	. 7164	39
22	. 9040	. 0960	.0392	. 6270	.7771	.1474	. 2850	. 7150	38
23	. 9065	. 0935	.0381	. 6309	.7759	.1476	. 2864	. 7136	37
24	. 9090	. 0910	.0370	. 6347	.7747	.1478	. 2879	. 7121	36
25	.49116	.50884	2.0360	.56385	1.7735	1.1480	.12893	.87107	35
26	. 9141	. 0859	.0349	. 6424	.7723	.1482	. 2907	. 7093	34
27	. 9166	. 0834	.0339	. 6462	.7711	.1484	. 2921	. 7078	33
28	. 9192	. 0808	.0329	. 6500	.7699	.1486	. 2936	. 7064	32
29	. 9217	. 0783	.0318	. 6539	.7687	.1488	. 2950	. 7050	31
30	.49242	.50758	2.0308	.56577	1.7675	1.1489	.12964	.87035	30
31	. 9268	. 0732	.0297	. 6616	.7663	.1491	. 2979	. 7021	29
32	. 9293	. 0707	.0287	. 6654	.7651	.1493	. 2993	. 7007	28
33	. 9318	. 0682	.0276	. 6692	.7639	.1495	. 3007	. 6992	27
34	. 9343	. 0656	.0266	. 6731	.7627	.1497	. 3022	. 6978	26
35	.49369	.50631	2.0256	.56769	1.7615	1.1499	.13036	.86964	25
36	. 9394	. 0606	.0245	. 6808	.7603	.1501	. 3050	. 6949	24
37	. 9419	. 0580	.0235	. 6846	.7591	.1503	. 3065	. 6935	23
38	. 9445	. 0555	.0224	. 6885	.7579	.1505	. 3079	. 6921	22
39	. 9470	. 0530	.0214	. 6923	.7567	.1507	. 3094	. 6906	21
40	.49495	.50505	2.0204	.56962	1.7555	1.1508	.13108	.86892	20
41	. 9521	. 0479	.0194	. 7000	.7544	.1510	. 3122	. 6877	19
42	. 9546	. 0454	.0183	. 7039	.7532	.1512	. 3137	. 6863	18
43	. 9571	. 0429	.0173	. 7077	.7520	.1514	. 3151	. 6849	17
44	. 9596	. 0404	.0163	. 7116	.7508	.1516	. 3166	. 6834	16
45	.49622	.50378	2.0152	.57155	1.7496	1.1518	.13180	.86820	15
46	. 9647	. 0353	.0142	. 7193	.7484	.1520	. 3194	. 6805	14
47	. 9672	. 0328	.0132	. 7232	.7473	.1522	. 3209	. 6791	13
48	. 9697	. 0303	.0122	. 7270	.7461	.1524	. 3223	. 6776	12
49	. 9723	. 0277	.0111	. 7309	.7449	.1526	. 3238	. 6762	11
50	.49748	.50252	2.0101	.57348	1.7437	1.1528	.13252	.86748	10
51	. 9773	. 0227	.0091	. 7386	.7426	.1530	. 3267	. 6733	9
52	. 9798	. 0202	.0081	. 7425	.7414	.1531	. 3281	. 6719	8
53	. 9823	. 0176	.0071	. 7464	.7402	.1533	. 3296	. 6704	7
54	. 9849	. 0151	.0061	. 7502	.7390	.1535	. 3310	. 6690	6
55	.49874	.50126	2.0050	.57541	1.7379	1.1537	.13325	.86675	5
56	. 9899	. 0101	.0040	. 7580	.7367	.1539	. 3339	. 6661	4
57	. 9924	. 0076	.0030	. 7619	.7355	.1541	. 3354	. 6646	3
58	. 9950	. 0050	.0020	. 7657	.7344	.1543	. 3368	. 6632	2
59	. 9975	. 0025	.0010	. 7696	.7332	.1545	. 3383	. 6617	1
60	.50000	. 0000	.0000	. 7735	.7320	.1547	. 3397	. 6602	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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30° **Natural Trigonometrical Functions.** **149°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.50000	.50000	2.0000	.57735	1.7320	1.1547	.13397	.86602	60
1	. 0025	.49975	1.9990	. 7774	.7309	.1549	. 3412	. 6588	59
2	. 0050	. 9950	.9980	. 7813	.7297	.1551	. 3426	. 6573	58
3	. 0075	. 9924	.9970	. 7851	.7286	.1553	. 3441	. 6559	57
4	. 0101	. 9899	.9960	. 7890	.7274	.1555	. 3456	. 6544	56
5	.50126	.49874	1.9950	.57929	1.7262	1.1557	.13470	.86530	55
6	. 0151	. 9849	.9940	. 7968	.7251	.1559	. 3485	. 6515	54
7	. 0176	. 9824	.9930	. 8007	.7239	.1561	. 3499	. 6500	53
8	. 0201	. 9799	.9920	. 8046	.7228	.1562	. 3514	. 6486	52
9	. 0226	. 9773	.9910	. 8085	.7216	.1564	. 3529	. 6471	51
10	.50252	.49748	1.9900	.58123	1.7205	1.1566	.13543	.86457	50
11	. 0277	. 9723	.9890	. 8162	.7193	.1568	. 3558	. 6442	49
12	. 0302	. 9698	.9880	. 8201	.7182	.1570	. 3572	. 6427	48
13	. 0327	. 9673	.9870	. 8240	.7170	.1572	. 3587	. 6413	47
14	. 0352	. 9648	.9860	. 8279	.7159	.1574	. 3602	. 6398	46
15	.50377	.49623	1.9850	.58318	1.7147	1.1576	.13616	.86383	45
16	. 0402	. 9597	.9840	. 8357	.7136	.1578	. 3631	. 6369	44
17	. 0428	. 9572	.9830	. 8396	.7124	.1580	. 3646	. 6354	43
18	. 0453	. 9547	.9820	8435	.7113	.1582	. 3660	. 6339	42
19	. 0478	. 9522	.9811	. 8474	.7101	.1584	. 3675	. 6325	41
20	.50503	.49497	1.9801	.58513	1.7090	1.1586	.13690	.86310	40
21	. 0528	. 9472	.9791	. 8552	.7079	.1588	. 3704	. 6295	39
22	. 0553	. 9447	.9781	. 8591	.7067	.1590	. 3719	. 6281	38
23	. 0578	. 9422	.9771	. 8630	.7056	.1592	. 3734	. 6266	37
24	. 0603	. 9397	.9761	. 8670	.7044	.1594	. 3749	. 6251	36
25	.50628	.49371	1.9752	.58709	1.7033	1.1596	.13763	.86237	35
26	. 0653	. 9346	.9742	. 8748	.7022	.1598	. 3778	. 6222	34
27	. 0679	. 9321	.9732	. 8787	.7010	.1600	. 3793	. 6207	33
28	. 0704	. 9296	.9722	. 8826	.6999	.1602	. 3807	. 6192	32
29	. 0729	. 9271	.9713	. 8865	.6988	.1604	. 3822	. 6178	31
30	.50754	.49246	1.9703	.58904	1.6977	1.1606	.13837	.86163	30
31	. 0779	. 9221	.9693	. 8944	.6965	.1608	. 3852	. 6148	29
32	. 0804	. 9196	.9683	. 8983	.6954	.1610	. 3867	. 6133	28
33	. 0829	. 9171	.9674	. 9022	.6943	.1612	. 3881	. 6118	27
34	. 0854	. 9146	.9664	. 9061	.6931	.1614	. 3896	. 6104	26
35	.50879	.49121	1.9654	.59100	1.6920	1.1616	.13911	.86089	25
36	. 0904	. 9096	.9645	. 9140	.6909	.1618	. 3926	. 6074	24
37	. 0929	. 9071	.9635	. 9179	.6898	.1620	. 3941	. 6059	23
38	. 0954	. 9046	.9625	. 9218	.6887	.1622	. 3955	. 6044	22
39	. 0979	. 9021	.9616	. 9258	.6875	.1624	. 3970	. 6030	21
40	.51004	.48996	1.9606	.59297	1.6864	1.1626	.13985	.86015	20
41	. 1029	. 8971	.9596	. 9336	.6853	.1628	. 4000	. 6000	19
42	. 1054	. 8946	.9587	. 9376	.6842	.1630	. 4015	. 5985	18
43	. 1079	. 8921	.9577	. 9415	.6831	.1632	. 4030	. 5970	17
44	. 1104	. 8896	.9568	. 9454	.6820	.1634	. 4044	. 5955	16
45	.51129	.48871	1.9558	.59494	1.6808	1.1636	.14059	.85941	15
46	. 1154	. 8846	.9549	. 9533	.6797	.1638	. 4074	. 5926	14
47	. 1179	. 8821	.9539	. 9572	.6786	.1640	. 4089	. 5911	13
48	. 1204	. 8796	.9530	. 9612	.6775	.1642	. 4104	. 5896	12
49	. 1229	. 8771	.9520	. 9651	.6764	.1644	. 4119	. 5881	11
50	.51254	.48746	1.9510	.59691	1.6753	1.1646	.14134	.85866	10
51	. 1279	. 8721	.9501	. 9730	.6742	.1648	. 4149	. 5851	9
52	. 1304	. 8696	.9491	. 9770	.6731	.1650	. 4164	. 5836	8
53	. 1329	. 8671	.9482	. 9809	.6720	.1652	. 4178	. 5821	7
54	. 1354	. 8646	.9473	. 9849	.6709	.1654	. 4193	. 5806	6
55	.51379	.48621	1.9463	.59888	1.6698	1.1656	.14208	.85791	5
56	. 1404	. 8596	.9454	. 9928	.6687	.1658	. 4223	. 5777	4
57	. 1429	. 8571	.9444	. 9967	.6676	.1660	. 4238	. 5762	3
58	. 1454	. 8546	.9435	.60007	.6665	.1662	. 4253	. 5747	2
59	. 1479	. 8521	.9425	. 0046	.6654	.1664	. 4268	. 5732	1
60	. 1504	. 8496	.9416	. 0086	.6643	.1666	. 4283	. 5717	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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31° **Natural Trigonometrical Functions.** 148°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.51504	.48496	1.9416	.60086	1.6643	1.1666	.14283	.85717	60
1	. 1529	. 8471	.9407	. 0126	.6632	.1668	. 4298	. 5702	59
2	. 1554	. 8446	.9397	. 0165	.6621	.1670	. 4313	. 5687	58
3	. 1578	. 8421	.9388	. 0205	.6610	.1672	. 4328	. 5672	57
4	. 1603	. 8396	.9378	. 0244	.6599	.1674	. 4343	. 5657	56
5	.51628	.48371	1.9369	.60284	1.6588	1.1676	.14358	.85642	55
6	. 1653	. 8347	.9360	. 0324	.6577	.1678	. 4373	. 5627	54
7	. 1678	. 8322	.9350	. 0363	.6566	.1681	. 4388	. 5612	53
8	. 1703	. 8297	.9341	. 0403	.6555	.1683	. 4403	. 5597	52
9	. 1728	. 8272	.9332	. 0443	.6544	.1685	. 4418	. 5582	51
10	.51753	.48247	1.9322	.60483	1.6534	1.1687	.14433	.85566	50
11	. 1778	. 8222	.9313	. 0522	.6523	.1689	. 4448	. 5551	49
12	. 1803	. 8197	.9304	. 0562	.6512	.1691	. 4463	. 5536	48
13	. 1827	. 8172	.9295	. 0602	.6501	.1693	. 4479	. 5521	47
14	. 1852	. 8147	.9285	. 0642	.6490	.1695	. 4494	. 5506	46
15	.51877	.48123	1.9276	.60681	1.6479	1.1697	.14509	.85491	45
16	. 1902	. 8098	.9267	. 0721	.6469	.1699	. 4524	. 5476	44
17	. 1927	. 8073	.9258	. 0761	.6458	.1701	. 4539	. 5461	43
18	. 1952	. 8048	.9248	. 0801	.6447	.1703	. 4554	. 5446	42
19	. 1977	. 8023	.9239	. 0841	.6436	.1705	. 4569	. 5431	41
20	.52002	.47998	1.9230	.60881	1.6425	1.1707	.14584	.85416	40
21	. 2026	. 7973	.9221	. 0920	.6415	.1709	. 4599	. 5400	39
22	. 2051	. 7949	.9212	. 0960	.6404	.1712	. 4615	. 5385	38
23	. 2076	. 7924	.9203	. 1000	.6393	.1714	. 4630	. 5370	37
24	. 2101	. 7899	.9193	. 1040	.6383	.1716	. 4645	. 5355	36
25	.52126	.47874	1.9184	.61080	1.6372	1.1718	.14660	.85340	35
26	. 2151	. 7849	.9175	. 1120	.6361	.1720	. 4675	. 5325	34
27	. 2175	. 7824	.9166	. 1160	.6350	.1722	. 4690	. 5309	33
28	. 2200	. 7800	.9157	. 1200	.6340	.1724	. 4706	. 5294	32
29	. 2225	. 7775	.9148	. 1240	.6329	.1726	. 4721	. 5279	31
30	.52250	.47750	1.9139	.61280	1.6318	1.1728	.14736	.85264	30
31	. 2275	. 7725	.9130	. 1320	.6308	.1730	. 4751	. 5249	29
32	. 2299	. 7700	.9121	. 1360	.6297	.1732	. 4766	. 5234	28
33	. 2324	. 7676	.9112	. 1400	.6286	.1734	. 4782	. 5218	27
34	. 2349	. 7651	.9102	. 1440	.6276	.1737	. 4797	. 5203	26
35	.52374	.47626	1.9093	.61480	1.6265	1.1739	.14812	.85188	25
36	. 2398	. 7601	.9084	. 1520	.6255	.1741	. 4827	. 5173	24
37	. 2423	. 7577	.9075	. 1560	.6244	.1743	. 4842	. 5157	23
38	. 2448	. 7552	.9066	. 1601	.6233	.1745	. 4858	. 5142	22
39	. 2473	. 7527	.9057	. 1641	.6223	.1747	. 4873	. 5127	21
40	.52498	.47502	1.9048	.61681	1.6212	1.1749	.14888	.85112	20
41	. 2522	. 7477	.9039	. 1721	.6202	.1751	. 4904	. 5096	19
42	. 2547	. 7453	.9030	. 1761	.6191	.1753	. 4919	. 5081	18
43	. 2572	. 7428	.9021	. 1801	.6181	.1756	. 4934	. 5066	17
44	. 2597	. 7403	.9013	. 1842	.6170	.1758	. 4949	. 5050	16
45	.52621	.47379	1.9004	.61882	1.6160	1.1760	.14965	.85035	15
46	. 2646	. 7354	.8995	. 1922	.6149	.1762	. 4980	. 5020	14
47	. 2671	. 7329	.8986	. 1962	.6139	.1764	. 4995	. 5004	13
48	. 2695	. 7304	.8977	. 2004	.6128	.1766	. 5011	. 4989	12
49	. 2720	. 7280	.8968	. 2043	.6118	.1768	. 5026	. 4974	11
50	.52745	.47255	1.8959	.62083	1.6107	1.1770	.15041	.84959	10
51	. 2770	. 7230	.8950	. 2123	.6097	.1772	. 5057	. 4943	9
52	. 2794	. 7205	.8941	. 2164	.6086	.1775	. 5072	. 4928	8
53	. 2819	. 7181	.8932	. 2204	.6076	.1777	. 5087	. 4912	7
54	. 2844	. 7156	.8924	. 2244	.6066	.1779	. 5103	. 4897	6
55	.52868	.47131	1.8915	.62285	1.6055	1.1781	.15118	.84882	5
56	. 2893	. 7107	.8906	. 2325	.6045	.1783	. 5133	. 4866	4
57	. 2918	. 7082	.8897	. 2366	.6034	.1785	. 5149	. 4851	3
58	. 2942	. 7057	.8888	. 2406	.6024	.1787	. 5164	. 4836	2
59	. 2967	. 7033	.8879	. 2446	.6014	.1790	. 5180	. 4820	1
60	. 2992	. 7008	.8871	. 2487	.6003	.1792	5195	. 4805	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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32° **Natural Trigonometrical Functions.** 147°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.52992	.47008	1.8871	.62487	1.6003	1.1792	.15195	.84805	60
1	. 3016	. 6983	.8862	. 2527	.5993	.1794	. 5211	. 4789	59
2	. 3041	. 6959	.8853	. 2568	.5983	.1796	. 5226	. 4774	58
3	. 3066	. 6934	.8844	. 2608	.5972	.1798	. 5241	. 4758	57
4	. 3090	. 6909	.8836	. 2649	.5962	.1800	. 5257	. 4743	56
5	.53115	.46885	1.8827	.62689	1.5952	1.1802	.15272	.84728	55
6	. 3140	. 6860	.8318	. 2730	.5941	.1805	. 5288	. 4712	54
7	. 3164	. 6835	.8809	. 2770	.5931	.1807	. 5303	. 4697	53
8	. 3189	. 6811	.8801	. 2811	.5921	.1809	. 5319	. 4681	52
9	. 3214	. 6786	.8792	. 2851	.5910	.1811	. 5334	. 4666	51
10	.53238	.46762	1.8783	.62892	1.5900	1.1813	.15350	.84650	50
11	. 3263	. 6737	.8775	. 2933	.5890	.1815	. 5365	. 4635	49
12	. 3288	. 6712	.8766	. 2973	.5880	.1818	. 5381	. 4619	48
13	. 3312	. 6688	.8757	. 3014	.5869	.1820	. 5396	. 4604	47
14	. 3337	. 6663	.8749	. 3055	.5859	.1822	. 5412	. 4588	46
15	.53361	.46638	1.8740	.63095	1.5849	1.1824	.15427	.84573	45
16	. 3386	. 6614	.8731	. 3136	.5839	.1826	. 5443	. 4557	44
17	. 3411	. 6589	.8723	. 3177	.5829	.1828	. 5458	. 4542	43
18	. 3435	. 6565	.8714	. 3217	.5818	.1831	. 5474	. 4526	42
19	. 3460	. 6540	.8706	. 3258	.5808	.1833	. 5489	. 4511	41
20	.53484	.46516	1.8697	.63299	1.5798	1.1835	.15505	.84495	40
21	. 3509	. 6491	.8688	. 3339	.5788	.1837	. 5520	. 4479	39
22	. 3533	. 6466	.8680	. 3380	.5778	.1839	. 5536	. 4464	38
23	. 3558	. 6442	.8671	. 3421	.5768	.1841	. 5552	. 4448	37
24	. 3583	. 6417	.8663	. 3462	.5757	.1844	. 5567	. 4433	36
25	.53607	.46393	1.8654	.63503	1.5747	1.1846	.15583	.84417	35
26	. 3632	. 6368	.8646	. 3543	.5737	.1848	. 5598	. 4402	34
27	. 3656	. 6344	.8637	. 3584	.5727	.1850	. 5614	. 4386	33
28	. 3681	. 6319	.8629	. 3625	.5717	.1852	. 5630	. 4370	32
29	. 3705	. 6294	.8620	. 3666	.5707	.1855	. 5645	. 4355	31
30	.53730	.46270	1.8611	.63707	1.5697	1.1857	.15661	.84339	30
31	. 3754	. 6245	.8603	. 3748	.5687	.1859	. 5676	. 4323	29
32	. 3779	. 6221	.8595	. 3789	.5677	.1861	. 5692	. 4308	28
33	. 3803	. 6196	.8586	. 3830	.5667	.1863	. 5708	. 4292	27
34	. 3828	. 6172	.8578	. 3871	.5657	.1866	. 5723	. 4276	26
35	.53852	.46147	1.8569	.63912	1.5646	1.1868	.15739	.84261	25
36	. 3877	. 6123	.8561	. 3953	.5636	.1870	. 5755	. 4245	24
37	. 3901	. 6098	.8552	. 3994	.5626	.1872	. 5770	. 4229	23
38	. 3926	. 6074	.8544	. 4035	.5616	.1874	. 5786	. 4214	22
39	. 3950	. 6049	.8535	. 4076	.5606	.1877	. 5802	. 4198	21
40	.53975	.46025	1.8527	.64117	1.5596	1.1879	.15817	.84182	20
41	. 3999	. 6000	.8519	. 4158	.5586	.1881	. 5833	. 4167	19
42	. 4024	. 5976	.8510	. 4199	.5577	.1883	. 5849	. 4151	18
43	. 4048	. 5951	.8502	. 4240	.5567	.1886	. 5865	. 4135	17
44	. 4073	. 5927	.8493	. 4281	.5557	.1888	. 5880	. 4120	16
45	.54097	.45902	1.8485	.64322	1.5547	1.1890	.15896	.84104	15
46	. 4122	. 5878	.8477	. 4363	.5537	.1892	. 5912	. 4088	14
47	. 4146	. 5854	.8468	. 4404	.5527	.1894	. 5927	. 4072	13
48	. 4171	. 5829	.8460	. 4446	.5517	.1897	. 5943	. 4057	12
49	. 4195	. 5805	.8452	. 4487	.5507	.1899	. 5959	. 4041	11
50	.54220	.45780	1.8443	.64528	1.5497	1.1901	.15975	.84025	10
51	. 4244	. 5756	.8435	. 4569	.5487	.1903	. 5991	. 4009	9
52	. 4268	. 5731	.8427	. 4610	.5477	.1906	. 6006	. 3993	8
53	. 4293	. 5707	.8418	. 4652	.5467	.1908	. 6022	. 3978	7
54	. 4317	. 5682	.8410	. 4693	.5458	.1910	. 6038	. 3962	6
55	.54342	.45658	1.8402	.64734	1.5448	1.1912	.16054	.83946	5
56	. 4366	. 5634	.8394	. 4775	.5438	.1915	. 6070	. 3930	4
57	. 4391	. 5609	.8385	. 4817	.5428	.1917	. 6085	. 3914	3
58	. 4415	. 5585	.8377	. 4858	.5418	.1919	. 6101	. 3899	2
59	. 4439	. 5560	.8369	. 4899	.5408	.1921	. 6117	. 3883	1
60	. 4464	. 5536	.8361	. 4941	.5399	.1922	. 6133	. 3867	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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33° **Natural Trigonometrical Functions.** **146°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.54464	.45536	1.8361	.64941	1.5399	1.1924	.16133	.83867	60
1	.4488	.5512	.8352	.4982	.5389	.1926	.6149	.3851	59
2	.4513	.5487	.8344	.5023	.5379	.1928	.6165	.3835	58
3	.4537	.5463	.8336	.5065	.5369	.1930	.6180	.3819	57
4	.4561	.5438	.8328	.5106	.5359	.1933	.6196	.3804	56
5	.54586	.45414	1.8320	.65148	1.5350	1.1935	.16212	.83788	55
6	.4610	.5390	.8311	.5189	.5340	.1937	.6228	.3772	54
7	.4634	.5365	.8303	.5231	.5330	.1939	.6244	.3756	53
8	.4659	.5341	.8295	.5272	.5320	.1942	.6260	.3740	52
9	.4683	.5317	.8287	.5314	.5311	.1944	.6276	.3724	51
10	.54708	.45292	1.8279	.65355	1.5301	1.1946	.16292	.83708	50
11	.4732	.5268	.8271	.5397	.5291	.1948	.6308	.3692	49
12	.4756	.5244	.8263	.5438	.5282	.1951	.6323	.3676	48
13	.4781	.5219	.8255	.5480	.5272	.1953	.6339	.3660	47
14	.4805	.5195	.8246	.5521	.5262	.1955	.6355	.3644	46
15	.54829	.45171	1.8238	.65563	1.5252	1.1958	.16371	.83629	45
16	.4854	.5146	.8230	.5604	.5243	.1960	.6387	.3613	44
17	.4878	.5122	.8222	.5646	.5233	.1962	.6403	.3597	43
18	.4902	.5098	.8214	.5688	.5223	.1964	.6419	.3581	42
19	.4926	.5073	.8206	.5729	.5214	.1967	.6435	.3565	41
20	.54951	.45049	1.8198	.65771	1.5204	1.1969	.16451	.83549	40
21	.4975	.5025	.8190	.5813	.5195	.1971	.6467	.3533	39
22	.4999	.5000	.8182	.5854	.5185	.1974	.6483	.3517	38
23	.5024	.4976	.8174	.5896	.5175	.1976	.6499	.3501	37
24	.5048	.4952	.8166	.5938	.5166	.1978	.6515	.3485	36
25	.55072	.44928	1.8158	.65980	1.5156	1.1980	.16531	.83469	35
26	.5097	.4903	.8150	.6021	.5147	.1983	.6547	.3453	34
27	.5121	.4879	.8142	.6063	.5137	.1985	.6563	.3437	33
28	.5145	.4855	.8134	.6105	.5127	.1987	.6579	.3421	32
29	.5169	.4830	.8126	.6147	.5118	.1990	.6595	.3405	31
30	.55194	.44806	1.8118	.66188	1.5108	1.1992	.16611	.83388	30
31	.5218	.4782	.8110	.6230	.5099	.1994	.6627	.3372	29
32	.5242	.4758	.8102	.6272	.5089	.1997	.6643	.3356	28
33	.5266	.4733	.8094	.6314	.5080	.1999	.6660	.3340	27
34	.5291	.4709	.8086	.6356	.5070	.2001	.6676	.3324	26
35	.55315	.44685	1.8078	.66398	1.5061	1.2004	.16692	.83308	25
36	.5339	.4661	.8070	.6440	.5051	.2006	.6708	.3292	24
37	.5363	.4637	.8062	.6482	.5042	.2008	.6724	.3276	23
38	.5388	.4612	.8054	.6524	.5032	.2010	.6740	.3260	22
39	.5412	.4588	.8047	.6566	.5023	.2013	.6756	.3244	21
40	.55436	.44564	1.8039	.66608	1.5013	1.2015	.16772	.83228	20
41	.5460	.4540	.8031	.6650	.5004	.2017	.6788	.3211	19
42	.5484	.4515	.8023	.6692	.4994	.2020	.6804	.3195	18
43	.5509	.4491	.8015	.6734	.4985	.2022	.6821	.3179	17
44	.5533	.4467	.8007	.6776	.4975	.2024	.6837	.3163	16
45	.55557	.44443	1.7999	.66818	1.4966	1.2027	.16853	.83147	15
46	.5581	.4419	.7992	.6860	.4957	.2029	.6869	.3131	14
47	.5605	.4395	.7984	.6902	.4947	.2031	.6885	.3115	13
48	.5629	.4370	.7976	.6944	.4938	.2034	.6901	.3098	12
49	.5654	.4346	.7968	.6986	.4928	.2036	.6918	.3082	11
50	.55678	.44322	1.7960	.67028	1.4919	1.2039	.16934	.83066	10
51	.5702	.4298	.7953	.7071	.4910	.2041	.6950	.3050	9
52	.5726	.4274	.7945	.7113	.4900	.2043	.6966	.3034	8
53	.5750	.4250	.7937	.7155	.4891	.2046	.6982	.3017	7
54	.5774	.4225	.7929	.7197	.4881	.2048	.6999	.3001	6
55	.55799	.44201	1.7921	.67239	1.4872	1.2050	.17015	.82985	5
56	.5823	.4177	.7914	.7282	.4863	.2053	.7031	.2969	4
57	.5847	.4153	.7906	.7324	.4853	.2055	.7047	.2952	3
58	.5871	.4129	.7898	.7366	.4844	.2057	.7064	.2936	2
59	.5895	.4105	.7891	.7408	.4835	.2060	.7080	.2920	1
60	.5919	.4081	.7883	.7451	.4826	.2062	.7096	.2904	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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34° **Natural Trigonometrical Functions.** 145°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.55919	.44081	1.7883	.67451	1.4826	1.2062	.17096	.82904	60
1	. 5943	. 4057	.7875	. 7493	.4816	.2064	. 7112	. 2887	59
2	. 5967	. 4032	.7867	. 7535	.4807	.2067	. 7129	. 2871	58
3	. 5992	. 4008	.7860	. 7578	.4798	.2069	. 7145	. 2855	57
4	. 6016	. 3984	.7852	. 7620	.4788	.2072	. 7161	. 2839	56
5	.56040	.43960	1.7844	.67663	1.4779	1.2074	.17178	.82822	55
6	. 6064	. 3936	.7837	. 7705	.4770	.2076	. 7194	. 2806	54
7	. 6088	. 3912	.7829	. 7747	.4761	.2079	. 7210	. 2790	53
8	. 6112	. 3888	.7821	. 7790	.4751	.2081	. 7227	. 2773	52
9	. 6136	. 3864	.7814	. 7832	.4742	.2083	. 7243	. 2757	51
10	.56160	.43840	1.7806	.67875	1.4733	1.2086	.17259	.82741	50
11	. 6184	. 3816	.7798	. 7917	.4724	.2088	. 7276	. 2724	49
12	. 6208	. 3792	.7791	. 7960	.4714	.2091	. 7292	. 2708	48
13	. 6232	. 3768	.7783	. 8002	.4705	.2093	. 7308	. 2692	47
14	. 6256	. 3743	.7776	. 8045	.4696	.2095	. 7325	. 2675	46
15	.56280	.43719	1.7768	.68087	1.4687	1.2098	.17341	.82659	45
16	. 6304	. 3695	.7760	. 8130	.4678	.2100	. 7357	. 2643	44
17	. 6328	. 3671	.7753	. 8173	.4669	.2103	. 7374	. 2626	43
18	. 6353	. 3647	.7745	. 8215	.4659	.2105	. 7390	. 2610	42
19	. 6377	. 3623	.7738	. 8258	.4650	.2107	. 7406	. 2593	41
20	.56401	.43599	1.7730	.68301	1.4641	1.2110	.17423	.82577	40
21	. 6425	. 3575	.7723	. 8343	.4632	.2112	. 7439	. 2561	39
22	. 6449	. 3551	.7715	. 8386	.4623	.2115	. 7456	. 2544	38
23	. 6473	. 3527	.7708	. 8429	.4614	.2117	. 7472	. 2528	37
24	. 6497	. 3503	.7700	. 8471	.4605	.2119	. 7489	. 2511	36
25	.56521	.43479	1.7693	.68514	1.4595	1.2122	.17505	.82495	35
26	. 6545	. 3455	.7685	. 8557	.4586	.2124	. 7521	. 2478	34
27	. 6569	. 3431	.7678	. 8600	.4577	.2127	. 7538	. 2462	33
28	. 6593	. 3407	.7670	. 8642	.4568	.2129	. 7554	. 2445	32
29	. 6617	. 3383	.7663	. 8685	.4559	.2132	. 7571	. 2429	31
30	.56641	.43359	1.7655	.68728	1.4550	1.2134	.17587	.82413	30
31	. 6664	. 3335	.7648	. 8771	.4541	.2136	. 7604	. 2396	29
32	. 6688	. 3311	.7640	. 8814	.4532	.2139	. 7620	. 2380	28
33	. 6712	. 3287	.7633	. 8857	.4523	.2141	. 7637	. 2363	27
34	. 6736	. 3263	.7625	. 8899	.4514	.2144	. 7653	. 2347	26
35	.56760	.43239	1.7618	.68942	1.4505	1.2146	.17670	.82330	25
36	. 6784	. 3216	.7610	. 8985	.4496	.2149	. 7686	. 2314	24
37	. 6808	. 3192	.7603	. 9028	.4487	.2151	. 7703	. 2297	23
38	. 6832	. 3168	.7596	. 9071	.4478	.2153	. 7719	. 2280	22
39	. 6856	. 3144	.7588	. 9114	.4469	.2156	. 7736	. 2264	21
40	.56880	.43120	1.7581	.69157	1.4460	1.2158	.17752	.82247	20
41	. 6904	. 3096	.7573	. 9200	.4451	.2161	. 7769	. 2231	19
42	. 6928	. 3072	.7566	. 9243	.4442	.2163	. 7786	. 2214	18
43	. 6952	. 3048	.7559	. 9286	.4433	.2166	. 7802	. 2198	17
44	. 6976	. 3024	.7551	. 9329	.4424	.2168	. 7819	. 2181	16
45	.57000	.43000	1.7544	.69372	1.4415	1.2171	.17835	.82165	15
46	. 7023	. 2976	.7537	. 9415	.4406	.2173	. 7852	. 2148	14
47	. 7047	. 2952	.7529	. 9459	.4397	.2175	. 7868	. 2131	13
48	. 7071	. 2929	.7522	. 9502	.4388	.2178	. 7885	. 2115	12
49	. 7095	. 2905	.7514	. 9545	.4379	.2180	. 7902	. 2098	11
50	.57119	.42881	1.7507	.69588	1.4370	1.2183	.17918	.82082	10
51	. 7143	. 2857	.7500	. 9631	.4361	.2185	. 7935	. 2065	9
52	. 7167	. 2833	.7493	. 9674	.4352	.2188	. 7951	. 2048	8
53	. 7191	. 2809	.7485	. 9718	.4343	.2190	. 7968	. 2032	7
54	. 7214	. 2785	.7478	. 9761	.4335	.2193	. 7985	. 2015	6
55	.57238	.42761	1.7471	.69804	1.4326	1.2195	.18001	.81998	5
56	. 7262	. 2738	.7463	. 9847	.4317	.2198	. 8018	. 1982	4
57	. 7286	. 2714	.7456	. 9891	.4308	.2200	. 8035	. 1965	3
58	. 7310	. 2690	.7449	. 9934	.4299	.2203	. 8051	. 1948	2
59	. 7334	. 2666	.7442	. 9977	.4290	.2205	. 8068	. 1932	1
60	. 7358	. 2642	.7434	.70021	.4281	.2208	. 8085	. 1915	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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35° **Natural Trigonometrical Functions.** **144°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.57358	.42642	1.7434	.70021	1.4281	1.2208	.18085	.81915	60
1	. 7381	. 2618	.7427	. 0064	.4273	.2210	. 8101	. 1898	59
2	. 7405	. 2595	.7420	. 0107	.4264	.2213	. 8118	. 1882	58
3	. 7429	. 2571	.7413	. 0151	.4255	.2215	. 8135	. 1865	57
4	. 7453	. 2547	.7405	. 0194	.4246	.2218	. 8151	. 1848	56
5	.57477	.42523	1.7398	.70238	1.4237	1.2220	.18168	.81832	55
6	. 7500	. 2499	.7391	. 0281	.4228	.2223	. 8185	. 1815	54
7	. 7524	. 2476	.7384	. 0325	.4220	.2225	. 8202	. 1798	53
8	. 7548	. 2452	.7377	. 0368	.4211	.2228	. 8218	. 1781	52
9	. 7572	. 2428	.7369	. 0412	.4202	.2230	. 8235	. 1765	51
10	.57596	.42404	1.7362	.70455	1.4193	1.2233	.18252	.81748	50
11	. 7619	. 2380	.7355	. 0499	.4185	.2235	. 8269	. 1731	49
12	. 7643	. 2357	.7348	. 0542	.4176	.2238	. 8285	. 1714	48
13	. 7667	. 2333	.7341	. 0586	.4167	.2240	. 8302	. 1698	47
14	. 7691	. 2309	.7334	. 0629	.4158	.2243	. 8319	. 1681	46
15	.57714	.42285	1.7327	.70673	1.4150	1.2245	.18336	.81664	45
16	. 7738	. 2262	.7319	. 0717	.4141	.2248	. 8353	. 1647	44
17	. 7762	. 2238	.7312	. 0760	.4132	.2250	. 8369	. 1630	43
18	. 7786	. 2214	.7305	. 0804	.4123	.2253	. 8386	. 1614	42
19	. 7809	. 2190	.7298	. 0848	.4115	.2255	. 8403	. 1597	41
20	.57833	.42167	1.7291	.70891	1.4106	1.2258	.18420	.81580	40
21	. 7857	. 2143	.7284	. 0935	.4097	.2260	. 8437	. 1563	39
22	. 7881	. 2119	.7277	. 0979	.4089	.2263	. 8453	. 1546	38
23	. 7904	. 2096	.7270	. 1022	.4080	.2265	. 8470	. 1530	37
24	. 7928	. 2072	.7263	. 1066	.4071	.2268	. 8487	. 1513	36
25	.57952	.42048	1.7256	.71110	1.4063	1.2270	.18504	.81496	35
26	. 7975	. 2024	.7249	. 1154	.4054	.2273	. 8521	. 1479	34
27	. 7999	. 2001	.7242	. 1198	.4045	.2276	. 8538	. 1462	33
28	. 8023	. 1977	.7234	. 1241	.4037	.2278	. 8555	. 1445	32
29	. 8047	. 1953	.7227	. 1285	.4028	.2281	. 8571	. 1428	31
30	.58070	.41930	1.7220	.71329	1.4019	1.2283	.18588	.81411	30
31	. 8094	. 1906	.7213	. 1373	.4011	.2286	. 8605	. 1395	29
32	. 8118	. 1882	.7206	. 1417	.4002	.2288	. 8622	. 1378	28
33	. 8141	. 1859	.7199	. 1461	.3994	.2291	. 8639	. 1361	27
34	. 8165	. 1835	.7192	. 1505	.3985	.2293	. 8656	. 1344	26
35	.58189	.41811	1.7185	.71549	1.3976	1.2296	.18673	.81327	25
36	. 8212	. 1788	.7178	. 1593	.3968	.2298	. 8690	. 1310	24
37	. 8236	. 1764	.7171	. 1637	.3959	.2301	. 8707	. 1293	23
38	. 8259	. 1740	.7164	. 1681	.3951	.2304	. 8724	. 1276	22
39	. 8283	. 1717	.7157	. 1725	.3942	.2306	. 8741	. 1259	21
40	.58307	.41693	1.7151	.71769	1.3933	1.2309	.18758	.81242	20
41	. 8330	. 1669	.7144	. 1813	.3925	.2311	. 8775	. 1225	19
42	. 8354	. 1646	.7137	. 1857	.3916	.2314	. 8792	. 1208	18
43	. 8378	. 1622	.7130	. 1901	.3908	.2316	. 8809	. 1191	17
44	. 8401	. 1599	.7123	. 1945	.3899	.2319	. 8826	. 1174	16
45	.58425	.41575	1.7116	.71990	1.3891	1.2322	.18843	.81157	15
46	. 8448	. 1551	.7109	. 2034	.3882	.2324	. 8860	. 1140	14
47	. 8472	. 1528	.7102	. 2078	.3874	.2327	. 8877	. 1123	13
48	. 8496	. 1504	.7095	. 2122	.3865	.2329	. 8894	. 1106	12
49	. 8519	. 1481	.7088	. 2166	.3857	.2332	. 8911	. 1089	11
50	.58543	.41457	1.7081	.72211	1.3848	1.2335	.18928	.81072	10
51	. 8566	. 1433	.7075	. 2255	.3840	.2337	. 8945	. 1055	9
52	. 8590	. 1410	.7068	. 2299	.3831	.2340	. 8962	. 1038	8
53	. 8614	. 1386	.7061	. 2344	.3823	.2342	. 8979	. 1021	7
54	. 8637	. 1363	.7054	. 2388	.3814	.2345	. 8996	. 1004	6
55	.58661	.41339	1.7047	.72432	1.3806	1.2348	.19013	.80987	5
56	. 8684	. 1316	.7040	. 2477	.3797	.2350	. 9030	. 0970	4
57	. 8708	. 1292	.7033	. 2521	.3789	.2353	. 9047	. 0953	3
58	. 8731	. 1268	.7027	. 2565	.3781	.2355	. 9064	. 0936	2
59	. 8755	. 1245	.7020	. 2610	.3772	.2358	. 9081	. 0919	1
60	. 8778	. 1221	.7013	. 2654	.3764	.2361	. 9098	. 0902	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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36° **Natural Trigonometrical Functions.** **143°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.58778	.41221	1.7013	.72654	1.3764	1.2361	.19098	.80902	60
1	. 8802	. 1198	.7006	. 2699	.3755	.2363	. 9115	. 0885	59
2	. 8825	. 1174	.6999	. 2743	.3747	.2366	. 9132	. 0867	58
3	. 8849	. 1151	.6993	. 2788	.3738	.2368	. 9150	. 0850	57
4	. 8873	. 1127	.6986	. 2832	.3730	.2371	. 9167	. 0833	56
5	.58896	.41104	1.6979	.72877	1.3722	1.2374	.19184	.80816	55
6	. 8920	. 1080	.6972	. 2921	.3713	.2376	. 9201	. 0799	54
7	. 8943	. 1057	.6965	. 2966	.3705	.2379	. 9218	. 0782	53
8	. 8967	. 1033	.6959	. 3010	.3697	.2382	. 9235	. 0765	52
9	. 8990	. 1010	.6952	. 3055	.3688	.2384	. 9252	. 0747	51
10	.59014	.40986	1.6945	.73100	1.3680	1.2387	.19270	.80730	50
11	. 9037	. 0963	.6938	. 3144	.3672	.2389	. 9287	. 0713	49
12	. 9060	. 0939	.6932	. 3189	.3663	.2392	. 9304	. 0696	48
13	. 9084	. 0916	.6925	. 3234	.3655	.2395	. 9321	. 0679	47
14	. 9107	. 0892	.6918	. 3278	.3647	.2397	. 9338	. 0662	46
15	.59131	.40869	1.6912	.73323	1.3638	1.2400	.19355	.80644	45
16	. 9154	. 0845	.6905	. 3368	.3630	.2403	. 9373	. 0627	44
17	. 9178	. 0822	.6898	. 3412	.3622	.2405	. 9390	. 0610	43
18	. 9201	. 0799	.6891	. 3457	.3613	.2408	. 9407	. 0593	42
19	. 9225	. 0775	.6885	. 3502	.3605	.2411	. 9424	. 0576	41
20	.59248	.40752	1.6878	.73547	1.3597	1.2413	.19442	.80558	40
21	. 9272	. 0728	.6871	. 3592	.3588	.2416	. 9459	. 0541	39
22	. 9295	. 0705	.6865	. 3637	.3580	.2419	. 9476	. 0524	38
23	. 9318	. 0681	.6858	. 3681	.3572	.2421	. 9493	. 0507	37
24	. 9342	. 0658	.6851	. 3726	.3564	.2424	. 9511	. 0489	36
25	.59365	.40635	1.6845	.73771	1.3555	1.2427	.19528	.80472	35
26	. 9389	. 0611	.6838	. 3816	.3547	.2429	. 9545	. 0455	34
27	. 9412	. 0588	.6831	. 3861	.3539	.2432	. 9562	. 0437	33
28	. 9435	. 0564	.6825	. 3906	.3531	.2435	. 9580	. 0420	32
29	. 9459	. 0541	.6818	. 3951	.3522	.2437	. 9597	. 0403	31
30	.59482	.40518	1.6812	.73996	1.3514	1.2440	.19614	.80386	30
31	. 9506	. 0494	.6805	. 4041	.3506	.2443	. 9632	. 0368	29
32	. 9529	. 0471	.6798	. 4086	.3498	.2445	. 9649	. 0351	28
33	. 9552	. 0447	.6792	. 4131	.3489	.2448	. 9666	. 0334	27
34	. 9576	. 0424	6785	. 4176	.3481	.2451	. 9683	. 0316	26
35	.59599	.40401	1.6779	.74221	1.3473	1.2453	.19701	.80299	25
36	. 9622	. 0377	.6772	. 4266	.3465	.2456	. 9718	. 0282	24
37	. 9646	. 0354	.6766	. 4312	.3457	.2459	. 9736	. 0264	23
38	. 9669	. 0331	.6759	. 4357	.3449	.2461	. 9753	. 0247	22
39	. 9692	. 0307	.6752	. 4402	.3440	.2464	. 9770	. 0230	21
40	.59716	.40284	1.6746	.74447	1.3432	1.2467	.19788	.80212	20
41	. 9739	. 0261	.6739	. 4492	.3424	.2470	. 9805	. 0195	19
42	. 9762	. 0237	.6733	. 4538	.3416	.2472	. 9822	. 0177	18
43	. 9786	. 0214	.6726	. 4583	.3408	.2475	. 9840	. 0160	17
44	. 9809	. 0191	.6720	. 4628	.3400	.2478	. 9857	. 0143	16
45	.59832	.40167	1.6713	.74673	1.3392	1.2480	.19875	.80125	15
46	. 9856	. 0144	.6707	. 4719	.3383	.2483	. 9892	. 0108	14
47	. 9879	. 0121	.6700	. 4764	.3375	.2486	. 9909	. 0090	13
48	. 9902	. 0098	.6694	. 4809	.3367	.2488	. 9927	. 0073	12
49	. 9926	. 0074	.6687	. 4855	.3359	.2491	. 9944	. 0056	11
50	.59949	.40051	1.6681	.74900	1.3351	1.2494	.19962	.80038	10
51	. 9972	. 0028	.6674	. 4946	.3343	.2497	. 9979	. 0021	9
52	. 9995	. 0004	.6668	. 4991	.3335	.2499	. 9997	. 0003	8
53	.60019	.39981	.6661	. 5037	.3327	.2502	.20014	.79986	7
54	. 0042	. 9958	.6655	. 5082	.3319	.2505	. 0031	. 9968	6
55	.60065	.39935	1.6648	.75128	1.3311	1.2508	.20049	.79951	5
56	. 0088	. 9911	.6642	. 5173	.3303	.2510	. 0066	. 9933	4
57	. 0112	. 9888	.6636	. 5219	.3294	.2513	. 0084	. 9916	3
58	. 0135	. 9865	.6629	. 5264	.3286	.2516	. 0101	. 9898	2
59	. 0158	. 9842	.6623	. 5310	.3278	.2519	. 0119	. 9881	1
60	. 0181	. 9818	.6616	. 5355	.3270	.2521	. 0136	. 9863	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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37° **Natural Trigonometrical Functions.** **142°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.60181	.39818	1.6616	.75355	1.3270	1.2521	.20136	.79863	60
1	. 0205	. 9795	.6610	. 5401	.3262	.2524	. 0154	. 9846	59
2	. 0228	. 9772	.6603	. 5447	.3254	.2527	. 0171	. 9828	58
3	. 0251	. 9749	.6597	. 5492	.3246	.2530	. 0189	. 9811	57
4	. 0274	. 9726	.6591	. 5538	.3238	.2532	. 0206	. 9793	56
5	.60298	.39702	1.6584	.75584	1.3230	1.2535	.20224	.79776	55
6	. 0320	. 9679	.6578	. 5629	.3222	.2538	. 0242	. 9758	54
7	. 0344	. 9656	.6572	. 5675	.3214	.2541	. 0259	. 9741	53
8	. 0367	. 9633	.6565	. 5721	.3206	.2543	. 0277	. 9723	52
9	. 0390	. 9610	.6559	. 5767	.3198	.2546	. 0294	. 9706	51
10	.60413	.39586	1.6552	.75812	1.3190	1.2549	.20312	.79688	50
11	. 0437	. 9563	.6546	. 5858	.3182	.2552	. 0329	. 9670	49
12	. 0460	. 9540	.6540	. 5904	.3174	.2554	. 0347	. 9653	48
13	. 0483	. 9517	.6533	. 5950	.3166	.2557	. 0365	. 9635	47
14	. 0506	. 9494	.6527	. 5996	.3159	.2560	. 0382	. 9618	46
15	.60529	.39471	1.6521	.76042	1.3151	1.2563	.20400	.79600	45
16	. 0552	. 9447	.6514	. 6088	.3143	.2565	. 0417	. 9582	44
17	. 0576	. 9424	.6508	. 6134	.3135	.2568	. 0435	. 9565	43
18	. 0599	. 9401	.6502	. 6179	.3127	.2571	. 0453	. 9547	42
19	. 0622	. 9378	.6496	. 6225	.3119	.2574	. 0470	. 9530	41
20	.60645	.39355	1.6489	.76271	1.3111	1.2577	.20488	.79512	40
21	. 0668	. 9332	.6483	. 6317	.3103	.2579	. 0505	. 9494	39
22	. 0691	. 9309	.6477	. 6364	.3095	.2582	. 0523	. 9477	38
23	. 0714	. 9285	.6470	. 6410	.3087	.2585	. 0541	. 9459	37
24	. 0737	. 9262	.6464	. 6456	.3079	.2588	. 0558	. 9441	36
25	.60761	.39239	1.6458	.76502	1.3071	1.2591	.20576	.79424	35
26	. 0784	. 9216	.6452	. 6548	.3064	.2593	. 0594	. 9406	34
27	. 0807	. 9193	.6445	. 6594	.3056	.2596	. 0611	. 9388	33
28	. 0830	. 9170	.6439	. 6640	.3048	.2599	. 0629	. 9371	32
29	. 0853	. 9147	.6433	. 6686	.3040	.2602	. 0647	. 9353	31
30	.60876	.39124	1.6427	.76733	1.3032	1.2605	.20665	.79335	30
31	. 0899	. 9101	.6420	. 6779	.3024	.2607	. 0682	. 9318	29
32	. 0922	. 9078	.6414	. 6825	.3016	.2610	. 0700	. 9300	28
33	. 0945	. 9055	.6408	. 6871	.3009	.2613	. 0718	. 9282	27
34	. 0963	. 9031	.6402	. 6918	.3001	.2616	. 0735	. 9264	26
35	.60991	.39008	1.6396	.76964	1.2993	1.2619	.20753	.79247	25
36	. 1014	. 8985	.6389	. 7010	.2985	.2622	. 0771	. 9229	24
37	. 1037	. 8962	.6383	. 7057	.2977	.2624	. 0789	. 9211	23
38	. 1061	. 8939	.6377	. 7103	.2970	.2627	. 0806	. 9193	22
39	. 1084	. 8916	.6371	. 7149	.2962	.2630	. 0824	. 9176	21
40	.61107	.38893	1.6365	.77196	1.2954	1.2633	.20842	.79158	20
41	. 1130	. 8870	.6359	. 7242	.2946	.2636	. 0860	. 9140	19
42	. 1153	. 8847	.6352	. 7289	.2938	.2639	. 0878	. 9122	18
43	. 1176	. 8824	.6346	. 7335	.2931	.2641	. 0895	. 9104	17
44	. 1199	. 8801	.6340	. 7382	.2923	.2644	. 0913	. 9087	16
45	.61222	.38778	1.6334	.77428	1.2915	1.2647	.20931	.79069	15
46	. 1245	. 8755	.6328	. 7475	.2907	.2650	. 0949	. 9051	14
47	. 1268	. 8732	.6322	. 7521	.2900	.2653	. 0967	. 9033	13
48	. 1290	. 8709	.6316	. 7568	.2892	.2656	. 0984	. 9015	12
49	. 1314	. 8686	.6309	. 7614	.2884	.2659	. 1002	. 8998	11
50	.61337	.38663	1.6303	.77661	1.2876	1.2661	.21020	.78980	10
51	. 1360	. 8640	.6297	. 7708	.2869	.2664	. 1038	. 8962	9
52	. 1383	. 8617	.6291	. 7754	.2861	.2667	. 1056	. 8944	8
53	. 1405	. 8594	.6285	. 7801	.2853	.2670	. 1074	. 8926	7
54	. 1428	. 8571	.6279	. 7848	.2845	.2673	. 1091	. 8908	6
55	.61451	.38548	1.6273	.77895	1.2838	1.2676	.21109	.78890	5
56	. 1474	. 8525	.6267	. 7941	.2830	.2679	. 1127	. 8873	4
57	. 1497	. 8503	.6261	. 7988	.2822	.2681	. 1145	. 8855	3
58	. 1520	. 8480	.6255	. 8035	.2815	.2684	. 1163	. 8837	2
59	. 1543	. 8457	.6249	. 8082	.2807	.2687	. 1181	. 8819	1
60	. 1566	. 8434	.6243	. 8128	.2799	.2690	. 1199	. 8801	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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38° **Natural Trigonometrical Functions.** 141°

M.	Sine.	Vrs: cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.61566	.38434	1.6243	.78128	1.2799	1.2690	.21199	.78801	60
1	. 1589	. 8411	.6237	. 8175	.2792	.2693	. 1217	. 8783	59
2	. 1612	. 8388	.6231	. 8222	.2784	.2696	. 1235	. 8765	58
3	. 1635	. 8365	.6224	. 8269	.2776	.2699	. 1253	. 8747	57
4	. 1658	. 8342	.6218	. 8316	.2769	.2702	. 1271	. 8729	56
5	.61681	.38319	1.6212	.78363	1.2761	1.2705	.21288	.78711	55
6	. 1703	. 8296	.6206	. 8410	.2753	.2707	. 1306	. 8693	54
7	. 1726	. 8273	.6200	. 8457	.2746	.2710	. 1324	. 8675	53
8	. 1749	. 8251	.6194	. 8504	.2738	.2713	. 1342	. 8657	52
9	. 1772	. 8228	.6188	. 8551	.2730	.2716	. 1360	. 8640	51
10	.61795	.38205	1.6182	.78598	1.2723	1.2719	.21378	.78622	50
11	. 1818	. 8182	.6176	. 8645	.2715	.2722	. 1396	. 8604	49
12	. 1841	. 8159	.6170	. 8692	.2708	.2725	. 1414	. 8586	48
13	. 1864	. 8136	.6164	. 8739	.2700	.2728	. 1432	. 8568	47
14	. 1886	. 8113	.6159	. 8786	.2692	.2731	. 1450	. 8550	46
15	.61909	.38091	1.6153	.78834	1.2685	1.2734	.21468	.78532	45
16	. 1932	. 8068	.6147	. 8881	.2677	.2737	. 1486	. 8514	44
17	. 1955	. 8045	.6141	. 8928	.2670	.2739	. 1504	. 8496	43
18	. 1978	. 8022	.6135	. 8975	.2662	.2742	. 1522	. 8478	42
19	. 2001	. 7999	.6129	. 9022	.2655	.2745	. 1540	. 8460	41
20	.62023	.37976	1.6123	.79070	1.2647	1.2748	.21558	.78441	40
21	. 2046	. 7954	.6117	. 9117	.2639	.2751	. 1576	. 8423	39
22	. 2069	. 7931	.6111	. 9164	.2632	.2754	. 1594	. 8405	38
23	. 2092	. 7908	.6105	. 9212	.2624	.2757	. 1612	. 8387	37
24	. 2115	. 7885	.6099	. 9259	.2617	.2760	. 1631	. 8369	36
25	.62137	.37862	1.6093	.79306	1.2609	1.2763	.21649	.78351	35
26	. 2160	. 7840	.6087	. 9354	.2602	.2766	. 1667	. 8333	34
27	. 2183	. 7817	.6081	. 9401	.2594	.2769	. 1685	. 8315	33
28	. 2206	. 7794	.6077	. 9449	.2587	.2772	. 1703	. 8297	32
29	. 2229	. 7771	.6070	. 9496	.2579	.2775	. 1721	. 8279	31
30	.62251	.37748	1.6064	.79543	1.2572	1.2778	.21739	.78261	30
31	. 2274	. 7726	.6058	. 9591	.2564	.2781	. 1757	. 8243	29
32	. 2297	. 7703	.6052	. 9639	.2557	.2784	. 1775	. 8224	28
33	. 2320	. 7680	.6046	. 9686	.2549	.2787	. 1793	. 8206	27
34	. 2342	. 7657	.6040	. 9734	.2542	.2790	. 1812	. 8188	26
35	.62365	.37635	1.6034	.79781	1.2534	1.2793	.21830	.78170	25
36	. 2388	. 7612	.6029	. 9829	.2527	.2795	. 1848	. 8152	24
37	. 2411	. 7589	.6023	. 9876	.2519	.2798	. 1866	. 8134	23
38	. 2433	. 7566	.6017	. 9924	.2512	.2801	. 1884	. 8116	22
39	. 2456	. 7544	.6011	. 9972	.2504	.2804	. 1902	. 8097	21
40	.62479	.37521	1.6005	.80020	1.2497	1.2807	.21921	.78079	20
41	. 2501	. 7498	.6000	. 0067	.2489	.2810	. 1939	. 8061	19
42	. 2524	. 7476	.5994	. 0115	.2482	.2813	. 1957	. 8043	18
43	. 2547	. 7453	.5988	. 0163	.2475	.2816	. 1975	. 8025	17
44	. 2570	. 7430	.5982	. 0211	.2467	.2819	. 1993	. 8007	16
45	.62592	.37408	1.5976	.80258	1.2460	1.2822	.22011	.77988	15
46	. 2615	. 7385	.5971	. 0306	.2452	.2825	. 2030	. 7970	14
47	. 2638	. 7362	.5965	. 0354	.2445	.2828	. 2048	. 7952	13
48	. 2660	. 7340	.5959	. 0402	.2437	.2831	. 2066	. 7934	12
49	. 2683	. 7317	.5953	. 0450	.2430	.2834	. 2084	. 7915	11
50	.62706	.37294	1.5947	.80498	1.2423	1.2837	.22103	.77897	10
51	. 2728	. 7272	.5942	. 0546	.2415	.2840	. 2121	. 7879	9
52	. 2751	. 7249	.5936	. 0594	.2408	.2843	. 2139	. 7861	8
53	. 2774	. 7226	.5930	. 0642	.2400	.2846	. 2157	. 7842	7
54	. 2796	. 7204	.5924	. 0690	.2393	.2849	. 2176	. 7824	6
55	.62819	.37181	1.5919	.80738	1.2386	1.2852	.22194	.77806	5
56	. 2841	. 7158	.5913	. 0786	.2378	.2855	. 2212	. 7788	4
57	. 2864	. 7136	.5907	. 0834	.2371	.2858	. 2230	. 7769	3
58	. 2887	. 7113	.5901	. 0882	.2364	.2861	. 2249	. 7751	2
59	. 2909	. 7090	.5896	. 0930	.2356	.2864	. 2267	. 7733	1
60	. 2932	. 7068	.5890	. 0978	.2349	.2867	. 2285	. 7715	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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39° **Natural Trigonometrical Functions.** 140°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.62932	.37068	1.5890	.80978	1.2349	1.2867	.22285	.77715	60
1	. 2955	. 7045	.5884	. 1026	.2342	.2871	. 2304	. 7696	59
2	. 2977	. 7023	.5879	. 1075	.2334	.2874	. 2322	. 7678	58
3	. 3000	. 7000	.5873	. 1123	.2327	.2877	. 2340	. 7660	57
4	. 3022	. 6977	.5867	. 1171	.2320	.2880	. 2359	. 7641	56
5	.63045	.36955	1.5862	.81219	1.2312	1.2883	.22377	.77623	55
6	. 3067	. 6932	.5856	. 1268	.2305	.2886	. 2395	. 7605	54
7	. 3090	. 6910	.5850	. 1316	.2297	.2889	. 2414	. 7586	53
8	. 3113	. 6887	.5845	. 1364	.2290	.2892	. 2432	. 7568	52
9	. 3135	. 6865	.5839	. 1413	.2283	.2895	. 2450	. 7549	51
10	.63158	.36842	1.5833	.81461	1.2276	1.2898	.22469	.77531	50
11	. 3180	. 6820	.5828	. 1509	.2268	.2901	. 2487	. 7513	49
12	. 3203	. 6797	.5822	. 1558	.2261	.2904	. 2505	. 7494	48
13	. 3225	. 6774	.5816	. 1606	.2254	.2907	. 2524	. 7476	47
14	. 3248	. 6752	.5811	. 1655	.2247	.2910	. 2542	. 7458	46
15	.63270	.36729	1.5805	.81703	1.2239	1.2913	.22561	.77439	45
16	. 3293	. 6707	.5799	. 1752	.2232	.2916	. 2579	. 7421	44
17	. 3315	. 6684	.5794	. 1800	.2225	.2919	. 2597	. 7402	43
18	. 3338	. 6662	.5788	. 1849	.2218	.2922	. 2616	. 7384	42
19	. 3360	. 6639	.5783	. 1898	.2210	.2926	. 2634	. 7365	41
20	.63383	.36617	1.5777	.81946	1.2203	1.2929	.22653	.77347	40
21	. 3405	. 6594	.5771	. 1995	.2196	.2932	. 2671	. 7329	39
22	. 3428	. 6572	.5766	. 2043	.2189	.2935	. 2690	. 7310	38
23	. 3450	. 6549	.5760	. 2092	.2181	.2938	. 2708	. 7292	37
24	. 3473	. 6527	.5755	. 2141	.2174	.2941	. 2727	. 7273	36
25	.63495	.36504	1.5749	.82190	1.2167	1.2944	.22745	.77255	35
26	. 3518	. 6482	.5743	. 2238	.2160	.2947	. 2763	. 7236	34
27	. 3540	. 6459	.5738	. 2287	.2152	.2950	. 2782	. 7218	33
28	. 3563	. 6437	.5732	. 2336	.2145	.2953	. 2800	. 7199	32
29	. 3585	. 6415	.5727	. 2385	.2138	.2956	. 2819	. 7181	31
30	.63608	.36392	1.5721	.82434	1.2131	1.2960	.22837	.77162	30
31	. 3630	. 6370	.5716	. 2482	.2124	.2963	. 2856	. 7144	29
32	. 3653	. 6347	.5710	. 2531	.2117	.2966	. 2874	. 7125	28
33	. 3675	. 6325	.5705	. 2580	.2109	.2969	. 2893	. 7107	27
34	. 3697	. 6302	.5699	. 2629	.2102	.2972	. 2912	. 7088	26
35	.63720	.36280	1.5694	.82678	1.2095	1.2975	.22930	.77070	25
36	. 3742	. 6258	.5688	. 2727	.2088	.2978	. 2949	. 7051	24
37	. 3765	. 6235	.5683	. 2776	.2081	.2981	. 2967	. 7033	23
38	. 3787	. 6213	.5677	. 2825	.2074	.2985	. 2986	. 7014	22
39	. 3810	. 6190	.5672	. 2874	.2066	.2988	. 3004	. 6996	21
40	.63832	.36168	1.5666	.82923	1.2059	1.2991	.23023	.76977	20
41	. 3854	. 6146	.5661	. 2972	.2052	.2994	. 3041	. 6958	19
42	. 3877	. 6123	.5655	. 3022	.2045	.2997	. 3060	. 6940	18
43	. 3899	. 6101	.5650	. 3071	.2038	.3000	. 3079	. 6921	17
44	. 3921	. 6078	.5644	. 3120	.2031	.3003	. 3097	. 6903	16
45	.63944	.36056	1.5639	.83169	1.2024	1.3006	.23116	.76884	15
46	. 3966	. 6034	.5633	. 3218	.2016	.3010	. 3134	. 6865	14
47	. 3989	. 6011	.5628	. 3267	.2009	.3013	. 3153	. 6847	13
48	. 4011	. 5989	.5622	. 3317	.2002	.3016	. 3172	. 6828	12
49	. 4033	. 5967	.5617	. 3366	.1995	.3019	. 3190	. 6810	11
50	.64056	.35944	1.5611	.83415	1.1988	1.3022	.23209	.76791	10
51	. 4078	. 5922	.5606	. 3465	.1981	.3025	. 3227	. 6772	9
52	. 4100	. 5900	.5600	. 3514	.1974	.3029	. 3246	. 6754	8
53	. 4123	. 5877	.5595	. 3563	.1967	.3032	. 3265	. 6735	7
54	. 4145	. 5855	.5590	. 3613	.1960	.3035	. 3283	. 6716	6
55	.64167	.35833	1.5584	.83662	1.1953	1.3038	.23302	.76698	5
56	. 4189	. 5810	.5579	. 3712	.1946	.3041	. 3321	. 6679	4
57	. 4212	. 5788	.5573	. 3761	.1939	.3044	. 3339	. 6660	3
58	. 4234	. 5766	.5568	. 3811	.1932	.3048	. 3358	. 6642	2
59	. 4256	. 5743	.5563	. 3860	.1924	.3051	. 3377	. 6623	1
60	. 4279	. 5721	.5557	. 3910	.1917	.3054	. 3395	. 6604	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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40° **Natural Trigonometrical Functions.** **139°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.64279	.35721	1.5557	.83910	1.1917	1.3054	.23395	.76604	60
1	. 4301	. 5699	.5552	. 3959	.1910	.3057	. 3414	. 6586	59
2	. 4323	. 5677	.5546	. 4009	.1903	.3060	. 3433	. 6567	58
3	. 4345	. 5654	.5541	. 4059	.1896	.3064	. 3452	. 6548	57
4	. 4368	. 5632	.5536	. 4108	.1889	.3067	. 3470	. 6530	56
5	.64390	.35610	1.5530	.84158	1.1882	1.3070	.23489	.76511	55
6	. 4412	. 5588	.5525	. 4208	.1875	.3073	. 3508	. 6492	54
7	. 4435	. 5565	.5520	. 4257	.1868	.3076	. 3527	. 6473	53
8	. 4457	. 5543	.5514	. 4307	.1861	.3080	. 3545	. 6455	52
9	. 4479	. 5521	.5509	. 4357	.1854	.3083	. 3564	. 6436	51
10	.64501	.35499	1.5503	.84407	1.1847	1.3086	.23583	.76417	50
11	. 4523	. 5476	.5498	. 4457	.1840	.3089	. 3602	. 6398	49
12	. 4546	. 5454	.5493	. 4506	.1833	.3092	. 3620	. 6380	48
13	. 4568	. 5432	.5487	. 4556	.1826	.3096	. 3639	. 6361	47
14	. 4590	. 5410	.5482	. 4606	.1819	.3099	. 3658	. 6342	46
15	.64612	.35388	1.5477	.84656	1.1812	1.3102	.23677	.76323	45
16	. 4635	. 5365	.5471	. 4706	.1805	.3105	. 3695	. 6304	44
17	. 4657	. 5343	.5466	. 4756	.1798	.3109	. 3714	. 6286	43
18	. 4679	. 5321	.5461	. 4806	.1791	.3112	. 3733	. 6267	42
19	. 4701	. 5299	.5456	. 4856	.1785	.3115	. 3752	. 6248	41
20	.64723	.35277	1.5450	.84906	1.1778	1.3118	.23771	.76229	40
21	. 4745	. 5254	.5445	. 4956	.1771	.3121	. 3790	. 6210	39
22	. 4768	. 5232	.5440	. 5006	.1764	.3125	. 3808	. 6191	38
23	. 4790	. 5210	.5434	. 5056	.1757	.3128	. 3827	. 6173	37
24	. 4812	. 5188	.5429	. 5107	.1750	.3131	. 3846	. 6154	36
25	.64834	.35166	1.5424	.85157	1.1743	1.3134	.23865	.76135	35
26	. 4856	. 5144	.5419	. 5207	.1736	.3138	. 3884	. 6116	34
27	. 4878	. 5121	.5413	. 5257	.1729	.3141	. 3903	. 6097	33
28	. 4900	. 5099	.5408	. 5307	.1722	.3144	. 3922	. 6078	32
29	. 4923	. 5077	.5403	. 5358	.1715	.3148	. 3940	. 6059	31
30	.64945	.35055	1.5398	.85408	1.1708	1.3151	.23959	.76041	30
31	. 4967	. 5033	.5392	. 5458	.1702	.3154	. 3978	. 6022	29
32	. 4989	. 5011	.5387	. 5509	.1695	.3157	. 3997	. 6003	28
33	. 5011	. 4989	.5382	. 5559	.1688	.3161	. 4016	. 5984	27
34	. 5033	. 4967	.5377	. 5609	.1681	.3164	. 4035	. 5965	26
35	.65055	.34945	1.5371	.85660	1.1674	1.3167	.24054	.75946	25
36	. 5077	. 4922	.5366	. 5710	.1667	.3170	. 4073	. 5927	24
37	. 5099	. 4900	.5361	. 5761	.1660	.3174	. 4092	. 5908	23
38	. 5121	. 4878	.5356	. 5811	.1653	.3177	. 4111	. 5889	22
39	. 5144	. 4856	.5351	. 5862	.1647	.3180	. 4130	. 5870	21
40	.65166	.34834	1.5345	.85912	1.1640	1.3184	.24149	.75851	20
41	. 5188	. 4812	.5340	. 5963	.1633	.3187	. 4168	. 5832	19
42	. 5210	. 4790	.5335	. 6013	.1626	.3190	. 4186	. 5813	18
43	. 5232	. 4768	.5330	. 6064	.1619	.3193	. 4205	. 5794	17
44	. 5254	. 4746	.5325	. 6115	.1612	.3197	. 4224	. 5775	16
45	.65276	.34724	1.5319	.86165	1.1605	1.3200	.24243	.75756	15
46	. 5298	. 4702	.5314	. 6216	.1599	.3203	. 4262	. 5737	14
47	. 5320	. 4680	.5309	. 6267	.1592	.3207	. 4281	. 5718	13
48	. 5342	. 4658	.5304	. 6318	.1585	.3210	. 4300	. 5699	12
49	. 5364	. 4636	.5299	. 6368	.1578	.3213	. 4319	. 5680	11
50	.65386	.34614	1.5294	.86419	1.1571	1.3217	.24338	.75661	10
51	. 5408	. 4592	.5289	. 6470	.1565	.3220	. 4357	. 5642	9
52	. 5430	. 4570	.5283	. 6521	.1558	.3223	. 4376	. 5623	8
53	. 5452	. 4548	.5278	. 6572	.1551	.3227	. 4396	. 5604	7
54	. 5474	. 4526	.5273	. 6623	.1544	.3230	. 4415	. 5585	6
55	.65496	.34504	1.5268	.86674	1.1537	1.3233	.24434	.75566	5
56	. 5518	. 4482	.5263	. 6725	.1531	.3237	. 4453	. 5547	4
57	. 5540	. 4460	.5258	. 6775	.1524	.3240	. 4472	. 5528	3
58	. 5562	. 4438	.5253	. 6826	.1517	.3243	. 4491	. 5509	2
59	. 5584	. 4416	.5248	. 6878	.1510	.3247	. 4510	. 5490	1
60	. 5606	. 4394	.5242	. 6929	.1504	.3250	. 4529	. 5471	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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41° **Natural Trigonometrical Functions.** 138°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.65606	.34394	1.5242	.86929	1.1504	1.3250	.24529	.75471	60
1	. 5628	. 4372	.5237	. 6980	.1497	.3253	. 4548	. 5452	59
2	. 5650	. 4350	.5232	. 7031	.1490	.3257	. 4567	. 5433	58
3	. 5672	. 4328	.5227	. 7082	.1483	.3260	. 4586	. 5414	57
4	. 5694	. 4306	.5222	. 7133	.1477	.3263	. 4605	. 5394	56
5	.65716	.34284	1.5217	.87184	1.1470	1.3267	.24624	.75375	55
6	. 5737	. 4262	.5212	. 7235	.1463	.3270	. 4644	. 5356	54
7	. 5759	. 4240	.5207	. 7287	.1456	.3274	. 4663	. 5337	53
8	. 5781	. 4219	.5202	. 7338	.1450	.3277	. 4682	. 5318	52
9	. 5803	. 4197	.5197	. 7389	.1443	.3280	. 4701	. 5299	51
10	.65825	.34175	1.5192	.87441	1.1436	1.3284	.24720	.75280	50
11	. 5847	. 4153	.5187	. 7492	.1430	.3287	. 4739	. 5261	49
12	. 5869	. 4131	.5182	. 7543	.1423	.3290	. 4758	. 5241	48
13	. 5891	. 4109	.5177	. 7595	.1416	.3294	. 4778	. 5222	47
14	. 5913	. 4087	.5171	. 7646	.1409	.3297	. 4797	. 5203	46
15	.65934	.34065	1.5166	.87698	1.1403	1.3301	.24816	.75184	45
16	. 5956	. 4043	.5161	. 7749	.1396	.3304	. 4835	. 5165	44
17	. 5978	. 4022	.5156	. 7801	.1389	.3307	. 4854	. 5146	43
18	. 6000	. 4000	.5151	. 7852	.1383	.3311	. 4873	. 5126	42
19	. 6022	. 3978	.5146	. 7904	.1376	.3314	. 4893	. 5107	41
20	.66044	.33956	1.5141	.87955	1.1369	1.3318	.24912	.75088	40
21	. 6066	. 3934	.5136	. 8007	.1363	.3321	. 4931	. 5069	39
22	. 6087	. 3912	.5131	. 8058	.1356	.3324	. 4950	. 5049	38
23	. 6109	. 3891	.5126	. 8110	.1349	.3328	. 4970	. 5030	37
24	. 6131	. 3869	.5121	. 8162	.1343	.3331	. 4989	. 5011	36
25	.66153	.33847	1.5116	.88213	1.1336	1.3335	.25008	.74992	35
26	. 6175	. 3825	.5111	. 8265	.1329	.3338	. 5027	. 4973	34
27	. 6197	. 3803	.5106	. 8317	.1323	.3342	. 5047	. 4953	33
28	. 6218	. 3781	.5101	. 8369	.1316	.3345	. 5066	. 4934	32
29	. 6240	. 3760	.5096	. 8421	.1309	.3348	. 5085	. 4915	31
30	.66262	.33738	1.5092	.88472	1.1303	1.3352	.25104	.74895	30
31	. 6284	. 3716	.5087	. 8524	.1296	.3355	. 5124	. 4876	29
32	. 6305	. 3694	.5082	. 8576	.1290	.3359	. 5143	. 4857	28
33	. 6327	. 3673	.5077	. 8628	.1283	.3362	. 5162	. 4838	27
34	. 6349	. 3651	.5072	. 8680	.1276	.3366	. 5181	. 4818	26
35	.66371	.33629	1.5067	.88732	1.1270	1.3369	.25201	.74799	25
36	. 6393	. 3607	.5062	. 8784	.1263	.3372	. 5220	. 4780	24
37	. 6414	. 3586	.5057	. 8836	.1257	.3376	. 5239	. 4760	23
38	. 6436	. 3564	.5052	. 8888	.1250	.3379	. 5259	. 4741	22
39	. 6458	. 3542	.5047	. 8940	.1243	.3383	. 5278	. 4722	21
40	.66479	.33520	1.5042	.88992	1.1237	1.3386	.25297	.74702	20
41	. 6501	. 3499	.5037	. 9044	.1230	.3390	. 5317	. 4683	19
42	. 6523	. 3477	.5032	. 9097	.1224	.3393	. 5336	. 4664	18
43	. 6545	. 3455	.5027	. 9149	.1217	.3397	. 5355	. 4644	17
44	. 6566	. 3433	.5022	. 9201	.1211	.3400	. 5375	. 4625	16
45	.66588	.33412	1.5018	.89253	1.1204	1.3404	.25394	.74606	15
46	. 6610	. 3390	.5013	. 9306	.1197	.3407	. 5414	. 4586	14
47	. 6631	. 3368	.5008	. 9358	.1191	.3411	. 5433	. 4567	13
48	. 6653	. 3347	.5003	. 9410	.1184	.3414	. 5452	. 4548	12
49	. 6675	. 3325	.4998	. 9463	.1178	.3418	. 5472	. 4528	11
50	.66697	.33303	1.4993	.89515	1.1171	1.3421	.25491	.74509	10
51	. 6718	. 3282	.4988	. 9567	.1165	.3425	. 5510	. 4489	9
52	. 6740	. 3260	.4983	. 9620	.1158	.3428	. 5530	. 4470	8
53	. 6762	. 3238	.4979	. 9672	.1152	.3432	. 5549	. 4450	7
54	. 6783	. 3217	.4974	. 9725	.1145	.3435	. 5569	. 4431	6
55	.66805	.33195	1.4969	.89777	1.1139	1.3439	.25588	.74412	5
56	. 6826	. 3173	.4964	. 9830	.1132	.3442	. 5608	. 4392	4
57	. 6848	. 3152	.4959	. 9882	.1126	.3446	. 5627	. 4373	3
58	. 6870	. 3130	.4954	. 9935	.1119	.3449	. 5647	. 4353	2
59	. 6891	. 3108	.4949	. 9988	.1113	.3453	. 5666	. 4334	1
60	. 6913	. 3087	.4945	.90040	.1106	.3456	. 5685	. 4314	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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42° Natural Trigonometrical Functions. 137°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.66913	.33087	1.4945	.90040	1.1106	1.3456	.25685	.74314	60
1	. 6935	. 3065	.4940	. 0093	.1100	.3460	. 5705	. 4295	59
2	. 6956	. 3044	.4935	. 0146	.1093	.3463	. 5724	. 4275	58
3	. 6978	. 3022	.4930	. 0198	.1086	.3467	. 5744	. 4256	57
4	. 6999	. 3000	.4925	. 0251	.1080	.3470	. 5763	. 4236	56
5	.67021	.32979	1.4921	.90304	1.1074	1.3474	.25783	.74217	55
6	. 7043	. 2957	.4916	. 0357	.1067	.3477	. 5802	. 4197	54
7	. 7064	. 2936	.4911	. 0410	.1061	.3481	. 5822	. 4178	53
8	. 7086	. 2914	.4906	. 0463	.1054	.3485	. 5841	. 4158	52
9	. 7107	. 2893	.4901	. 0515	.1048	.3488	. 5861	. 4139	51
10	.67129	.32871	1.4897	.90568	1.1041	1.3492	.25880	.74119	50
11	. 7150	. 2849	.4892	. 0621	.1035	.3495	. 5900	. 4100	49
12	. 7172	. 2828	.4887	. 0674	.1028	.3499	. 5919	. 4080	48
13	. 7194	. 2806	.4882	. 0727	.1022	.3502	. 5939	. 4061	47
14	. 7215	. 2785	.4877	. 0780	.1015	.3506	. 5959	. 4041	46
15	.67237	.32763	1.4873	.90834	1.1009	1.3509	.25978	.74022	45
16	. 7258	. 2742	.4868	. 0887	.1003	.3513	. 5998	. 4002	44
17	. 7280	. 2720	.4863	. 0940	.0996	.3517	. 6017	. 3983	43
18	. 7301	. 2699	.4858	. 0993	.0990	.3520	. 6037	. 3963	42
19	. 7323	. 2677	.4854	. 1046	.0983	.3524	. 6056	. 3943	41
20	.67344	.32656	1.4849	.91099	1.0977	1.3527	.26076	.73924	40
21	. 7366	. 2634	.4844	. 1153	.0971	.3531	. 6096	. 3904	39
22	. 7387	. 2613	.4839	. 1206	.0964	.3534	. 6115	. 3885	38
23	. 7409	. 2591	.4835	. 1259	.0958	.3538	. 6135	. 3865	37
24	. 7430	. 2570	.4830	. 1312	.0951	.3542	. 6154	. 3845	36
25	.67452	.32548	1.4825	.91366	1.0945	1.3545	.26174	.73826	35
26	. 7473	. 2527	.4821	. 1419	.0939	.3549	. 6194	. 3806	34
27	. 7495	. 2505	.4816	. 1473	.0932	.3552	. 6213	. 3787	33
28	. 7516	. 2484	.4811	. 1526	.0926	.3556	. 6233	. 3767	32
29	. 7537	. 2462	.4806	. 1580	.0919	.3560	. 6253	. 3747	31
30	.67559	.32441	1.4802	.91633	1.0913	1.3563	.26272	.73728	30
31	. 7580	. 2419	.4797	. 1687	.0907	.3567	. 6292	. 3708	29
32	. 7602	. 2398	.4792	. 1740	.0900	.3571	. 6311	. 3688	28
33	. 7623	. 2377	.4788	. 1794	.0894	.3574	. 6331	. 3669	27
34	. 7645	. 2355	.4783	. 1847	.0888	.3578	. 6351	. 3649	26
35	.67666	.32334	1.4778	.91901	1.0881	1.3581	.26371	.73629	25
36	. 7688	. 2312	.4774	. 1955	.0875	.3585	. 6390	. 3610	24
37	. 7709	. 2291	.4769	. 2008	.0868	.3589	. 6410	. 3590	23
38	. 7730	. 2269	.4764	. 2062	.0862	.3592	. 6430	. 3570	22
39	. 7752	. 2248	.4760	. 2116	.0856	.3596	. 6449	. 3551	21
40	.67773	.32227	1.4755	.92170	1.0849	1.3600	.26469	.73531	20
41	. 7794	. 2205	.4750	. 2223	.0843	.3603	. 6489	. 3511	19
12	. 7816	. 2184	.4746	. 2277	.0837	.3607	. 6508	. 3491	18
43	. 7837	. 2163	.4741	. 2331	.0830	.3611	. 6528	. 3472	17
44	. 7859	. 2141	.4736	. 2385	.0824	.3614	. 6548	. 3452	16
45	.67880	.32120	1.4732	.92439	1.0818	1.3618	.26568	.73432	15
46	. 7901	. 2098	.4727	. 2493	.0812	.3622	. 6587	. 3412	14
47	. 7923	. 2077	.4723	. 2547	.0805	.3625	. 6607	. 3393	13
48	. 7944	. 2056	.4718	. 2601	.0799	.3629	. 6627	. 3373	12
49	. 7965	. 2034	.4713	. 2655	.0793	.3633	. 6647	. 3353	11
50	.67987	.32013	1.4709	.92709	1.0786	1.3636	.26666	.73333	10
51	. 8008	. 1992	.4704	. 2763	.0780	.3640	. 6686	. 3314	9
52	. 8029	. 1970	.4699	. 2817	.0774	.3644	. 6706	. 3294	8
53	. 8051	. 1949	.4695	. 2871	.0767	.3647	. 6726	. 3274	7
54	. 8072	. 1928	.4690	. 2926	.0761	.3651	. 6746	. 3254	6
55	.68093	.31907	1.4686	.92980	1.0755	1.3655	.26765	.73234	5
56	. 8115	. 1885	.4681	. 3034	.0749	.3658	. 6785	. 3215	4
57	. 8136	. 1864	.4676	. 3088	.0742	.3662	. 6805	. 3195	3
58	. 8157	. 1843	.4672	. 3143	.0736	.3666	. 6825	. 3175	2
59	. 8178	. 1821	.4667	3197	.0730	.3669	. 6845	. 3155	1
60	. 8200	. 1800	.4663	. 3251	.0724	.3673	. 6865	. 3135	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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43° **Natural Trigonometrical Functions.** 136°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.68200	.31800	1.4663	.93251	1.0724	1.3673	.26865	.73135	60
1	. 8221	. 1779	.4658	. 3306	.0717	.3677	. 6884	. 3115	59
2	. 8242	. 1758	.4654	. 3360	.0711	.3681	. 6904	. 3096	58
3	. 8264	. 1736	.4649	. 3415	.0705	.3684	. 6924	. 3076	57
4	. 8285	. 1715	.4644	. 3469	.0699	.3688	. 6944	. 3056	56
5	.68306	.31694	1.4640	.93524	1.0692	1.3692	.26964	.73036	55
6	. 8327	. 1673	.4635	. 3578	.0686	.3695	. 6984	. 3016	54
7	. 8349	. 1651	.4631	. 3633	.0680	.3699	. 7004	. 2996	53
8	. 8370	. 1630	.4626	. 3687	.0674	.3703	. 7023	. 2976	52
9	. 8391	. 1609	.4622	. 3742	.0667	.3707	. 7043	. 2956	51
10	.68412	.31588	1.4617	.93797	1.0661	1.3710	.27063	.72937	50
11	. 8433	. 1566	.4613	. 3851	.0655	.3714	. 7083	. 2917	49
12	. 8455	. 1545	.4608	. 3906	.0649	.3718	. 7103	. 2897	48
13	. 8476	. 1524	.4604	. 3961	.0643	.3722	. 7123	. 2877	47
14	. 8497	. 1503	.4599	. 4016	.0636	.3725	. 7143	. 2857	46
15	.68518	.31482	1.4595	.94071	1.0630	1.3729	.27163	.72837	45
16	. 8539	. 1460	.4590	. 4125	.0624	.3733	. 7183	. 2817	44
17	. 8561	. 1439	.4586	. 4180	.0618	.3737	. 7203	. 2797	43
18	. 8582	. 1418	.4581	. 4235	.0612	.3740	. 7223	. 2777	42
19	. 8603	. 1397	.4577	. 4290	.0605	.3744	. 7243	. 2757	41
20	.68624	.31376	1.4572	.94345	1.0599	1.3748	.27263	.72737	40
21	. 8645	. 1355	.4568	. 4400	.0593	.3752	. 7283	. 2717	39
22	. 8666	. 1333	.4563	. 4455	.0587	.3756	. 7302	. 2697	38
23	. 8688	. 1312	.4559	. 4510	.0581	.3759	. 7322	. 2677	37
24	. 8709	. 1291	.4554	. 4565	.0575	.3763	. 7342	. 2657	36
25	.68730	.31270	1.4550	.94620	1.0568	1.3767	.27362	.72637	35
26	. 8751	. 1249	.4545	. 4675	.0562	.3771	. 7382	. 2617	34
27	. 8772	. 1228	.4541	. 4731	.0556	.3774	. 7402	. 2597	33
28	. 8793	. 1207	.4536	. 4786	.0550	.3778	. 7422	. 2577	32
29	. 8814	. 1186	.4532	. 4841	.0544	.3782	. 7442	. 2557	31
30	.68835	.31164	1.4527	.94896	1.0538	1.3786	.27462	.72537	30
31	. 8856	. 1143	.4523	. 4952	.0532	.3790	. 7482	. 2517	29
32	. 8878	. 1122	.4518	. 5007	.0525	.3794	. 7503	. 2497	28
33	. 8899	. 1101	.4514	. 5062	.0519	.3797	. 7523	. 2477	27
34	. 8920	. 1080	.4510	. 5118	.0513	.3801	. 7543	. 2457	26
35	.68941	.31059	1.4505	.95173	1.0507	1.3805	.27563	.72437	25
36	. 8962	. 1038	.4501	. 5229	.0501	.3809	. 7583	. 2417	24
37	. 8983	. 1017	.4496	. 5284	.0495	.3813	. 7603	. 2397	23
38	. 9004	. 0996	.4492	. 5340	.0489	.3816	. 7623	. 2377	22
39	. 9025	. 0975	.4487	. 5395	.0483	.3820	. 7643	. 2357	21
40	.69046	.30954	1.4483	.95451	1.0476	1.3824	.27663	.72337	20
41	. 9067	. 0933	.4479	. 5506	.0470	.3828	. 7683	. 2317	19
42	. 9088	. 0912	.4474	. 5562	.0464	.3832	. 7703	. 2297	18
43	. 9109	. 0891	.4470	. 5618	.0458	.3836	. 7723	. 2277	17
44	. 9130	. 0870	.4465	. 5673	.0452	.3839	. 7743	. 2256	16
45	.69151	.30849	1.4461	.95729	1.0446	1.3843	.27764	.72236	15
46	. 9172	. 0828	.4457	. 5785	.0440	.3847	. 7784	. 2216	14
47	. 9193	. 0807	.4452	. 5841	.0434	.3851	. 7804	. 2196	13
48	. 9214	. 0786	.4448	. 5896	.0428	.3855	. 7824	. 2176	12
49	. 9235	. 0765	.4443	. 5952	.0422	.3859	. 7844	. 2156	11
50	.69256	.30744	1.4439	.96008	1.0416	1.3863	.27864	.72136	10
51	. 9277	. 0723	.4435	. 6064	.0410	.3867	. 7884	. 2115	9
52	. 9298	. 0702	.4430	. 6120	.0404	.3870	. 7904	. 2095	8
53	. 9319	. 0681	.4426	. 6176	.0397	.3874	. 7925	. 2075	7
54	. 9340	. 0660	.4422	. 6232	.0391	.3878	. 7945	. 2055	6
55	.69361	.30639	1.4417	.96288	1.0385	1.3882	.27965	.72035	5
56	. 9382	. 0618	.4413	. 6344	.0379	.3886	. 7985	. 2015	4
57	. 9403	. 0597	.4408	. 6400	.0373	.3890	. 8005	. 1994	3
58	. 9424	. 0576	.4404	. 6456	.0367	.3894	. 8026	. 1974	2
59	. 9445	. 0555	.4400	. 6513	.0361	.3898	. 8046	. 1954	1
60	. 9466	. 0534	.4395	. 6569	.0355	.3902	. 8066	. 1934	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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44° **Natural Trigonometrical Functions.** **135°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.69466	.30534	1.4395	.96569	1.0355	1.3902	.28066	.71934	60
1	.9487	.0513	.4391	.6625	.0349	.3905	.8086	.1914	59
2	.9508	.0492	.4387	.6681	.0343	.3909	.8106	.1893	58
3	.9528	.0471	.4382	.6738	.0337	.3913	.8127	.1873	57
4	.9549	.0450	.4378	.6794	.0331	.3917	.8147	.1853	56
5	.69570	.30430	1.4374	.96850	1.0325	1.3921	.28167	.71833	55
6	.9591	.0409	.4370	.6907	.0319	.3925	.8187	.1813	54
7	.9612	.0388	.4365	.6963	.0313	.3929	.8208	.1792	53
8	.9633	.0367	.4361	.7020	.0307	.3933	.8228	.1772	52
9	.9654	.0346	.4357	.7076	.0301	.3937	.8248	.1752	51
10	.69675	.30325	1.4352	.97133	1.0295	1.3941	.28268	.71732	50
11	.9696	.0304	.4348	.7189	.0289	.3945	.8289	.1711	49
12	.9716	.0283	.4344	.7246	.0283	.3949	.8309	.1691	48
13	.9737	.0263	.4339	.7302	.0277	.3953	.8329	.1671	47
14	.9758	.0242	.4335	.7359	.0271	.3957	.8349	.1650	46
15	.69779	.30221	1.4331	.97416	1.0265	1.3960	.28370	.71630	45
16	.9800	.0200	.4327	.7472	.0259	.3964	.8390	.1610	44
17	.9821	.0179	.4322	.7529	.0253	.3968	.8410	.1589	43
18	.9841	.0158	.4318	.7586	.0247	.3972	.8431	.1569	42
19	.9862	.0138	.4314	.7643	.0241	.3976	.8451	.1549	41
20	.69883	.30117	1.4310	.97699	1.0235	1.3980	.28471	.71529	40
21	.9904	.0096	.4305	.7756	.0229	.3984	.8492	.1508	39
22	.9925	.0075	.4301	.7813	.0223	.3988	.8512	.1488	38
23	.9945	.0054	.4297	.7870	.0218	.3992	.8532	.1468	37
24	.9966	.0034	.4292	.7927	.0212	.3996	.8553	.1447	36
25	.69987	.30013	1.4288	.97984	1.0206	1.4000	.28573	.71427	35
26	.70008	.29992	.4284	.8041	.0200	.4004	.8593	.1406	34
27	.0029	.9971	.4280	.8098	.0194	.4008	.8614	.1386	33
28	.0049	.9950	.4276	.8155	.0188	.4012	.8634	.1366	32
29	.0070	.9930	.4271	.8212	.0182	.4016	.8654	.1345	31
30	.70091	.29909	1.4267	.98270	1.0176	1.4020	.28675	.71325	30
31	.0112	.9888	.4263	.8327	.0170	.4024	.8695	.1305	29
32	.0132	.9867	.4259	.8384	.0164	.4028	.8716	.1284	28
33	.0153	.9847	.4254	.8441	.0158	.4032	.8736	.1264	27
34	.0174	.9826	.4250	.8499	.0152	.4036	.8756	.1243	26
35	.70194	.29805	1.4246	.98556	1.0146	1.4040	.28777	.71223	25
36	.0215	.9785	.4242	.8613	.0141	.4044	.8797	.1203	24
37	.0236	.9764	.4238	.8671	.0135	.4048	.8818	.1182	23
38	.0257	.9743	.4233	.8728	.0129	.4052	.8838	.1162	22
39	.0277	.9722	.4229	.8786	.0123	.4056	.8859	.1141	21
40	.70298	.29702	1.4225	.98843	1.0117	1.4060	.28879	.71121	20
41	.0319	.9681	.4221	.8901	.0111	.4065	.8899	.1100	19
42	.0339	.9660	.4217	.8958	.0105	.4069	.8920	.1080	18
43	.0360	.9640	.4212	.9016	.0099	.4073	.8940	.1059	17
44	.0381	.9619	.4208	.9073	.0093	.4077	.8961	.1039	16
45	.70401	.29598	1.4204	.99131	1.0088	1.4081	.28981	.71018	15
46	.0422	.9578	.4200	.9189	.0082	.4085	.9002	.0998	14
47	.0443	.9557	.4196	.9246	.0076	.4089	.9022	.0977	13
48	.0463	.9536	.4192	.9304	.0070	.4093	.9043	.0957	12
49	.0484	.9516	.4188	.9362	.0064	.4097	.9063	.0936	11
50	.70505	.29495	1.4183	.99420	1.0058	1.4101	.29084	.70916	10
51	.0525	.9475	.4179	.9478	.0052	.4105	.9104	.0895	9
52	.0546	.9454	.4175	.9536	.0047	.4109	.9125	.0875	8
53	.0566	.9433	.4171	.9593	.0041	.4113	.9145	.0854	7
54	.0587	.9413	.4167	.9651	.0035	.4117	.9166	.0834	6
55	.70608	.29392	1.4163	.99709	1.0029	1.4122	.29186	.70813	5
56	.0628	.9372	.4159	.9767	.0023	.4126	.9207	.0793	4
57	.0649	.9351	.4154	.9826	.0017	.4130	.9228	.0772	3
58	.0669	.9330	.4150	.9884	.0012	.4134	.9248	.0752	2
59	.0690	.9310	.4146	.9942	.0006	.4138	.9269	.0731	1
60	.0711	.9289	.4142	1.0000	.0000	.4142	.9289	.0711	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
1	1	1	1.0000	1.0000	3.142	0.7854
2	4	8	1.4142	1.2599	6.283	3.1416
3	9	27	1.7321	1.4422	9.425	7.0686
4	16	64	2.0000	1.5874	12.566	12.5664
5	25	125	2.2361	1.7100	15.708	19.6350
6	36	216	2.4495	1.8171	18.850	28.2743
7	49	343	2.6458	1.9129	21.991	38.4845
8	64	512	2.8284	2.0000	25.133	50.2655
9	81	729	3.0000	2.0801	28.274	63.6173
10	100	1000	3.1623	2.1544	31.416	78.5398
11	121	1331	3.3166	2.2240	34.558	95.033
12	144	1728	3.4641	2.2894	37.699	113.097
13	169	2197	3.6056	2.3513	40.841	132.732
14	196	2744	3.7417	2.4101	43.982	153.938
15	225	3375	3.8730	2.4662	47.124	176.715
16	256	4096	4.0000	2.5198	50.265	201.062
17	289	4913	4.1231	2.5713	53.407	226.980
18	324	5832	4.2426	2.6207	56.549	254.469
19	361	6859	4.3589	2.6684	59.690	283.529
20	400	8000	4.4721	2.7144	62.832	314.159
21	441	9261	4.5826	2.7589	65.973	346.361
22	484	10648	4.6904	2.8020	69.115	380.133
23	529	12167	4.7958	2.8439	72.257	415.476
24	576	13824	4.8990	2.8845	75.398	452.389
25	625	15625	5.0000	2.9240	78.540	490.874
26	676	17576	5.0990	2.9625	81.681	530.929
27	729	19683	5.1962	3.0000	84.823	572.555
28	784	21952	5.2915	3.0366	87.965	615.752
29	841	24389	5.3852	3.0723	91.106	660.520
30	900	27000	5.4772	3.1072	94.248	706.858
31	961	29791	5.5678	3.1414	90.389	754.768
32	1024	32768	5.6569	3.1748	100.531	804.248
33	1089	35937	5.7446	3.2075	103.673	855.299
34	1156	39304	5.8310	3.2396	106.814	907.920
35	1225	42875	5.9161	3.2711	109.956	962.113
36	1296	46656	6.0000	3.3019	113.097	1017.88
37	1369	50653	6.0828	3.3322	116.239	1075.21
38	1444	54872	6.1644	3.3620	119.381	1134.11
39	1521	59319	6.2450	3.3912	122.522	1194.59
40	1600	64000	6.3246	3.4200	125.660	1256.64

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle Circum.	Circle Area
41	1681	68921	6.4031	3.4482	128.81	1320.25
42	1764	74088	6.4807	3.4760	131.95	1385.44
43	1849	79507	6.5574	3.5034	135.09	1452.20
44	1936	85184	6.6332	3.5303	138.23	1520.53
45	2025	91125	6.7082	3.5569	141.37	1590.43
46	2116	97336	6.7823	3.5830	144.51	1661.90
47	2209	103823	6.8557	3.6088	147.65	1734.94
48	2304	110592	6.9282	3.6342	150.80	1809.56
49	2401	117649	7.0000	3.6593	153.94	1885.74
50	2500	125000	7.0711	3.6840	157.08	1963.50
51	2601	132651	7.1414	3.7084	160.22	2042.82
52	2704	140608	7.2111	3.7325	163.36	2123.72
53	2809	148877	7.2801	3.7563	166.50	2206.18
54	2916	157464	7.3485	3.7798	169.65	2290.22
55	3025	166375	7.4162	3.8030	172.79	2375.83
56	3136	175616	7.4833	3.8259	175.93	2463.01
57	3249	185193	7.5498	3.8485	179.07	2551.76
58	3364	195112	7.6158	3.8709	182.21	2642.08
59	3481	205379	7.6811	3.8930	185.35	2733.97
60	3600	216000	7.7460	3.9149	188.50	2827.43
61	3721	226981	7.8102	3.9365	191.64	2922.47
62	3844	238328	7.8740	3.9579	194.78	3019.07
63	3969	250047	7.9373	3.9791	197.92	3117.25
64	4096	262144	8.0000	4.0000	201.06	3216.99
65	4225	274625	8.0623	4.0207	204.20	3318.31
66	4356	287496	8.1240	4.0412	207.35	3421.19
67	4489	300763	8.1854	4.0615	210.49	3525.65
68	4624	314432	8.2462	4.0817	213.63	3631.68
69	4761	328509	8.3066	4.1016	216.77	3739.28
70	4900	343000	8.3666	4.1213	219.91	3848.45
71	5041	357911	8.4261	4.1408	223.05	3959.19
72	5184	373248	8.4853	4.1602	226.19	4071.50
73	5329	389017	8.5440	4.1793	229.34	4185.39
74	5476	405224	8.6023	4.1983	232.48	4300.84
75	5625	421875	8.6603	4.2172	235.62	4417.86
76	5776	438976	8.7178	4.2358	238.76	4536.46
77	5929	456533	8.7750	4.2543	241.90	4656.63
78	6084	474552	8.8318	4.2727	245.04	4778.36
79	6241	493039	8.8882	4.2908	248.19	4901.67
80	6400	512000	8.9443	4.3089	251.33	5026.55

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle Circum.	Circle Area
81	6561	531441	9.0000	4.3267	254.47	5153.00
82	6724	551368	9.0554	4.3445	257.61	5281.02
83	6889	571787	9.1104	4.3621	260.75	5410.61
84	7056	592704	9.1652	4.3795	263.89	5541.77
85	7225	614125	9.2195	4.3968	267.04	5674.50
86	7396	636056	9.2736	4.4140	270.18	5808.80
87	7569	658503	9.3274	4.4310	273.32	5944.68
88	7744	681472	9.3808	4.4480	276.46	6082.12
89	7921	704969	9.4340	4.4647	279.60	6221.14
90	8100	729000	9.4868	4.4814	282.74	6361.73
91	8281	753571	9.5394	4.4979	285.88	6503.88
92	8464	778688	9.5917	4.5144	289.03	6647.61
93	8649	804357	9.6437	4.5307	292.17	6792.91
94	8836	830584	9.6954	4.5468	295.31	6939.78
95	9025	857375	9.7468	4.5629	298.45	7088.22
96	9216	884736	9.7980	4.5789	301.59	7238.23
97	9409	912673	9.8489	4.5947	304.73	7389.81
98	9604	941192	9.8995	4.6104	307.88	7542.96
99	9801	970299	9.9499	4.6261	311.02	7697.69
100	10000	1000000	10.0000	4.6416	314.16	7853.98
101	10201	1030301	10.0499	4.6570	317.30	8011.85
102	10404	1061208	10.0995	4.6723	320.44	8171.28
103	10609	1092727	10.1489	4.6875	323.58	8332.29
104	10816	1124864	10.1980	4.7027	326.73	8494.87
105	11025	1157625	10.2470	4.7177	329.87	8659.01
106	11236	1191016	10.2956	4.7326	333.01	8824.73
107	11449	1225043	10.3441	4.7475	336.15	8992.02
108	11664	1259712	10.3923	4.7622	339.29	9160.88
109	11881	1295029	10.4403	4.7769	342.43	9331.32
110	12100	1331000	10.4881	4.7914	345.58	9503.32
111	12321	1367631	10.5357	4.8059	348.72	9676.89
112	12544	1404928	10.5830	4.8203	351.86	9852.03
113	12769	1442897	10.6301	4.8346	355.00	10028.7
114	12996	1481544	10.6771	4.8488	358.14	10207.0
115	13225	1520875	10.7238	4.8629	361.28	10386.9
116	13456	1560896	10.7703	4.8770	364.42	10568.3
117	13689	1601613	10.8167	4.8910	367.57	10751.3
118	13924	1643032	10.8628	4.9049	370.71	10935.9
119	14161	1685159	10.9087	4.9187	373.85	11122.0
120	14400	1728000	10.9545	4.9324	376.99	11309.7

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle Circum.	Circle Area
121	14641	1771561	11.0000	4.9461	380.13	11499.0
122	14884	1815848	11.0454	4.9597	383.27	11689.9
123	15129	1860867	11.0905	4.9732	386.42	11882.3
124	15376	1906624	11.1355	4.9866	389.56	12076.3
125	15625	1953125	11.1803	5.0000	392.70	12271.8
126	15876	2000376	11.2250	5.0133	395.84	12469.0
127	16129	2048383	11.2694	5.0265	398.98	12667.7
128	16384	2097152	11.3137	5.0397	402.12	12868.0
129	16641	2146689	11.3578	5.0528	405.27	13069.8
130	16900	2197000	11.4018	5.0658	408.41	13273.2
131	17161	2248091	11.4455	5.0788	411.55	13478.2
132	17424	2299968	11.4891	5.0916	414.69	13684.8
133	17689	2352637	11.5326	5.1045	417.83	13892.9
134	17956	2406104	11.5758	5.1172	420.97	14102.6
135	18225	2460375	11.6190	5.1299	424.12	14313.9
136	18496	2515456	11.6619	5.1426	427.26	14526.7
137	18769	2571353	11.7047	5.1551	430.40	14741.1
138	19044	2628072	11.7473	5.1676	433.54	14957.1
139	19321	2685619	11.7898	5.1801	436.68	15174.7
140	19600	2744000	11.8322	5.1925	439.82	15393.8
141	19881	2803221	11.8743	5.2048	442.96	15614.5
142	20164	2863288	11.9164	5.2171	446.11	15836.8
143	20449	2924207	11.9583	5.2293	449.25	16060.6
144	20736	2985984	12.0000	5.2415	452.39	16286.0
145	21025	3048625	12.0416	5.2536	455.53	16513.0
146	21316	3112136	12.0830	5.2656	458.67	16741.5
147	21609	3176523	12.1244	5.2776	461.81	16971.7
148	21904	3241792	12.1655	5.2896	464.96	17203.4
149	22201	3307949	12.2066	5.3015	468.10	17436.6
150	22500	3375000	12.2474	5.3133	471.24	17671.5
151	22801	3442951	12.2882	5.3251	474.38	17907.9
152	23104	3511808	12.3288	5.3368	477.52	18145.8
153	23409	3581577	12.3693	5.3485	480.66	18385.4
154	23716	3652264	12.4097	5.3601	483.81	18626.5
155	24025	3723875	12.4499	5.3717	486.95	18869.2
156	24336	3796416	12.4900	5.3832	490.09	19113.4
157	24649	3869893	12.5300	5.3947	493.23	19359.3
158	24964	3944312	12.5698	5.4061	496.37	19606.7
159	25281	4019679	12.6095	5.4175	499.51	19855.7
160	25600	4096000	12.6491	5.4288	502.65	20106.2

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
161	25921	4173281	12.6886	5.4401	505.80	20358.3
162	26244	4251528	12.7279	5.4514	508.94	20612.0
163	26569	4330747	12.7671	5.4626	512.08	20867.2
164	26896	4410944	12.8062	5.4737	515.22	21124.1
165	27225	4492125	12.8452	5.4848	518.36	21382.5
166	27556	4574296	12.8841	5.4959	521.50	21642.4
167	27889	4657463	12.9228	5.5069	524.65	21904.0
168	28224	4741632	12.9615	5.5178	527.79	22167.1
169	28561	4826809	13.0000	5.5288	530.93	22431.8
170	28900	4913000	13.0384	5.5397	534.07	22698.0
171	29241	5000211	13.0767	5.5505	537.21	22965.8
172	29584	5088448	13.1149	5.5613	540.35	23235.2
173	29929	5177717	13.1529	5.5721	543.50	23506.2
174	30276	5268024	13.1909	5.5828	546.64	23778.7
175	30625	5359375	13.2288	5.5934	549.78	24052.8
176	30976	5451776	13.2665	5.6041	552.92	24328.5
177	31329	5545233	13.3041	5.6147	556.06	24605.7
178	31684	5639752	13.3417	5.6252	559.20	24884.6
179	32041	5735339	13.3791	5.6357	562.35	25164.9
180	32400	5832000	13.4164	5.6462	565.49	25446.9
181	32761	5929741	13.4536	5.6567	568.63	25730.4
182	33124	6028568	13.4907	5.6671	571.77	26015.5
183	33489	6128487	13.5277	5.6774	574.91	26302.2
184	33856	6229504	13.5647	5.6877	578.05	26590.4
185	34225	6331625	13.6015	5.6980	581.19	26880.3
186	34596	6434856	13.6382	5.7083	584.34	27171.6
187	34969	6539203	13.6748	5.7185	587.48	27464.6
188	35344	6644672	13.7113	5.7287	590.62	27759.1
189	35721	6751269	13.7477	5.7388	593.76	28055.2
190	36100	6859000	15.7840	5.7489	596.90	28352.9
191	36481	6967871	13.8203	5.7590	600.04	28652.1
192	36864	7077888	13.8564	5.7690	603.19	28952.9
193	37249	7189057	13.8924	5.7790	606.33	29255.3
194	37636	7301384	13.9284	5.7890	609.47	29559.2
195	38025	7414875	13.9642	5.7989	612.61	29864.8
196	38416	7529536	14.0000	5.8088	615.75	30171.9
197	38809	7645373	14.0357	5.8186	618.89	30480.5
198	39204	7762392	14.0712	5.8285	622.04	30790.7
199	39601	7880599	14.1067	5.8383	625.18	31102.6
200	40000	8000000	14.1421	5.8480	628.32	31415.9

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
201	40401	8120601	14.1774	5.8578	631.46	31730.9
202	40804	8242408	14.2127	5.8675	634.60	32047.4
203	41209	8365427	14.2478	5.8771	637.74	32365.5
204	41616	8489664	14.2829	5.8868	640.89	32685.1
205	42025	8615125	14.3178	5.8964	644.03	33006.4
206	42436	8741816	14.3527	5.9059	647.17	33329.2
207	42849	8869743	14.3875	5.9155	650.31	33653.5
208	43264	8998912	14.4222	5.9250	653.45	33979.5
209	43681	9129329	14.4568	5.9345	656.59	34307.0
210	44100	9261000	14.4914	5.9439	659.73	34636.1
211	44521	9393931	14.5258	5.9533	662.88	34966.7
212	44944	9528128	14.5602	5.9627	666.02	35298.9
213	45369	9663597	14.5945	5.9721	669.16	35632.7
214	45796	9800344	14.6287	5.9814	672.30	35968.1
215	46225	9938375	14.6629	5.9907	675.44	36305.0
216	46656	10077696	14.6969	6.0000	678.58	36643.5
217	47089	10218313	14.7309	6.0092	681.73	36983.6
218	47524	10360232	14.7648	6.0185	684.87	37325.3
219	47961	10503459	14.7986	6.0277	688.01	37668.5
220	48400	10648000	14.8324	6.0368	691.15	38013.3
221	48841	10793861	14.8661	6.0459	694.29	38359.6
222	49284	10941048	14.8997	6.0550	697.43	38707.6
223	49729	11089567	14.9332	6.0641	700.58	39057.1
224	50176	11239424	14.9666	6.0732	703.72	39408.1
225	50625	11390625	15.0000	6.0822	706.86	39760.8
226	51076	11543176	15.0333	6.0912	710.00	40115.0
227	51529	11697083	15.0665	6.1002	713.14	40470.8
228	51984	11852352	15.0997	6.1091	716.28	40828.1
229	52441	12008989	15.1327	6.1180	719.42	41187.1
230	52900	12167000	15.1658	6.1269	722.57	41547.6
231	53361	12326391	15.1987	6.1358	725.71	41909.6
232	53824	12487168	15.2315	6.1446	728.85	42273.3
233	54289	12649337	15.2643	6.1534	731.99	42638.5
234	54756	12812904	15.2971	6.1622	735.13	43005.3
235	55225	12977875	15.3297	6.1710	738.27	43373.6
236	55696	13144256	15.3623	6.1797	741.42	43743.5
237	56169	13312053	15.3948	6.1885	744.56	44115.0
238	56644	13481272	15.4272	6.1972	747.70	44488.1
239	57121	13651919	15.4596	6.2058	750.84	44862.7
240	57600	13824000	15.4919	6.2145	753.98	45238.9

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
241	58081	13997521	15.5242	6.2231	757.12	45616.7
242	58564	14172488	15.5563	6.2317	760.27	45996.1
243	59049	14348907	15.5885	6.2403	763.41	46377.0
244	59536	14526784	15.6205	6.2488	766.55	46759.5
245	60025	14706125	15.6525	6.2573	769.69	47143.5
246	60516	14886936	15.6844	6.2658	772.83	47529.2
247	61009	15069223	15.7162	6.2743	775.97	47916.4
248	61504	15252992	15.7480	6.2828	779.12	48305.1
249	62001	15438249	15.7797	6.2912	782.26	48695.5
250	62500	15625000	15,8114	6.2996	785.40	49087.4
251	63001	15813251	15.8430	6.3080	788.54	49480.9
252	63504	16003008	15.8745	6.3164	791.68	49875.9
253	64009	16194277	15.9060	6.3247	794.82	50272.6
254	64516	16387064	15.9374	6.3330	797.96	50670.7
255	65025	16581375	15.9687	6.3413	801.11	51070.5
256	65536	16777216	16.0000	6.3496	804.25	51471.9
257	66049	16974593	16.0312	6.3579	807.39	51874.8
258	66564	17173512	16.0624	6.3661	810.53	52279.2
259	67081	17373979	16.0935	6.3743	813.67	52685.3
260	67600	17576000	16.1245	6.3825	816.81	53092.9
261	68121	17779581	16.1555	6.3907	819.96	53502.1
262	68644	17984728	16.1864	6.3988	823.10	53912.9
263	69169	18191447	16.2173	6.4070	826.24	54325.2
264	69696	18399744	16.2481	6.4151	829.38	54739.1
265	70225	18609625	16.2788	6.4232	832.52	55154.6
266	70756	18821096	16.3095	6.4312	835.66	55571.6
267	71289	19034163	16.3401	6.4393	838.81	55990.3
268	71824	19248832	16.3707	6.4473	841.95	56410.4
269	72361	19465109	16.4012	6.4553	845.09	56832.2
270	72900	19683000	16.4317	6.4633	848.23	57255.5
271	73441	19902511	16.4621	6.4713	851.37	57680.4
272	73984	20123648	16.4924	6.4792	854.51	58106.9
273	74529	20346417	16.5227	6.4872	857.66	58534.9
274	75076	20570824	16.5529	6.4951	860.80	58964.6
275	75625	20796875	16.5831	6.5030	863.94	59395.7
276	76176	21024576	16.6132	6.5108	867.08	59828.5
277	76729	21253933	16.6433	6.5187	870.22	60262.8
278	77284	21484952	16.6733	6.5265	873.36	60698.7
279	77841	21717639	16.7033	6.5343	876.50	61136.2
280	78400	21952000	16.7332	6.5421	879.65	61575.2

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
281	78961	22188041	16.7631	6.5499	882.79	62015.8
282	79524	22425768	16.7929	6.5577	885.93	62458.0
283	80089	22665187	16.8226	6.5654	889.07	62901.8
284	80656	22906304	16.8523	6.5731	892.21	63347.1
285	81225	23149125	16.8819	6.5808	895.35	63794.0
286	81796	23393656	16.9115	6.5885	898.50	64242.4
287	82369	23639903	16.9411	6.5962	901.64	64692.5
288	82944	23887872	16.9706	6.6039	904.78	65144.1
289	83521	24137569	17.0000	6.6115	907.92	65597.2
290	84100	24389000	17.0294	6.6191	911.06	66052.0
291	84681	24642171	17.0587	6.6267	914.20	66508.3
292	85264	24897088	17.0880	6.6343	917.35	66966.2
293	85849	25153757	17.1172	6.6419	920.49	67425.6
294	86436	25412184	17.1464	6.6494	923.63	67886.7
295	87025	25672375	17.1756	6.6569	926.77	68349.3
296	87616	25934336	17.2047	6.6644	929.91	68813.5
297	88209	26198073	17.2337	6.6719	933.05	69279.2
298	88804	26463592	17.2627	6.6794	936.19	69746.5
299	89401	26730899	17.2916	6.6869	939.34	70215.4
300	90000	27000000	17.3205	6.6943	942.48	70685.8
301	90601	27270901	17.3494	6.7018	945.62	71157.9
302	91204	27543608	17.3781	6.7092	948.76	71631.5
303	91809	27818127	17.4069	6.7166	951.90	72106.6
304	92416	28094464	17.4356	6.7240	955.04	72583.4
305	93025	28372625	17.4642	6.7313	958.19	73061.7
306	93636	28652616	17.4929	6.7387	961.33	73541.5
307	94249	28934443	17.5214	6.7460	964.47	74023.0
308	94864	29218112	17.5499	6.7533	967.61	74506.0
309	95481	29503629	17.5784	6.7606	970.75	74990.6
310	96100	29791000	17.6068	6.7679	973.89	75476.8
311	96721	30080231	17.6352	6.7752	977.04	75964.5
312	97344	30371328	17.6635	6.7824	980.18	76453.8
313	97969	30664297	17.6918	6.7897	983.32	76944.7
314	98596	30959144	17.7200	6.7969	986.46	77437.1
315	99225	31255875	17.7482	6.8041	989.60	77931.1
316	99856	31554496	17.7764	6.8113	992.74	78426.7
317	100489	31855013	17.8045	6.8185	995.88	78923.9
318	101124	32157432	17.8326	6.8256	999.03	79422.6
319	101761	32461759	17.8606	6.8328	1002.20	79922.9
320	102400	32768000	17.8885	6.8399	1005.30	80424.8

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle Circum.	Circle Area
321	103041	33076161	17.9165	6.8470	1008.5	80928.2
322	103684	33386248	17.9444	6.8541	1011.6	81433.2
323	104329	33698267	17.9722	6.8612	1014.7	81939.8
324	104976	34012224	18.0000	6.8683	1017.9	82448.0
325	105625	34328125	18.0278	6.8753	1021.0	82957.7
326	106276	34645976	18.0555	6.8824	1024.2	83469.0
327	106929	34965783	18.0831	6.8894	1027.3	83981.8
328	107584	35287552	18.1108	6.8964	1030.4	84496.3
329	108241	35611289	18.1384	6.9034	1033.6	85012.3
330	108900	35937000	18.1659	6.9104	1036.7	85529.9
331	109561	36264691	18.1934	6.9174	1039.9	86049.0
332	110224	36594368	18.2209	6.9244	1043.0	86569.7
333	110889	36926037	18.2483	6.9313	1046.2	87092.0
334	111556	37259704	18.2757	6.9382	1049.3	87615.9
335	112225	37595375	18.3030	6.9451	1052.4	88141.3
336	112896	37933056	18.3303	6.9521	1055.6	88668.3
337	113569	38272753	18.3576	6.9589	1058.7	89196.9
338	114244	38614472	18.3848	6.9658	1061.9	89727.0
339	114921	38958219	18.4120	6.9727	1065.0	90258.7
340	115600	39304000	18.4391	6.9795	1068.1	90792.0
341	116281	39651821	18.4662	6.9864	1071.3	91326.9
342	116964	40001688	18.4932	6.9932	1074.4	91863.3
343	117649	40353607	18.5203	7.0000	1077.6	92401.3
344	118336	40707584	18.5472	7.0068	1080.7	92940.9
345	119025	41063625	18.5742	7.0136	1083.8	93482.0
346	119716	41421736	18.6011	7.0203	1087.0	94024.7
347	120409	41781923	18.6279	7.0271	1090.1	94569.0
348	121104	42144192	18.6548	7.0338	1093.3	95114.9
349	121801	42508549	18.6815	7.0406	1096.4	95662.3
350	122500	42875000	18.7083	7.0473	1099.6	96211.3
351	123201	43243551	18.7350	7.0540	1102.7	96761.8
352	123904	43614208	18.7617	7.0607	1105.8	97314.0
353	124609	43986977	18.7883	7.0674	1109.0	97867.7
354	125316	44361864	18.8149	7.0740	1112.1	98423.0
355	126025	44738875	18.8414	7.0807	1115.3	98979.8
356	126736	45118016	18.8680	7.0873	1118.4	99538.2
357	127449	45499293	18.8944	7.0940	1121.5	100098
358	128164	45882712	18.9209	7.1006	1124.7	100660
359	128881	46268279	18.9473	7.1072	1127.8	101223
360	129600	46656000	18.9737	7.1138	1131.0	101788

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
361	130321	47045881	19.0000	7.1204	1134.1	102354
362	131044	47437928	19.0263	7.1269	1137.3	102922
363	131769	47832147	19.0526	7.1335	1140.4	103491
364	132496	48228544	19.0788	7.1400	1143.5	104062
365	133225	48627125	19.1050	7.1466	1146.7	104635
366	133956	49027896	19.1311	7.1531	1149.8	105209
367	134689	49430863	19.1572	7.1596	1153.0	105785
368	135424	49836032	19.1833	7.1661	1156.1	106362
369	136161	50243409	19.2094	7.1726	1159.2	106941
370	136900	50653000	19.2354	7.1791	1162.4	107521
371	137641	51064811	19.2614	7.1855	1165.5	108103
372	138384	51478848	19.2873	7.1920	1168.7	108687
373	139129	51895117	19.3132	7.1984	1171.8	109272
374	139876	52313624	19.3391	7.2048	1175.0	109858
375	140625	52734375	19.3649	7.2112	1178.1	110447
376	141376	53157376	19.3907	7.2177	1181.2	111036
377	142129	53582633	19.4165	7.2240	1184.4	111628
378	142884	54010152	19.4422	7.2304	1187.5	112221
379	143641	54439939	19.4679	7.2368	1190.7	112815
380	144400	54872000	19.4936	7.2432	1193.8	113411
381	145161	55306341	19.5192	7.2495	1196.9	114009
382	145924	55742968	19.5448	7.2558	1200.1	114608
383	146689	56181887	19.5704	7.2622	1203.2	115209
384	147456	56623104	19.5959	7.2685	1206.4	115812
385	148225	57066625	19.6214	7.2748	1209.5	116416
386	148996	57512456	19.6469	7.2811	1212.7	117021
387	149769	57960603	19.6723	7.2874	1215.8	117628
388	150544	58411072	19.6977	7.2936	1218.9	118237
389	151321	58863869	19.7231	7.2999	1222.1	118847
390	152100	59319000	19.7484	7.3061	1225.2	119459
391	152881	59776471	19.7737	7.3124	1228.4	120072
392	153664	60236288	19.7990	7.3186	1231.5	120687
393	154449	60698457	19.8242	7.3248	1234.6	121304
394	155236	61162984	19.8494	7.3310	1237.8	121922
395	156025	61629875	19.8746	7.3372	1240.9	122542
396	156816	62099136	19.8997	7.3434	1244.1	123163
397	157609	62570773	19.9249	7.3496	1247.2	123786
398	158404	63044792	19.9499	7.3558	1250.4	124410
399	159201	63521199	19.9750	7.3619	1253.5	125036
400	160000	64000000	20.0000	7.3684	1256.6	125664

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES, AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle	
					Circum.	Area
401	160801	64481201	20.0250	7.3742	1259.8	126293
402	161604	64964808	20.0499	7.3803	1262.9	126923
403	162409	65450827	20.0749	7.3864	1266.1	127556
404	163216	65939264	20.0998	7.3925	1269.2	128190
405	164025	66430125	20.1246	7.3986	1272.3	128825
406	164836	66923416	20.1494	7.4047	1275.5	129462
407	165649	67419143	20.1742	7.4108	1278.6	130100
408	166464	67917312	20.1990	7.4169	1281.8	130741
409	167281	68417929	20.2237	7.4229	1284.9	131382
410	168100	68921000	20.2485	7.4290	1288.1	132025
411	168921	69426531	20.2731	7.4350	1291.2	132670
412	169744	69934528	20.2978	7.4410	1294.3	133317
413	170569	70444997	20.3224	7.4470	1297.5	133965
414	171396	70957944	20.3470	7.4530	1300.6	134614
415	172225	71473375	20.3715	7.4590	1303.8	135265
416	173056	71991296	20.3961	7.4650	1306.9	135918
417	173889	72511713	20.4206	7.4710	1310.0	136572
418	174724	73034632	20.4450	7.4770	1313.2	137228
419	175561	73560059	20.4695	7.4829	1316.3	137885
420	176400	74088000	20.4939	7.4889	1319.5	138544
421	177241	74618461	20.5183	7.4948	1322.6	139205
422	178084	75151448	20.5426	7.5007	1325.8	139867
423	178929	75686967	20.5670	7.5067	1328.9	140531
424	179776	76225024	20.5913	7.5126	1332.0	141196
425	180625	76765625	20.6155	7.5185	1335.2	141863
426	181476	77308776	20.6398	7.5244	1338.3	142531
427	182329	77854483	20.6640	7.5302	1341.5	143201
428	183184	78402752	20.6882	7.5361	1344.6	143872
429	184041	78953589	20.7123	7.5420	1347.7	144545
430	184900	79507000	20.7364	7.5478	1350.9	145220
431	185761	80062991	20.7605	7.5537	1354.0	145896
432	186624	80621568	20.7846	7.5595	1357.2	146574
433	187489	81182737	20.8087	7.5654	1360.3	147254
434	188356	81746504	20.8327	7.5712	1363.5	147934
435	189225	82312875	20.8567	7.5770	1366.6	148617
436	190096	82881856	20.8806	7.5828	1369.7	149301
437	190969	83453453	20.9045	7.5886	1372.9	149987
438	191844	84027672	20.9284	7.5944	1376.0	150674
439	192721	84604519	20.9523	7.6001	1379.2	151363
440	193600	85184000	20.9762	7.6059	1382.3	152053

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SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, CIRCUMFERENCES AND CIRCULAR AREAS OF NOS. FROM 1 TO 520

No.	Square	Cube	Sq. Root	Cube Root	Circle Circum.	Circle Area
441	194481	85766121	21.0000	7.6117	1385.4	152745
442	195364	86350888	21.0238	7.6174	1388.6	153439
443	196249	86938307	21.0476	7.6232	1391.7	154134
444	197136	87528384	21.0713	7.6289	1394.9	154830
445	198025	88121125	21.0950	7.6346	1398.0	155528
446	198916	88716536	21.1187	7.6403	1401.2	156228
447	199809	89314623	21.1424	7.6460	1404.3	156930
448	200704	89915392	21.1660	7.6517	1407.4	157633
449	201601	90518849	21.1896	7.6574	1410.6	158337
450	202500	91125000	21.2132	7.6631	1413.7	159043
451	203401	91733851	21.2368	7.6688	1416.9	159751
452	204304	92345408	21.2603	7.6744	1420.0	160460
453	205209	92959677	21.2838	7.6801	1423.1	161171
454	206116	93576664	21.3073	7.6857	1426.3	161883
455	207025	94196375	21.3307	7.6914	1429.4	162597
456	207936	94818816	21.3542	7.6970	1432.6	163313
457	208849	95443993	21.3776	7.7026	1435.7	164030
458	209764	96071912	21.4009	7.7082	1438.9	164748
459	210681	96702579	21.4243	7.7138	1442.0	165468
460	211600	97336000	21.4476	7.7194	1445.1	166190
461	212521	97972181	21.4709	7.7250	1448.3	166914
462	213444	98611128	21.4942	7.7306	1451.4	167639
463	214369	99252847	21.5174	7.7362	1454.6	168365
464	215296	99897344	21.5407	7.7418	1457.7	169093
465	216225	100544625	21.5639	7.7473	1460.8	169823
466	217156	101194696	21.5870	7.7529	1464.0	170554
467	218089	101847563	21.6102	7.7584	1467.1	171287
468	219024	102503232	21.6333	7.7639	1470.3	172021
469	219961	103161709	21.6564	7.7695	1473.4	172757
470	220900	103823000	21.6795	7.7750	1476.5	173494
471	221841	104487111	21.7025	7.7805	1479.7	174234
472	222784	105154048	21.7256	7.7860	1482.8	174974
473	223729	105823817	21.7486	7.7915	1486.0	175716
474	224676	106496424	21.7715	7.7970	1489.1	176460
475	225625	107171875	21.7945	7.8025	1492.3	177205
476	226576	107850176	21.8174	7.8079	1495.4	177952
477	227529	108531333	21.8403	7.8134	1498.5	178701
478	228484	109215352	21.8632	7.8188	1501.7	179451
479	229441	109902239	21.8861	7.8243	1504.8	180203
480	230400	110592000	21.9089	7.8297	1508.0	180956

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Squares, Cubes, Square Roots, Cube Roots, Circumferences and Circular Areas of Nos. from 1 to 520

No.	Square	Cube	Sq. Root	Cube Root	Circle Circum.	Circle Area
481	231361	111284641	21.9317	7.8352	1511.1	181711
482	232324	111980168	21.9545	7.8406	1514.3	182467
483	233289	112678587	21.9773	7.8460	1517.4	183225
484	234256	113379904	22.0000	7.8514	1520.5	183984
485	235225	114084125	22.0227	7.8568	1523.7	184745
486	236196	114791256	22.0454	7.8622	1526.8	185508
487	237169	115501303	22.0681	7.8676	1530.0	186272
488	238144	116214272	22.0907	7.8730	1533.1	187038
489	239121	116930169	22.1133	7.8784	1536.2	187805
490	240100	117649000	22.1359	7.8837	1539.4	188574
491	241081	118370771	22.1585	7.8891	1542.5	189345
492	242064	119095488	22.1811	7.8944	1545.7	190117
493	243049	119823157	22.2036	7.8998	1548.8	190890
494	244036	120553784	22.2261	7.9051	1551.9	191665
495	245025	121287375	22.2486	7.9105	1555.1	192442
496	246016	122023936	22.2711	7.9158	1558.2	193221
497	247009	122763473	22.2935	7.9211	1561.4	194000
498	248004	123505992	22.3159	7.9264	1564.5	194782
499	249001	124251499	22.3383	7.9317	1567.7	195565
500	250000	125000000	22.3607	7.9370	1570.8	196350
501	251001	125751501	22.3830	7.9423	1573.9	197136
502	252004	126506008	22.4054	7.9476	1577.1	197923
503	253009	127263527	22.4277	7.9528	1580.2	198713
504	254016	128024064	22.4499	7.9581	1583.4	199504
505	255025	128787625	22.4722	7.9634	1586.5	200296
506	256036	129554216	22.4944	7.9686	1589.7	201090
507	257049	130323843	22.5167	7.9739	1592.8	201886
508	258064	131096512	22.5389	7.9791	1595.9	202683
509	259081	131872229	22.5610	7.9843	1599.1	203482
510	260100	132651000	22.5832	7.9896	1602.2	204282
511	261121	133432831	22.6053	7.9948	1605.4	205084
512	262144	134217728	22.6274	8.0000	1608.5	205887
513	263169	135005697	22.6495	8.0052	1611.6	206692
514	264196	135796744	22.6716	8.0104	1614.8	207499
515	265225	136590875	22.6936	8.0156	1617.9	208307
516	266256	137388096	22.7156	8.0208	1621.1	209117
517	267289	138188413	22.7376	8.0260	1624.2	209928
518	268324	138991832	22.7596	8.0311	1627.3	210741
519	269361	139798359	22.7816	8.0363	1630.5	211556
520	270400	140608000	22.8035	8.0415	1633.6	212372

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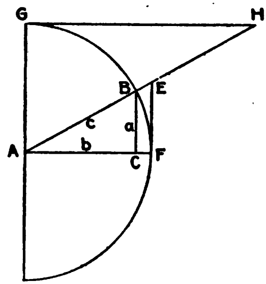
TABLE 5. TRIGONOMETRIC FUNCTIONS AND THE SOLUTION OF TRIANGLES



In the accompanying figure the trigonometric functions of the angle A between the lines $B\ A$ and $A\ C$ are as follows;

$$\sin A = B\,C$$
$$\cos A = A\,C$$
$$\tan A = E\,F$$
$$\cot A = G\,H$$
$$\sec A = A\,E$$
$$\text{cosec}\ A = A\,H$$
$$\text{ex-sec}\ A = B\,E$$

In the right-angled triangle $A\ B\ C$ let a equal the side $B\ C$ opposite the angle A; let b equal the side $A\ C$ opposite the angle B; let c equal $A\ B$, the side opposite the angle C.

Let $C = 90°$
The following formulæ apply to right-angled triangles:

Angles. $A + B + C = 180°$
$A + B = 90°$
$A = 90° - B$
$B = 90° - A$

Sides. $a = c \sin A = b \tan A$
$a = \sqrt{(c+b)\ (c-b)}$
$b = c \cos. A = \dfrac{a}{\tan A}$

$\sin A = \dfrac{a}{c}$

$\cos A = \dfrac{b}{c}$

$\tan A = \dfrac{a}{b}$

Area

$\text{area} = \dfrac{a\,b}{2}$

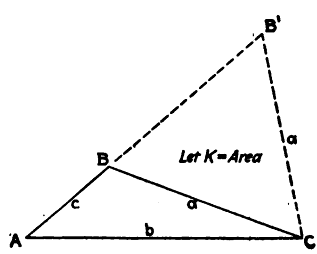
$b = \sqrt{(c+a)\ (c-a)}$

$c = \dfrac{a}{\sin A} = \dfrac{b}{\cos A}$

$c = \sqrt{a^2 + b^2}$

Oblique Triangles.

Note. Where an angle is more than 90° its sine, cosine, and tangent are equal to that of the angle (180° − the angle in question); that is, if the sine of 120° is desired take the sine of (180° − 120°) = 60°.



Given	Desired	Formulæ
A, B, a	C, b	$C = 180 - (A + B)$; $b = \frac{a}{\sin A} \sin B$
	c, K	$c = \frac{a}{\sin A} \sin (A + B)$; $K = \frac{a^2 \sin B \sin C}{2 \sin A}$
A, a, b	B, C	$\sin B = \frac{\sin A}{a} b$; $C = 180° - (A + B)$
	c	$c = \frac{a}{\sin A} \sin C$
		Two solutions are possible with B′ as an acute angle and B as an obtuse angle
C, a, b	$\frac{1}{2}(A+B)$	$\frac{1}{2}(A+B) = 90° - \frac{1}{2} C$
	$\frac{1}{2}(A-B)$	$\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B)$
	A B	$A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B)$ $B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B)$
	c	$c = (a - b) \frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)}$
	K	$K = \frac{1}{2} ab \sin C$
a, b, c	B	In the following formula $s = \frac{1}{2}(a + b + c)$
		$\sin \frac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{ac}}$
		$\sin B = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{ac}$
	K	$K = \sqrt{s(s-a)(s-b)(s-c)}$

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EXPLANATION OF TABLES.

TABLE I. LOGARITHMS OF NUMBERS.—The logarithm of any number to any base is the index of the power to which the base must be raised to equal the number. The logarithms given in Table I are Briggs or Common Logarithms in which the base is 10. Then $100 = 10^2$, and the logarithm of $100 = 2$. Also $200 = 10^{2.30103}$, and the logarithm of $200 = 2.30103$. The integer of a logarithm is called the *characteristic*, and is one less than the number of integers in the number. The decimal part of the logarithm is called the *mantissa* and is given in Table I.

The mantissae of the logarithms in Table I are given to five places; while the numbers are given to four significant figures. Where there are more than four significant figures in the number, the table of proportional parts may be used. The star opposite certain logarithms shows that the two figures at the left are to be taken from the line below.

The logarithm of 1 is 0, and the logarithm of any number less than unity will be negative. It is much more convenient to use positive mantissae, and logarithms of numbers less than unity are written as cologarithms or modified logarithms in which the negative logarithm is subtracted from a positive integer as 10, 20, etc., 100, 200, etc.; and the cologarithm or modified logarithm is written as a positive logarithm with the integer shown as subtracted from the logarithm. For example the logarithm of $0.2 =$ logarithm of $\frac{1}{5} = \log.\ 1 - \log.\ 5 = 0.00000 - 0.69893 = -0.69893$. The cologarithm or modified logarithm will be equal to the logarithm subtracted from 10 and is written $9.30103 - 10$. The logarithm of $.00625 = \log.\ \frac{5}{800} = \log.\ 5 - \log.\ 800 = 0.69897 - 2.90309 = -2.20412$, or as a cologarithm or modified logarithm $= 7.79588 - 10$. The mantissae of the cologarithms of numbers less than unity are given in Table I.

The following rules should be kept in mind in using the table of logarithms.

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1. The logarithm of a product is the sum of the logarithms of the factors.

2. The logarithm of a quotient is the difference of the logarithms of the dividend and divisor.

3. The logarithm of a power of a number is equal to the logarithm of the number multiplied by the index of the power.

4. The logarithm of a root of a number is equal to the logarithm of the number divided by the index of the root.

5. The logarithm of a fraction is equal to the logarithm of the numerator minus the logarithm of the denominator.

6. In dividing modified logarithms add a number to the positive and negative characteristics so that the resulting logarithm will have —10 following the logarithm. For example if 8.36748 — 10 is to be divided by 3, the logarithm should be written 28.36748 — 30; and dividing by 3 we have 9.45583 — 10.

Reverse the operation when multiplying modified logarithms.

7. The characteristic of the logarithm of an integer is always one less than the number of digits in the integral part of the number.

8. The characteristic of the cologarithm of a number less than unity (a decimal) is equal to 10 minus the number of the place to the right of the decimal point occupied by the first significant figure.

TABLE II. LOGARITHMIC FUNCTIONS OF ANGLES. —To avoid the use of negative characteristics the logarithms of the functions of angles are written as cologarithms, 10 being added to the characteristic of each logarithm. In adding the logarithms of the functions of angles the correct number of tens should be subtracted from the result.

For angles from 0° to 45° and from 135° to 180° the headings at the tops of the columns are to be used; while from 45° to 90° and from 90° to 135° the headings at the bottoms of the columns are to be used; the minutes being read from the top down on the left of the page, and from the bottom up on the right of the page.

In using the logarithmic functions of angles in connection with logarithms of numbers it should be remembered that the logarithmic functions of angles are cologarithms and that 10 should be subtracted from each logarithmic function.

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TABLE III. NATURAL FUNCTIONS OF ANGLES.— For angles from 0° to 45° and from 135° to 180° the headings at the tops of the columns are to be used; while from 45° to 90° and from 90° to 135° the headings at the bottoms of the columns are to be used; the minutes being read from the top down on the left of the page and from the bottom up on the right of the page.

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